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PRACTICAL ARITHMETIC

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PRACTICAL ARITHMETIC

COMPLETE FOR ALL ORDINARY PURPOSES.

A sound foundation; a sure superstructure.

BY

EDWIN C. HEWETT, LL. D.,

EX-PRESIDENT OF THE ILLINOIS STATE NORMAL UNIVERSITY.

CHICAGO AND NEW YORK:

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PREFACE.

In making this book, the aim has been to justify its title, *Practical* Arithmetic. In pursuing this end, fundamental principles and processes have received large attention, and they have been amply illustrated by examples and explanations. Few rules have been given, but the pupil's power to appreciate what is required, and to find his own rules or processes, has been cultivated at every step. It would seem that the plan of structure in many, perhaps most, of our text-books on Arithmetic is to give the pupils a rule or formula for solving problems after they have been classified and arranged under appropriate "cases." It appears to be taken for granted that the pupil will have the ability to arrange and classify the problems he will meet in actual life, and to bring them under the proper remembered rule or formula.

It need not be said that this book has not been made on that plan. The principles of Arithmetic are very few, and the fundamental processes are not numerous. It is of prime importance that the pupil should become thoroughly familiar with those principles and versed in those processes. He is then prepared to make the application directly to the practical problems in life as they arise, without waiting to find the class or "case" of the problem, and to evoke the appropriate rule from his memory. For this reason a large part of the problems in this book are given promiscuously, as they will arise in practical life.

What has been said will explain why much the larger part of the book is given to subjects which are often rather hurriedly and superficially treated in the earlier pages of some text-books. Furthermore, it will explain why comparatively little space is given to topics that often occupy a large place in the later pages of many books. No very large space is given to Percentage; no division into "cases" is made; no formulas for solution are given. Little is said about Partial Payments, Taxes, Duties, Partnership, or

Exchange. No room is given to Arbitration of Exchange, Banking, Equation of Accounts, etc. In none of these subjects are any new arithmetical principles involved, nor any new fundamental processes introduced. The pupil who is well grounded in the essentials of Arithmetic will readily learn their special applications in any of these subjects should he have occasion to do so. To very many such occasion will never arise. Besides, is it not true that thorough business men generally have little respect for what is usually taught in the schools, concerning the applications of Arithmetic to the more intricate operations of extensive business, or to its specific lines?

Business men require their clerks to be accurate and rapid in all the fundamental processes and operations of Arithmetic; and they desire them to be so well grounded in arithmetical principles that they can apply them understandingly to practical problems that may arise in any line of business. That is, they want them to be able to do, and to know why they do. But we think it is generally true that the application itself can be mastered more satisfactorily in the office than in the school. Besides, in very many cases — as in the extending of taxes, or the finding of interest in banks, or in making bills for lumber, for excavations, for mechanics' work, etc. — the results are not obtained by computation; they are taken from prepared tables. In view of all these reasons it does not seem wise to spend a great amount of time or effort on these applications in ordinary schools.

It is supposed that most of the pupils who use this book will be prepared for it by the use of the Primary of the same series. This will be much to their advantage. But it is believed that the book is so constructed that the pupil can use it profitably and intelligently, no matter where, or in what way, he may have received his earlier training.

Competent and careful observers often assert that too much time, proportionally, is given to Arithmetic in schools, and that too little accuracy and proficiency is gained. There is truth in both statements. And it is hoped that the use of this book will correct these evils.

E. C. H.

August 22, 1896.

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PRACTICAL ARITHMETIC.

NOTATION AND NUMERATION.

PRELIMINARY DEFINITIONS.

1. A **number** is one or a collection of ones of the same kind.

2. The **unit** of any given number is one of the collection which constitutes the number.

The unit of 16 oranges is 1 orange; of 16 dozen oranges is 1 dozen oranges; of 16 centuries is 1 century; of 16 thousand bricks is 1 thousand bricks; of 16 tens is 1 ten; of 16 thirds is 1 third. The unit of 16 is 1.

3. **"How many?" "What?"**—A number answers the question, "How many?" This is sometimes given as the definition of number. But our conception of a number is incomplete until another question is answered, namely, "What?" or "What kind?" "Five" has only half its meaning till we know of what kind the units are — boys, dozens, hundreds, tenths, or what not. Five sevenths is not a number that can be used in solving a problem till we know what it is five sevenths of — "John's money," "the sheep in the pasture," etc.

So it is of the greatest importance that we get into the habit of answering the question, "What?" or "Of what kind?" What is seventeen? It may be seventeen men, tenths, hundreds; or it may be one ten and seven ones of any one of these kinds, if we choose so to regard it. One hundred twenty-three may be thought of as so many ones, or as twelve tens and three ones; that is, it may be thought of as one number or two numbers. What is four and two sixths? This can not be thought of as *one* number; it is four units of one kind joined with two units of another kind.

4. Primary and Relative Units. — Every integral number, if expressed by more than one figure, and every fractional number of any kind, will include the thought of more than one kind of unit. In every such case the units are of *two classes.* For convenience we call them *primary* and *relative* units. The primary unit always has two distinguishing marks: first, when the number is applied to objects — is made concrete — the primary unit takes the name; second, when the number is expressed by figures the primary unit is always expressed in the first place at the left of the decimal point.

Any relative unit is some exact part of its primary unit, or it is some number of times that primary unit.

ILLUSTRATIONS: In the number one hundred twenty-three, there are three primary units, two relative units of one kind, and one of another kind. If the number were applied (made concrete), the name — apple, for instance — would be applied to the primary units only.

In a fractional number the primary unit may or may not be written. In such examples as four and two sixths, or five and three tenths, the primary unit is written if the expression be made in figures. But there would be no figures to express primary units if two sixths or three tenths were written alone in figures.

5. Concrete and Abstract Numbers.— When the objects to which numbers belong are named, the number is said to be *concrete;* as 2 apples, 27 cents, 368 miles.

When no objects are named, the number is *abstract;* as 2, 27, 368.

6. Simple and Compound Numbers.— When the relation between the primary and the relative units of a number is invariably expressed by *ten,* or a power of ten, the number is *Simple;* if any other number expresses that relation the given number is *Compound.* Hence, all the numbers in Federal Money, and in the Metric system, are simple numbers. Hence, also, a so-called mixed number, like four and one seventh, is just as truly a compound number as numbers expressing shillings and pence, or bushels and pecks.

7. What is Arithmetic?— Arithmetic teaches the theory of numbers and the art of computing by them.

8. Notation is the writing of numbers by means of characters.

There are two systems of notation in general use, the *Roman* and the *Arabic.*

9. Numeration is the reading of numbers expressed by characters.

ROMAN NOTATION.

10. The Romans employed seven letters to express numbers. They are:

Letters,	I,	V,	X,	L,	C,	D,	M.
Values,	One.	Five.	Ten.	Fifty.	One Hundred.	Five Hundred.	One Thousand.

11. Expressing Other Numbers.— Other numbers are expressed by putting two or more of these letters together, according to the following principles:

I. When a letter is written *after* another letter of the same or greater value, they express together the sum of their values. Thus, II=two; XV=fifteen; XXX=thirty; CX=one hundred ten.

II. When a letter is written *before* another letter of greater value, the two together express the difference of their values. Thus, IV=four; IX=nine; XL=forty; XC=ninety.

III. A bar placed over a letter multiplies its value by one thousand. Thus, $\overline{V}$=five thousand; $\overline{L}$=fifty thousand; $\overline{M}$=one million.

EXERCISES.

12. Read the following expressions:

III, VII, XIII, XIV, XVI, XIX, XX, XXV, XXVIII, XXXII, XXXIX, XL, XLIV, XLVI, L, LV, LXIII, LXVII, LXXI, LXXVIII, LXXX, LXXXIV, LXXXIX, XC, XCV, XCVII, C, CCC, D, DC, DCCCC, M, MM, MX, MC, MD, $\overline{X}$, $\overline{C}$, MDCCCXCVI.

13. Express the following numbers by the Roman notation: Twelve, fifteen, twenty-four, thirty-seven, forty-five, fifty-nine, sixty-three, seventy-six, eighty-eight, ninety-one, one hundred, one hundred nine, one hundred twenty-five, one hundred forty, two hundred seventy-four, four hundred fifty, five hundred ninety-nine, one thousand one hundred ten.

14. Roman numerals are generally used to number lessons, chapters, and volumes. They are used on the dials of time-pieces and in the titles of kings.

ARABIC NOTATION.

15. The Arabic System of Notation expresses numbers by means of characters called figures. These figures are 1 (one), 2 (two), 3 (three), 4 (four), 5 (five), 6 (six), 7 (seven), 8 (eight), 9 (nine). Sometimes these figures are called digits.

16. Grouping Numbers. — Nine is the largest number that can be expressed by a single figure. To express larger numbers, two or more figures must be combined. But in order to combine these figures in a systematic way, numbers must be thought of as in groups. The system of grouping employed in the Arabic notation is by tens. Ten ones are grouped into a larger collection and called one ten. Ten is *one* of a new kind, and the figure 1 is used to express it. This is done by putting the 1 in a new place; that is, the 1 is removed one place to the left.

17. The Decimal Point.— But, in order that we may *know* that the 1 is in the second place to the left, there must be something to count from. For this purpose a dot is used, and it is called a decimal point.

18. The Zero.— Further, in order to prevent any mistake as to the place of the 1 with reference to the decimal point, a character is needed to put in the *first* place whenever just one ten or some exact number of tens are written. The character 0 is used, and it is called zero. Hence, 10 expresses ten written in figures.

19. Writing Numbers.— Any number of tens from one to nine can be expressed by writing the figure whose shape denotes the number of tens to be expressed, in the second place to the left of the decimal point.

EXERCISES.

20. Write in figures: Two tens, three tens, four tens, five tens, six tens, seven tens, eight tens, nine tens.

Two tens (20) is read twenty, three tens (30) thirty, four tens (40) forty, etc.

Write in figures: Two tens and three ones, three tens and four ones, four tens and four ones, five tens and three ones, six tens and seven ones, seven tens and six ones, eight tens and five ones, nine tens and eight ones.

21. Writing Larger Numbers.— Nine tens and nine ones (99) is the largest number that can be expressed by two figures. The next higher number is ten tens. These are grouped into a larger group, and called one hundred. This is one of still another kind, and the figure 1 is used to express it. This is done by putting the 1 yet another place to the left, or in the third place, thus — 100. Any number of hundreds from one to nine can be expressed by writing the figure whose shape denotes the number of hundreds to be expressed, in the third place to the left of the decimal point. Use zeros if no lower denominations are given.

EXERCISES.

22. Write in figures: Two hundreds, three hundreds, four hundreds, five hundreds, six hundreds, seven hundreds, eight hundreds, nine hundreds.

Write in figures: Two hundreds three tens, three hundreds two tens four ones, four hundreds one ten three ones, five hundreds five tens, six hundreds four tens seven ones, seven hundreds two ones, eight hundreds six ones, nine hundreds nine tens nine ones.

23. Writing Numbers Greater than One Thousand.—Nine hundreds nine tens and nine ones (999) is the largest number that can be expressed by three figures. The next higher number is ten hundreds. These are put into one group and called one thousand. This is expressed in figures by putting the 1 in the fourth place to the left of the decimal point. In the same way thousands are grouped into tens of thousands, and tens of thousands into hundreds of thousands, and so on. Thus, the Arabic system of notation is a decimal system (*decem* means ten). Ten units of any order always make one unit of the next higher order.

24. Principles.—I. *Moving a figure one place toward the left makes its value ten times what it was before.*

II. *A zero is used to fill a place that would otherwise be vacant between some figure and the decimal point.*

The zero has no other use in arithmetic. Annexing zeros does not "multiply" or in any way change the value of figures. The value is changed by the *change of place* with respect to the decimal point; the annexing of zeros may show that the place has been changed.

III. *Figures have two kinds of value, viz., a shape-value and a place-value.*

The shape-value shows the number of ones in the number expressed. It answers the question, "How many?"

The place-value determines the *kind* of ones, as units, tens, tenths, etc. It answers the question, "What kind?"

The place-value is determined by the position of the figure with reference to the decimal point.

Instead of shape-value and place-value, it is common to say simple and local value.

IV. *The same figures in the same order always express the same number of units; and the name of the units expressed is determined by the position of the right-hand figure of the group with respect to the decimal point.*

ILLUSTRATION: In this expression, 1|34|67., the figures between the vertical lines mean thirty-four units; the value of these units is "hundreds," because 4, the right-hand figure, stands in the third place to the left of the decimal point, or in hundreds' place. If, now, the decimal point be changed, as in the following instances, 1|34|6.7, 1|34|670., the figures 34 express thirty-four units every time, but the name of the units changes with every change of the place of the 4 in relation to the decimal point. Read the 34 in these several expressions.

V. *In every expression by figures, a change in relation to the decimal point does not change the number of units expressed, but does change the name, or kind.*

In such a case, the answer to the question, "How many?" is constant; but the answer to the question, "What kind?" changes with each change in relation to the decimal point.

It must be remembered that the position of a figure with respect to the decimal point always determines the name of the number it represents.

25. Orders and Periods. — The place occupied by one figure is called an order. The first place is called units' order, the second is called tens' order, the third hundreds' order, etc.

The orders are grouped by threes. Beginning with the lowest or right-hand order, each group of three is called a *period;* and it takes its name from its lowest or right-hand order. Periods are separated by commas.

26. The following table shows the grouping and naming of orders and periods:

Numeration Table.

NAMES OF PERIODS,	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
NAMES OF ORDERS,	Hundred Quadrillions. Ten Quadrillions. Quadrillions.	Hundred Trillions. Ten Trillions. Trillions	Hundred Billions. Ten Billions. Billions.	Hundred Millions. Ten Millions. Millions.	Hundred Thousands. Ten Thousands. Thousands.	Hundreds. Tens. Ones.
	12,	500,	731,	049,	020,	860
PERIODS, - - - -	Sixth.	Fifth.	Fourth.	Third.	Second.	First.

The periods above quadrillions, in their order, are: Quintillions, sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, tredecillions, quatuordecillions, quindecillions, sexdecillions, septendecillions, octodecillions, novendecillions, vigintillions.

Observe the list of the names of periods as just given. Write the list down, beginning with trillions, and cut off the termination "illions" from each word. What remains of each name signifies a number; these numbers go on in regular order from three to twenty. Thus, tr=3, quadr=4, quint=5, sext=6, etc. (The first three names are omitted, as their significance is not so clear.)

Now "trillions" is the name of the fifth period, "quadrillions" of the sixth, "quintillions" of the seventh, etc. In other words—

The number of the period is always greater by two than the number signified by the first part of its name.

27. To Write a Number in Figures.—*Begin at the left hand and fill each period in order, using zeros where there is no number to express.*

28. To Read a Number.—*Begin at the right hand and point off the number into periods of three orders each. Beginning at the left, read the denominations in each period by themselves, pronouncing after them the name of the period.*

EXERCISES.

29. How many and what kind of units does each figure in the following numbers stand for? Begin at the right-hand side of each number:

1.	236	401	300	83	460	4001
2.	92	345	270	606	500	3010
3.	400	78	683	210	333	2100
4.	540	700	904	567	98	5040
5.	602	583	68	400	705	7208
6.	555	830	417	57	312	1011

Read the numbers written above.

Write in a column on your slates the first four numbers in each column above, in words.

30. Write the following numbers in figures:

1. Fifty-nine, one hundred, two hundred, five hundred ten, three hundred one.

2. Four hundred, two hundred eighteen, ninety-seven, five hundred twenty.

3. Six hundred eight, two hundred forty-six, seventy-two, three hundred fifty.

4. Eighty-six, four hundred eleven, two hundred twenty-two, five hundred sixty.

5. Five hundred, seven hundred seven, two hundred seventy-three, eighty.

6. Seven hundred fifty-nine, nine hundred four, sixty-seven, five hundred seventeen.

7. Eight hundred eighty, six hundred six, four hundred thirty-four, ninety-nine.

8. Three hundred fourteen, six hundred twelve, eight hundred eighty-eight, seventy-six.

9. One thousand, three thousand one, five thousand seventy, eight thousand sixty-five.

10. Four thousand four hundred, two thousand ten, nine thousand sixteen, six thousand six hundred six, six thousand two hundred fifty-eight, three thousand five hundred twelve.

EXERCISES.

31. Copy, point off, and read:

1.	10000	100000	1000000
2.	81001	405060	4005006
3.	60700	300400	7020040
4.	5040	610305	15700307
5.	14014	427200	28046094
6.	37121	370037	45004684
7.	41786	700485	62400093
8.	31814	513000	43720581
9.	60432	1896007	126048100

10.	91337	4681200	290840317
11.	49131	8304614	347000000
12.	94326	7560057	500500500
13.	13258	3174108	1307800735
14.	86532	9095470	5830016702
15.	75812	6713270	10374050613
16.	60006	5111419	38062709340
17.	140008637475		2510836079409
18.	3716047509200		81605094207354
19.	9000405000708		327000000400019
20.	7216000091000		4000917327000801

32. Write in figures:

1. Two thousand twenty-five, forty-one thousand three hundred, eighteen thousand fourteen.

2. Seven thousand four hundred seventy-four, thirty-six thousand fifty, fifty-three thousand five hundred eight.

3. Fifteen thousand four hundred eighty-seven, ten thousand seventy, sixty-five thousand three hundred twelve.

4. Forty-eight thousand eight hundred, seventy-one thousand seven hundred one, twelve thousand fifty-six.

5. Eighty-seven thousand eighty-seven, ninety-two thousand nine hundred eleven, three hundred thousand forty-five.

6. Ninety-nine thousand four hundred twenty-six, five hundred thirteen thousand, six hundred thousand four hundred.

7. Three hundred two thousand seventy-six, two hundred thousand sixteen, one hundred eighteen thousand six hundred.

8. Four hundred forty-four thousand four hundred forty-four, six hundred thousand six, nine hundred thousand.

9. Seven hundred thousand, two hundred thousand two hundred two, five hundred fifteen thousand five hundred.

10. Six hundred seventy thousand seventy, three hundred thirty thousand seven hundred, seven million two hundred.

11. Three million eighty-one thousand, nine million five hundred thousand, twelve million fifty-three.

12. Ten million ten thousand ten, twenty-seven million four hundred thousand eleven, eighty-nine million eight hundred six thousand forty-seven.

13. Three hundred million three hundred, five hundred seven million three hundred thousand nine.

14. Nine hundred ninety-seven million three thousand fourteen, one billion three hundred million, five billion one hundred fifty-four, thirteen billion four hundred six thousand.

15. Three hundred eighteen billion one million, six hundred three billion, eight hundred thirty-one billion eighty thousand.

16. Four trillion, thirty-seven trillion sixty-four thousand, four hundred five trillion two million, nine quadrillion nine billion nine.

NOTE: Begin at the left hand every time.

ADDITION.

33. *1.* Henry caught 4 trout and Albert caught 3. How many trout did both boys catch?

2. In my garden are 5 cherry trees and 3 pear trees. How many trees are there in my garden?

3. A farmer sold a sheep for 6 dollars and a pig for 4 dollars. How many dollars did he receive for both?

4. Ella practiced her music for 6 hours yesterday and 5 hours to-day. How many hours has she practiced in 2 days?

5. In a certain class are 6 boys and 7 girls. How many children are there in the class?

6. A father gave to one of his children 5 cents, to another 3 cents, and to another 6 cents. How many cents did he give to all three of them?

7. How many are 3 oranges, 5 oranges, 4 oranges, and 2 oranges?

What do you call the work you have been doing?

34. Addition is the finding of a number which shall contain as many ones as two or more numbers taken together.

The number found is called the *sum* of the other numbers.

The sign + means *and;* the sign = means *are,* or *equal.*

Read: 7 plums + 4 plums = 11 plums; 8 cents + 9 cents = 17 cents.

Can you tell how many 3 apples and 4 cows are? Why not?

Only units of the *same kind* can be added.

35. Combinations of Two Numbers.— There may seem to be a very great number of possible combinations of two numbers in addition, but they are all given in the following tables. How many are there?

Tables.

SUMS NOT EXCEEDING TEN.

1+1=2	2+2=4	3+3=6	4+4=8
1+2=3	2+3=5	3+4=7	4+5=9
1+3=4	2+4=6	3+5=8	4+6=10
1+4=5	2+5=7	3+6=9	
1+5=6	2+6=8	3+7=10	
1+6=7	2+7=9		5+5=10
1+7=8	2+8=10		
1+8=9			
1+9=10			

SUMS FROM ELEVEN TO EIGHTEEN.

6+5=11	7+4=11	8+3=11	9+2=11
6+6=12	7+5=12	8+4=12	9+3=12
6+7=13	7+7=14	8+5=13	9+4=13
6+8=14	7+8=15	8+8=16	9+5=14
6+9=15	7+9=16	8+9=17	9+9=18

These forty-five combinations of the tables include all that are necessary to any addition whatever. The addition of large numbers is no more difficult than the addition of these small ones. In adding fifty-six and eight, for instance, we have simply to add the six and eight. According to the tables, their sum is one ten and four, or fourteen; hence, the result will be six tens and four, or sixty-four. The sum of seventy and twenty is simply the sum of seven tens and two tens. These illustrations show that the tables contain all the combinations we need to make, in the separate steps of any addition. For this reason the tables ought to be thoroughly committed to memory. It must be done before any real progress in addition is possible.

ADDENDS.

36. Order of Addends.—Although there are only forty-five combinations, in all but nine of them the numbers to be added may occur in two orders, thus: 6+5=11; 5+6=11; 7+8=15; 8+7=15, etc.

The orders in which the forty-five combinations may occur are promiscuously given below in two forms. Practice upon them until you can name each sum the instant your eye rests upon the two figures that represent the numbers to be added:

2+2=	9+8=	8+2=	2+5=	2+7=
3+5=	4+2=	3+4=	7+8=	3+9=
7+1=	8+4=	9+1=	6+2=	1+2=
4+3=	1+9=	1+5=	3+7=	7+7=
6+7=	2+6=	2+4=	8+5=	8+6=
5+5=	7+5=	6+3=	7+2=	1+8=
9+2=	6+8=	5+7=	8+1=	2+9=
2+8=	9+9=	7+9=	9+7=	3+6=
4+6=	7+3=	6+4=	5+6=	5+7=

8+9=	4+5=	9+6=	1+6=	1+4=
7+4=	3+2=	1+3=	2+3=	5+2=
3+3=	4+1=	4+7=	3+1=	9+3=
5+9=	1+7=	8+8=	4+8=	6+5=
7+6=	5+8=	5+3=	5+4=	4+9=
5+1=	6+6=	6+1=	6+9=	2+1=
3+8=	9+5=	9+4=	8+3=	4+4=

1	8	3	9	1	2	6	5	7	6	9	1	4	8
1	2	4	1	5	4	3	7	9	4	6	3	7	8
5	6	9	8	6	5	4	3	2	1	5	9	8	7
3	1	4	3	9	4	8	1	3	6	6	7	1	2
8	3	6	7	2	4	2	4	6	9	5	1	8	3
5	7	2	8	5	4	1	9	5	3	2	4	7	6
2	1	8	7	1	3	2	9	6	5	1	4	3	4
9	8	6	7	2	9	7	5	6	8	7	1	2	5
7	9	6	7	2	1	8	4	9	2	3	7	4	6
2	9	8	5	6	9	4	2	8	2	5	1	3	7
5	9	2	4	8	7	3	5	7	3				
5	2	8	6	9	4	3	9	6	8				

SIGHT EXERCISES.

37. Name at sight the sums of the following numbers:

1.	11	14	25	12	28	30	13
	6	3	2	9	5	8	7

2.	35	58	41	19	72	16	44
	4	8	8	6	8	7	9
3.	20	17	84	33	49	59	65
	7	7	6	5	8	4	6
4.	85	53	27	92	36	34	68
	9	8	5	7	6	8	7
5.	56	71	38	45	79	24	47
	8	9	9	7	5	7	9
6.	83	77	56	39	75	63	97
	9	8	5	9	5	6	4
7.	67	40	69	96	91	29	86
	6	9	7	8	7	3	3

EXERCISES.

38. Practice the following additions until the results can be named rapidly and accurately:

1. Add by 2's from 1 to 71, thus: 1, 3, 5, 7, etc.

2. By 3's from 1 to 100; from 0 to 99.

3. By 4's from 1 to 101; from 2 to 102.

4. By 5's from 1 to 101; from 4 to 99.

5. By 6's from 1 to 97; from 2 to 98.

6. By 6's from 3 to 99; from 6 to 96.

7. By 7's from 1 to 99; from 2 to 100.

8. By 7's from 5 to 103; from 7 to 98.

9. By 8's from 1 to 97; from 2 to 98.

10. By 8's from 4 to 100; from 5 to 101.

11. By 8's from 7 to 95; from 8 to 96.

12. By 9's from 1 to 100; from 2 to 101.

13. By 9's from 8 to 98; from 9 to 99.

14. Add 5 to 8, 18, 28, 38, 48, 58; thus: Eight and five are thirteen; eighteen and five are twenty-three, etc.

15. Add 5 to 9, 19, 29, 39, 49, 59, 69, 79.

16. Add 6 to 5, 15, 25, 35, 45, 55, 65, 75.

17. Add 6 to 7, 17, 27, 37, 47, 57, 67, 77.

18. Add 7 to 5, 15, 25, 35, 45, 55, 65, 75.

19. Add 7 to 7, 17, 27, 37, 47, 57, 67, 77.

20. Add 7 to 8, 18, 28, 38, 48, 58, 68, 78.

21. Add 8 to 4, 14, 24, 34, 44, 54, 64, 74.

22. Add 8 to 6, 16, 26, 36, 46, 56, 66, 76.

23. Add 8 to 9, 19, 29, 39, 49, 59, 69, 79.

24. Add 9 to 3, 13, 23, 33, 43, 53, 63, 73.

25. Add 9 to 6, 16, 26, 36, 46, 56, 66, 76.

26. Add 9 to 8, 18, 28, 38, 48, 58, 68, 78.

39. Add each of the following columns of numbers from the bottom upward and from the top downward.

Do not copy them. In adding, name results only.

Thus, in the first column, say: "Twelve, seventeen, twenty-four, thirty-two," etc. If the addition of the next number does not give another ten, speak only the units in the next result; for instance, finish the addition of the first column by saying "Eight" (not thirty-eight), "forty-one." When the sum of the next two or three numbers is exactly ten, train the eye to observe the fact, and add the ten at once; as in the second column, say: "Eleven, twenty, thirty, seven, forty-two."

1.	3	5	4	7	5	9	8	6	3	5
	6	7	7	2	9	8	5	3	5	9
	8	6	3	5	1	2	5	1	2	3
	7	4	5	6	6	5	7	5	7	6
	5	9	8	4	3	6	6	8	6	2
	8	3	9	8	7	3	3	7	4	7
	4	8	6	3	8	5	9	4	8	4
	—	—	—	—	—	—	—	—	—	—
2.	2	7	4	9	6	8	3	5	5	7
	9	8	3	7	8	5	9	4	9	6
	6	2	9	3	5	4	9	3	8	5
	4	6	1	8	1	2	6	8	2	4
	7	5	6	6	3	7	7	6	6	8
	5	9	8	2	7	6	8	7	8	9
	3	4	3	5	9	6	5	9	4	2
	—	—	—	—	—	—	—	—	—	—
3.	6	2	8	4	3	7	9	5	8	4
	4	8	1	7	8	4	1	9	5	9
	8	3	5	8	1	3	6	4	6	3
	5	6	7	2	7	6	8	6	3	5
	7	1	2	5	4	8	5	8	7	5
	9	4	9	6	6	2	3	3	4	7
	6	7	3	9	5	5	7	6	9	8
	—	—	—	—	—	—	—	—	—	—

40. Find the sums of the following by columns, and by lines from left to right and from right to left:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*j.*	*k.*	*l.*
1.	6	8	3	5	1	9	5	7	2	1	4	6
2.	3	9	5	6	4	7	2	8	5	9	1	4
3.	7	2	8	9	3	4	7	1	6	7	3	9
4.	5	4	8	3	7	6	5	9	4	7	6	1
5.	8	5	2	9	6	3	7	4	8	6	4	5
6.	4	5	7	9	9	3	5	2	6	4	7	8
7.	2	9	4	6	5	2	8	1	7	9	2	6
8.	9	2	8	5	7	3	2	6	5	5	8	7
9.	1	7	5	8	2	7	9	6	4	3	8	7
10.	7	6	9	2	6	8	4	7	1	5	6	3
11.	6	9	3	5	4	1	9	5	8	2	5	9
12.	5	1	5	7	9	6	2	9	4	7	3	8

41. Directions for Adding Numbers Expressed by Two or more Figures.

1. Write the figures in vertical columns. Why?

2. Write units of the same kind in the same column. Why?

3. Begin with the lowest or right-hand order. Why?

4. If there are any tens from the addition of the lower order, put them with next order.

42. Copy and find the sums:

1.	*2.*	*3.*	*4.*	*5.*	*6.*	*7.*	*8.*
27	16	43	38	65	87	39	64
36	59	72	86	34	23	87	37
19	31	98	24	56	39	44	93
45	42	73	39	83	66	9	86
23	8	83	61	74	37	81	94
24	25	64	25	55	54	65	83
51	34	71	37	38	86	72	27

9.	*10.*	*11.*	*12.*	*13.*	*14.*
437	843	617	777	538	358
394	384	394	813	324	972
517	736	476	426	476	329
654	857	245	691	973	666
826	249	837	245	618	857
789	632	582	357	295	596

15.	*16.*	*17.*	*18.*	*19.*
7384	8137	7867	7	3468
4265	3278	24	58	324
3854	4651	6835	613	95
6783	3785	439	8376	8
9124	7351	5762	4265	76
5738	4286	8	6835	539
8462	3875	5843	8600	7214
5964	1894	6937	7169	9486

20.	*21.*	*22.*	*23.*
259618	936517	721568	476295
871465	387649	597436	578578
538794	754823	348952	864369
216537	647195	653789	357942
983624	476936	737373	639476
465973	589674	578643	756834

43. Tests for the Accuracy of the Result.

1. Add in the opposite direction. Why is this a test?

2. Add from the left-hand order.

36
27
34
97

ILLUSTRATION: Take the example in the margin. The sum of the tens is eight, but to justify the 9 written in the answer one ten must have come from the units; adding the units we have the needed ten, and the seven units written in the answer.

ORAL EXERCISES.

44. *1.* In Lucy's class are 8 boys and 9 girls. How many children are there in the class?

2. George bought for his mother 6 cents' worth of matches and 12 cents' worth of soap. How many cents did he pay for both?

3. There were 11 mowers in the field, and 8 boys turned the grass. How many persons were working in the field?

4. Mr. Osgood paid his grocer $8 for a barrel of flour, and enough for a half-barrel of sugar to make the cost of both flour and sugar $20. How many dollars did he pay for the sugar?

5. A farmer sold 14 bushels of new potatoes to one dealer and 6 bushels to another. How many bushels did he sell to both?

6. How many trees are there in an orchard which has 23 apple trees and 7 peach trees?

7. If Laura sells 9 chickens to one man and 18 to another, how many chickens does she sell?

8. Of daily papers a newsboy sold 27 morning papers and 9 evening papers. How many papers did he sell all together?

9. Frank broke a window in Mr. Lovett's shop, and had to pay 24 cents for a glass and 8 cents to get it put in place. How much did he have to pay for his carelessness?

10. One express package weighed 20 pounds and another weighed 15 pounds. How many pounds did both weigh?

11. Mr. Jones has 13 Jersey cows and 14 Durham cows. How many cows has he?

12. A lady paid $26 for a dress, $8 for a bonnet, and $5 for an umbrella. How many dollars did she pay for all?

13. One piece of ice weighs 34 pounds and a smaller piece weighs 12 pounds. How many pounds do both pieces weigh?

14. A merchant sold 19 yards of muslin to one customer, 9 yards to another, and 10 yards to another. How many yards did he sell to all three customers?

15. Mr. Adam paid a stock-dealer $46 for a Holstein cow and $9 for her calf. How many dollars did he pay for both?

16. If I pay 40 cents for a tie and 15 cents for a collar, how many cents do I pay for both?

17. Harold paid $26 for a desk. He paid $6 to have it painted, and sold it for $7 more than it cost him. How much did he get for it?

18. A farmer raised 14 loads of hay in one field, 12 loads in another, and 8 loads in another. How many loads of hay did he raise?

19. A lady has 13 hens and 20 ducks. How many fowls has she?

20. A grocer sold 23 pounds of sugar to one customer, 13 pounds to another, and 8 pounds to a third. How many pounds did he sell to all three?

21. One monument is 17 feet high, a second is 8 feet higher than the first, and a third is 9 feet higher than the second. How high is the third?

22. A girl has read 25 pages in a primer and 40 pages in a first reader. How many pages has she read?

23. Mary has a 25-cent piece, a 10-cent piece, and a 2-cent piece. How many cents has she?

24. There are 24 cherries on one branch and 30 on another. How many cherries are there on both branches?

25. A boy selling newspapers, made 23 cents on Monday and 21 cents on Tuesday. How many cents did he make on both days?

26. One family used 28 quarts of milk in a week, and another used 22 quarts. How many quarts did both families use in a week?

27. Harry has earned 30 cents selling papers. How many more cents must he earn to have 50 cents?

28. A farmer sold three loads of potatoes. The first contained 25 bushels, the second 25 bushels, and the third 20 bushels. How many bushels did he sell?

29. From a peddler I bought a bushel of tomatoes for 70 cents, a half-bushel of turnips for 20 cents, and a half-peck of peas for 10 cents. How many cents did I pay for all?

30. After spending 55 cents for a book, Albert found he had 20 cents left. How many cents had he at first?

31. There are 25 ducks in one pond and 24 in another pond. How many ducks in both ponds?

32. There are 30 boys in the first class in geography and 42 in the second class. How many boys are there in both geography classes?

33. A farmer took 40 eggs to market on Monday and 24 eggs on Tuesday. How many eggs did he take both days?

34. A farmer has 40 acres of corn planted in one field and 36 acres in another field. How many acres of corn has he all together?

35. Carl earned 44 cents in one day and 56 cents the next day. How many cents did he earn in both days?

WRITTEN EXERCISES.

45. *36.* Find the sum of eight hundred ninety-six, five thousand six hundred seven, fifty-nine, seven thousand four hundred twelve, nine hundred sixty-eight.

37. Find the sum of thirty-six thousand seven hundred eight, eight thousand ninety-four, twenty-seven thousand six hundred eight, six hundred four thousand five hundred eighty, fifty-five thousand four hundred nineteen.

38. Find the sum of eight hundred twenty-six thousand seven hundred fifteen, eight thousand ninety-six, forty-nine thousand eight hundred seven, four million nine hundred thousand six hundred, seventeen million sixty-eight thousand forty-six.

39. Find the sum of nine million six hundred thirty-seven thousand four hundred eighty-five, forty-six million eight hundred five thousand six hundred nineteen, eighty-four million eight hundred eighty-six, nine hundred thirteen million ninety-eight thousand nine hundred, seven hundred seven million four hundred ten thousand eight hundred eleven.

40. Find the sum of eight hundred seven million four hundred six thousand nine hundred five, five hundred eighty-two million seven hundred fifty-eight, six hundred million seven hundred ninety-five thousand, fourteen billion eighty-seven million nineteen thousand six hundred eighty, seventy-five billion four hundred eighty-six.

41. A farmer sold five fat oxen. The first weighed 1,397 pounds; the second, 1,578 pounds; the third, 1,272 pounds; the fourth, 1,485 pounds; and the fifth, 1,508 pounds. What was the weight of all five together?

42. An orchard contains 135 cherry trees, 119 more pear trees than cherry trees, as many apple trees as there are pear trees and cherry trees, and 146 more plum trees than pear trees. How many trees are there in the orchard?

43. Mr. Brown, in his will, gave each of his three daughters $3,475, to each of his two sons $4,865, and to his wife as much as to his two sons and one daughter. What was the value of his property?

44. At the battle of Gettysburg, the loss in the Union army was 2,834 men killed and 13,790 wounded; and in the Confederate army, 4,500 killed and 26,500 wounded. What was the whole loss in both armies?

45. A contractor bought four loads of paving bricks. In the first load there were 3,985 bricks; in the second, 4,198; in the third, 3,899; and in the fourth, 4,218. What was the total number of bricks in all the loads?

46. In going from Boston to San Francisco a man traveled 150 miles in Massachusetts, 336 miles in New York, 236 miles in Canada, 222 miles in Michigan, 41 miles in Indiana, 227 miles in Illinois, 276 miles in Iowa, 371 miles in Nebraska, 263 miles in Colorado, 473 miles in Utah, 461 miles in Nevada, and 282 miles in California. By that route, how many miles is it from Boston to San Francisco?

47. The sales of a business firm for the first week in January were $6,372; for the second week, $5,956; for the third week, $7,285; and for the fourth week, $6,739. What was the amount of the sales for the four weeks?

48. A grain dealer bought 3,485 bushels of wheat on Monday, 4,538 bushels on Tuesday, 3,864 bushels on Wednesday, 3,956 bushels on Thursday, 4,327 bushels on Friday, and 4,415 bushels on Saturday. How many bushels did he buy during the week?

49. The height of Mount Vesuvius is 3,948 feet; Mount Washington is 2,340 feet higher than Mount Vesuvius; Mount Etna, 4,547 feet higher than Mount Washington; Pike's Peak, 3,312 feet higher than Mount Etna; Mont Blanc, 1,639 feet higher than Pike's Peak;

Mount Logan 2,714 feet higher than Mont Blanc; and Mount Everest, 10,562 feet higher than Mount Logan. What is the height of Mount Everest?

50. Lake Ontario covers a surface of 6,950 square miles; Lake Erie, 7,750 square miles; Lake Huron, 21,000 square miles; Lake Michigan, 22,400 square miles; and Lake Superior, 31,500 square miles. How many square miles in the surface of the five lakes?

51. From the nail works at Pittsburg there were shipped 7,890 kegs on Monday; 613 kegs more on Tuesday than on Monday; 8,760 kegs on Wednesday, and the same number of kegs on Thursday; on Friday, 7,950 kegs; and 1,375 kegs more on Saturday than on Friday. How many kegs were shipped during the week?

52. The length of the Mississippi River proper is 3,160 miles; of the Missouri to its junction with the Mississippi is 3,100 miles; Arkansas River, 2,170 miles; Red, 1,600 miles; Ohio, 975 miles; Wabash, 520 miles; Illinois, 435 miles; Des Moines, 400 miles; Rock, 330 miles. What is the combined length of the Mississippi and the tributaries named above?

53. The area of the North Atlantic States in square miles is: Maine, 33,040; New Hampshire, 9,305; Vermont, 9,565; Massachusetts, 8,315; Rhode Island, 1,250; Connecticut, 4,990; New York, 49,170; New Jersey, 7,815; Pennsylvania, 45,215. What is the total area of this division?

54. In 1890 the population of the six largest cities in the United States was: New York, 1,515,301; Chicago, 1,099,850; Philadelphia, 1,046,964; Brooklyn, 806,343; St. Louis, 451,770; Boston, 448,477. What was the total population of these six cities?

55. A freight train left Rutland, Vt., with eight cars loaded with marble as follows: The first car contained 37,850 pounds; the second car, 37,395 pounds; third car, 36,976 pounds; fourth car 37,415 pounds; fifth car, 38,226 pounds; sixth car, 36,844 pounds; seventh car, 37,785 pounds; eighth car, 38,210 pounds. How many pounds of marble were carried on the train?

56. The capacity of St. Peter's, Rome, is 27,000 more than St. Paul's, London; of St. Paul's, 23,000 more than of the Mormon Temple, Salt Lake City; of the Mormon Temple, 2,000 more than Spurgeon's Tabernacle; of Spurgeon's Tabernacle, 1,176 more than Music Hall, Cincinnati; of Music Hall, 1,824 more than Washington Hall, Paterson, N. J.; of Washington Hall, 135 more than the Academy of Music, Philadelphia; and of the Academy of Music, 280 more than Music Hall, Boston, whose capacity is 2,585. What is the capacity of Music Hall, Cincinnati? Of the Mormon Temple? Of St. Peter's?

57. In 1890 Louisiana produced 75,645,433 pounds of rice; South Carolina, 30,338,951 pounds; Georgia, 14,556,432 pounds; North Carolina, 5,846,404 pounds; Florida, 1,011,805 pounds; Mississippi, 676,746 pounds; Alabama, 399,270 pounds; Texas, 108,423 pounds; Arkansas, 7,110 pounds. What was the total number of pounds of rice produced by these States? How much did Louisiana and South Carolina together produce?

58. The States which produced cotton were: Virginia, 5,375 bales; North Carolina, 336,261 bales; South Carolina, 747,190 bales; Georgia, 1,191,846 bales; Florida, 57,928 bales; Missouri, 15,856 bales; Kansas, 212 bales; Kentucky, 873 bales; Tennessee, 190,579 bales; Alabama,

915,210 bales; Mississippi, 1,154,725 bales; Louisiana, 659,180 bales; Texas, 1,471,242 bales; Oklahoma Territory, 425 bales; Arkansas, 691,494 bales. What was the whole number of bales produced?

(How many pounds in a bale of cotton?)

59. In 1889 the production of iron in each of the following States was: New York, 1,247,537 tons; Pennsylvania, 1,560,234 tons; Alabama, 1,570,319 tons; Michigan, 5,856,169 tons; Ohio, 254,294 tons; Wisconsin, 837,399 tons; Minnesota, 864,508 tons; New Jersey, 415,510 tons; Virginia, 511,255 tons; Tennessee, 473,294 tons; Missouri, 265,718 tons; Colorado, 109,136 tons. What was the total production in these States?

60. The number of carp planted in each of the following States during the decade 1880 to 1890 was: Pennsylvania, 765,919; Indiana, 756,719; North Carolina, 201,293; Illinois, 202,831; Kansas, 169,682; Missouri, 175,211; New York, 102,901; Tennessee, 151,968; Virginia, 194,089; Georgia, 187,043; Colorado, 140,973; Iowa, 72,758; Kentucky, 59,084; Maryland, 51,741; Nebraska, 86,840; Wisconsin, 65,619. What was the whole number planted in these States?

(What are carp? What are they good for?)

46. Add and test the results:

1. 423, 713, 254, 653, 836.

2. 718, 424, 763, 576, 683.

3. 415, 654, 592, 708, 947, 97.

4. 321, 585, 94, 645, 300, 563.

5. 735, 327, 419, 26, 75, 894, 107.

6. 237, 693, 49, 871, 316, 354.

7. 564, 85, 4327, 872, 396, 483.

8. 576, 8427, 58, 76, 4291, 653, 318.

9. 516, 795, 7, 86, 4818, 927.

10. 723, 905, 45, 148, 6402, 8537.

11. 97, 613, 4, 4275, 659, 7, 436, 78.

12. 814, 3642, 7854, 95, 347, 8, 684, 562.

13. 619, 37, 4685, 7856, 54, 3912, 8, 5723, 865.

14. 583, 397, 3915, 6834, 49, 56, 4543, 7816.

15. 2318, 954, 78, 3735, 646, 493, 66, 587, 294.

16. 913, 319, 471, 85, 6426, 7178, 642, 896.

17. 497, 684, 9, 71, 9157, 584, 375, 832, 169.

18. 88, 437, 726, 8947, 511, 478, 647, 358.

19. 666, 777, 888, 999, 5555, 444, 333, 222.

20. 468, 7812, 34, 86, 954, 313, 627, 9, 4786.

21.	22.	23.
853472	726834	486537
596327	493685	937496
347415	659478	697865
765997	382967	843257
481743	578169	369482
826579	826794	576849
649382	987698	768594
276458	754863	678876
956710	694586	952758
483569	478967	386475
168634	347695	875964
715286	876543	494949
295867	234567	768786
947657	947638	586789

SUBTRACTION.

47. *1.* There were 9 trees in front of Mr. Cole's house. After 4 of them were cut down, how many remained?

2. A man had $12 and paid $4 for a railroad ticket. How many dollars had he left?

3. In a park one monument is 14 feet high and another is 8 feet high. How many feet higher is one monument than the other?

4. Anna is 13 years old and her brother is 5 years younger. How old is her brother?

5. Lewis picked 15 quarts of berries, which was 6 quarts more than Elmer picked. How many quarts of berries did Elmer pick?

6. From a board 16 feet long a carpenter cut off a piece 6 feet long. How many feet were left?

7. A baker received an order for 14 loaves of bread. He had only 9 loaves baked. How many loaves must he bake before he can fill the order?

8. Fred had 9 cents. He earned enough more cents, by picking the stones off a neighbor's lawn, to buy a book which cost 17 cents. How many cents did he earn?

What do you call the work you have been doing?

When we wish to find how many more units in one than in another of two given numbers, or when we

wish to know how many units put with a given number will produce another given number, or when we wish to know how many units of a given number will be left if another given number be taken out of it, we have a problem in *Subtraction.*

Under which of these three forms does each of the preceding problems belong?

Always, in subtraction:

The larger given number is the Minuend.

The smaller given number is the Subtrahend.

The result is the Difference, or Remainder.

In each of the problems given, which number is the minuend? Which number is the subtrahend? Which is the remainder, or difference?

The *sign* of subtraction is a short dash —, and it is read minus, or *less.* Read: 9—3=6; 11—6=5.

Can 5 oranges be taken from 9 plums? Why not?

Subtraction is possible only when the units are of the same kind.

5 and 4 are how many? 4 and 5 are how many? 9 minus 5 are how many? 9 minus 4 are how many?

Make these four statements for each of the forty-five combinations on page 23, thus: 6 and 5 are 11; 5 and 6 are 11; 11 minus 5 are 6; 11 minus 6 are 5.

Eighty-one of these statements are differences. These are all the differences there are; for in the subtraction of larger numbers we simply find one of these differences at each step. Hence, these eighty-one differences must be thoroughly familiar before any real progress in subtraction is possible.

EXERCISES.

48. Practice upon the following exercises until the differences can be named instantly and without effort:

2	3	3	4	4	5	5	6	6	7	7	8
1	1	2	1	3	1	4	1	5	1	6	1
—	—	—	—	—	—	—	—	—	—	—	—
8	9	9	10	10	4	5	5	6	6	7	7
7	1	8	1	9	2	2	3	2	4	2	5
—	—	—	—	—	—	—	—	—	—	—	—
8	8	9	9	10	10	11	11	6	7	7	8
2	6	2	7	2	8	2	9	3	3	4	3
—	—	—	—	—	—	—	—	—	—	—	—
8	9	9	10	10	11	11	12	12	8	9	9
5	3	6	3	7	3	8	3	9	4	4	5
—	—	—	—	—	—	—	—	—	—	—	—
10	10	11	11	12	12	13	13	10	11	11	12
4	6	4	7	4	8	4	9	5	5	6	5
—	—	—	—	—	—	—	—	—	—	—	—
12	13	13	14	14	12	13	13	14	14	15	15
7	5	8	5	9	6	6	7	6	8	6	9
—	—	—	—	—	—	—	—	—	—	—	—
14	15	15	16	16	16	17	17	18			
7	7	8	7	9	8	8	9	9			
—	—	—	—	—	—	—	—	—			

1.	*2.*	*3.*	*4.*	*5.*
6−1=	11−5=	11−4=	11−8=	8−7=
7−2=	15−9=	10−2=	18−9=	4−1=
8−5=	8−1=	9−8=	12−5=	12−9=
10−4=	6−2=	6−3=	15−6=	9−3=
13−9=	10−3=	16−7=	10−7=	3−2=
12−3=	5−4=	13−5=	7−5=	16−9=
14−7=	4−2=	8−4=	8−3=	14−6=
9−2=	11−9=	7−6=	10−1=	3−1=
15−7=	16−8=	5−2=	6−4=	10−6=
12−4=	14−5=	17−9=	4−3=	13−4=
8−6=	7−4=	10−8=	11−7=	7−1=
10−9=	12−7=	9−5=	9−7=	15−8=
11−3=	10−5=	11−2=	17−8=	5−1=
13−7=	13−8=	7−3=	5−3=	9−4=
9−6=	12−6=	6−5=	8−2=	14−9=
12−8=	14−8=	9−1=	11−6=	13−6=

EXERCISES.

49. Subtract:

1. By 2's from 60 to 0, thus: 60, 58, 56, etc.

2. By 2's from 67 to 1.

3. By 3's from 69 to 0; from 67 to 1.

4. By 4's from 68 to 0; from 67 to 3.

5. By 5's from 69 to 4; from 66 to 1.

6. By 6's from 68 to 2; from 70 to 4.

7. By 7's from 77 to 0; from 80 to 3.

8. By 8's from 89 to 1; from 92 to 4.

9. By 9's from 95 to 5; from 98 to 8.

10. Subtract 6 from 12, 22, 32, 42, 52, 62, 72, thus: twelve minus six are six; twenty-two minus six are sixteen, etc.

11. Subtract 7 from 15, 25, 35, 45, 55, 65, 75.

12. Subtract 7 from 14, 24, 34, 44, 54, 64, 74.

13. Subtract 5 from 13, 23, 33, 43, 53, 63, 73.

14. Subtract 5 from 12, 22, 32, 42, 52, 62, 72.

15. Subtract 4 from 10, 20, 30, 40, 50, 60, 70.

16. Subtract 4 from 11, 21, 31, 41, 51, 61, 71.

17. Subtract 8 from 15, 25, 35, 45, 55, 65, 75.

18. Subtract 8 from 14, 24, 34, 44, 54, 64, 74.

19. Subtract 9 from 18, 28, 38, 48, 58, 68, 78.

20. Subtract 9 from 16, 26, 36, 46, 56, 66, 76.

EXERCISES.

50. Subtract. Read the differences at sight from the left:

1.	*2.*	*3.*	*4.*	*5.*	*6.*
28	19	37	26	49	38
5	6	3	4	5	2
7.	*8.*	*9.*	*10.*	*11.*	*12.*
17	47	65	76	58	89
6	13	32	41	35	56
13.	*14.*	*15.*	*16.*	*17.*	*18.*
94	63	594	768	489	647
72	42	321	430	145	220
19.	*20.*	*21.*	*22.*	*23.*	
952	749	896	348	619	
612	307	230	16	318	
24.	*25.*	*26.*	*27.*	*28.*	
836	574	759	463	689	
105	200	459	103	578	
29.	*30.*	*31.*	*32.*	*33.*	
4961	5387	9596	7469	6387	
2730	3245	5500	3464	6050	
34.	*35.*	*36.*	*37.*	*38.*	
2462	72541	54287	76384	84590	
2412	32141	51234	22311	34260	
39.	*40.*	*41.*	*42.*	*43.*	
69758	362154	753684	978600	893274	
34200	112000	453654	225000	340024	

44. 8+7−4+6−3−5+9−6+3+8−9+7=?

45. 6+9+5−3+8−5+4+7−9+6−5+8=?

46. 3+8+6−5+9+8−6+7−2+10−7−4=?

47. 5+7+8−4+6−7+5+8−9+7+6−10=?

48. 9+5+7−6+8+10−9+7+4−6−8−4=?

49. 4+7+8+6+9+5−8−6−4−9+7+8=?

50. 7+9+5−6+8+4−7+8+10−3−7+9=?

51. 6+8−5+9+9+5−8+7−9+6−10+3=?

52. 9+8+7+6−4+9−5+8−4−6+7+10=?

53. 8+7+10+9−6+8−7+9−8+5+8−9=?

54. 4+9+7+5+8−4−5−7+9+6−8−8=?

EXERCISES.

51. *1.* Subtract 268 from 547.

547
268
279

Which number is the subtrahend? Write it under the minuend, with units of the same kind in the same column. Why?

Draw a horizontal line beneath the figures. Why?

Begin with the lowest or right-hand order to subtract. Why? What is there in this problem that is new?

8 ones can not be taken from 7 ones. Take 1 ten from the 4 tens. 1 ten equals 10 ones. 10 ones and 7 ones are 17 ones. 17 ones minus 8 ones are 9 ones. Write 9 for these ones under the column of ones. Why?

Since 1 ten was taken from 4 tens, only 3 tens remain. 6 tens can not be taken from 3 tens. Take 1 hundred from 5 hundreds. 1 hundred equals 10 tens. 10 tens and 3 tens are 13 tens. 13 tens minus 6 tens are 7 tens. Write 7 for the tens under the column of tens. Why?

Since 1 hundred was taken from 5 hundreds, only 4 hundreds remain. 4 hundreds minus 2 hundreds are 2 hundreds. Write 2 for hundreds under the column of hundreds. The difference, or remainder, is 279.

The sum of the subtrahend and difference must equal the minuend. Why?

Hence, to prove a problem in subtraction, find the sum of the subtrahend and difference. If their sum equals the minuend, the subtraction is correct.

Subtract and prove:

2.	436—278	*18.*	964—556	*34.*	5814—2965
3.	524—137	*19.*	249—93	*35.*	3439—647
4.	361—233	*20.*	126—9	*36.*	7585—3609
5.	653—462	*21.*	514—205	*37.*	2831—927
6.	127—49	*22.*	966—673	*38.*	2673—493
7.	872—567	*23.*	485—190	*39.*	8435—5728
8.	744—396	*24.*	672—287	*40.*	6524—2417
9.	567—258	*25.*	333—239	*41.*	1467—978
10.	943—486	*26.*	758—49	*42.*	1358—849
11.	625—549	*27.*	540—170	*43.*	7460—5380
12.	382—199	*28.*	860—536	*44.*	5245—237
13.	796—386	*29.*	209—74	*45.*	9600—8900
14.	849—489	*30.*	914—326	*46.*	6437—6389
15.	357—208	*31.*	671—265	*47.*	8352—7349
16.	645—245	*32.*	6846—3979	*48.*	9087—6938
17.	478—389	*33.*	9357—4768	*49.*	4692—1876

EXERCISES.

52. *1.* Find the difference between 9,000 and 4,765. hat is there new in this problem?

00
65
35

Since there are no ones, no tens, and no hundreds, take 1 thousand from 9 thousand. 1 thousand equals 10 hundreds. Now take 1 hundred from the 10 hundreds. 9 hundreds are left. 1 hundred equals 10 tens. Again ɛe 1 ten from the 10 tens. 9 tens are left. 1 ten equals 10 ɛs. From what do you take 5? From what do you take 6? ɔm what do you take 7?

Subtract:

2.	73450—34672	*16.*	444444—60006
3.	56300—17411	*17.*	351806—1859
4.	65000—47325	*18.*	912016—273407
5.	30000—15476	*19.*	501510—7463
6.	43406—27537	*20.*	600000—333333
7.	84034—39345	*21.*	704006—193007
8.	70461—29764	*22.*	800650—70034
9.	94003—6527	*23.*	400072—3496
10.	58014—9345	*24.*	300400—672
11.	15205—4306	*25.*	900009—300700
12.	60048—786	*26.*	201012—8809
13.	20006—5008	*27.*	533444—233658
14.	17111—13060	*28.*	100902—5555
15.	48000—9999	*29.*	808606—183576

ORAL EXERCISES.

53. *1.* George paid $17 for a second-hand bicycle, and he sold it for $9. How many dollars did he lose?

2. It took 15 minutes to skate up to the bridge against the wind, but only 9 minutes to get back. How many more minutes was I in going than in returning?

3. A boy had to go to a town 14 miles away. He rode 8 miles, but had to walk the rest of the way. How many miles did he walk?

4. There were in the room 10 chairs and 18 persons. How many persons had to stand?

5. In a ball game our nine made 17 runs and the other nine only 8 runs. How many more runs did our nine make than the other?

6. Ellen has 18 lines of poetry to learn. How many lines will she still have to learn after she has learned 11 lines?

7. Ralph had 19 papers to deliver. He has delivered 13. How many more has he to deliver?

8. John is 20 years old and Nellie is 14 years old. How many years younger is Nellie than John?

9. One part of 21 is 9. What is the other part?

10. I borrowed $22 and afterward paid $8 on the debt. How much did I still owe?

11. One package of tea weighs 24 ounces, and another weighs 16 ounces. How much heavier is the first than the second package?

12. Albert earned 26 cents selling papers. He paid 13 cents for fruit. How many cents had he left?

13. William had 28 cherries. He gave Frank 9 and Hugh 7. How many had he left?

14. In a schoolroom 27 boys sit in three rows. There are 8 boys in one row and 10 in the next row. How many boys in the third row?

15. In a grove are 30 trees. 9 are maple, 13 are walnut, and the rest are ash. How many ash trees are there?

16. A well was sunk through sand and clay to a depth of 32 feet. There were 8 feet of sand. How many feet of clay were there?

17. The sum of three numbers is 36. Two of the numbers are 12 and 15. What is the other number?

18. A man paid $6 for a pair of shoes, $18 for a coat and vest, $7 for a pair of pantaloons. He gave the clerk $40. How many dollars ought the clerk to return?

19. Flora had 16 cents. Henry gave her 12 cents more, and her mother enough to make 38 cents. How many cents did her mother give her?

20. How old are you? From this how do you find in what year you were born?

21. How much deeper is a well that is 39 feet deep than one that is 31 feet deep?

22. I gave a hotel clerk $40. He gave me back $12. How much was my bill?

23. A garden has 54 hills of cucumbers and squashes. There are 14 hills of squashes. How many hills of cucumbers are there?

24. Fred has $25 toward a bicycle, for which he is to pay $45. How many more dollars does he need?

25. Harry is 9 inches shorter than his brother Walter. Walter is 63 inches tall. How tall is Harry?

26. A lady paid 75 cents for some tea and coffee. The coffee cost 50 cents. What was the cost of the tea?

27. A boy added two numbers and his answer was 60. If one of the numbers was 26, what was the other?

28. Jane wants a reader that costs 50 cents. She has only 25 cents. How many more cents must she get?

29. When Carl saves 35 cents more he will have 70 cents. How many cents has he?

30. A man paid $95 for a horse and cart. What did the horse cost if the cart cost $20?

31. A class has 60 minutes for reading and spelling. How many minutes do the pupils have for reading if they take 20 minutes for spelling?

32. Mr. Henry's parlor is 97 inches high. The wainscoting above which it is to be papered is 33 inches high. Into what lengths must the paper be cut?

33. A farmer sold a grocer 78 dollars' worth of hay. He took in payment, 16 dollars' worth of flour, 10 dollars' worth of sugar, 8 dollars' worth of other groceries, and the rest in money. How much money did he get?

34. A man earns $85 a month. He pays $18 for house-rent, $12 for meat, and $15 for groceries. How many dollars has he left?

35. 65 boys worked on an example. 48 boys found the correct answer. How many have a wrong answer?

36. A farmer raised 84 bushels of potatoes. He kept 28 bushels. How many bushels did he sell?

WRITTEN EXERCISES.

54. *37.* A man bought land for $2,560, and sold it for $4,080. How much did he gain?

38. A man sold a piece of property for $4,750 and gained $875. How much did the property cost him?

39. A man bought wheat for $39,600. How much must he gain to receive $42,750 for it?

40. An army of 76,397 men is required by a general who finds he has only 57,698 men. How many more men must he have?

41. Columbus discovered America in 1492. How many years from that time to the landing of the Pilgrims in 1620?

42. The sum of two numbers is 86,457. One of them is 63,589. What is the other number?

43. What number added to two hundred sixty-nine thousand eight hundred seventy-five, will make one million two hundred seventy-four thousand three hundred fifty-four?

44. Henry Hudson sailed up the Hudson River in 1609. How many years ago was that?

45. The difference is five hundred twenty-eight thousand seven hundred forty-six, and the minuend is nine hundred forty-seven thousand six hundred twenty-eight. What is the subtrahend?

46. The difference is three hundred forty-eight thousand six hundred fifty-two, and the larger number is seven million one hundred twenty-six thousand four hundred ninety-two. What is the smaller number?

47. What number subtracted from sixty-two million three hundred seventy-four thousand eight hundred ninety-six, will leave six million five hundred ninety-four thousand five hundred seventy-three?

48. 4,368,798−487,695=?

49. What length of time elapsed between the birth of Sir Francis Bacon, in 1561, and the birth of Benjamin Franklin, in 1706?

50. In 1890, the State having the greatest number of miles of railroad was Illinois, with 10,071 miles. Kansas was next, with 8,806 miles. How many more miles had Illinois than Kansas?

51. Texas, the largest State, has an area of 265,780 square miles; California, the next largest State, has an area of 158,360 square miles. How much larger is Texas than California?

52. The value of the mirrors manufactured in this country was $304,000 in 1880, and $6,000,533 in 1890. How much greater was the value in 1890 than in 1880?

53. The value of the matches manufactured in 1890 was $2,193,638, and in 1880, $4,668,446. How much did the value decrease in ten years?

54. The number of tons of freight moved on the Ohio River in 1890 was 7,770,565, and on the Mississippi River, 8,861,182. How many more tons were moved on the Mississippi River than on the Ohio?

55. A boy working an example used the number 2,306,400, which he afterward found was too large by 28,875. What was the correct number?

56. In 1890, New York's population, which was the largest of any State, was 5,997,853. Pennsylvania's population was 5,258,014, which was the next largest. How much greater was the population of New York than that of Pennsylvania?

57. In 1889, Colorado produced 18,375,551 ounces of silver, and Montana produced 13,511,455 ounces. How many more ounces did Colorado produce than Montana?

58. New York, in 1890, produced 24,616,736 bushels of potatoes; Iowa, 18,068,311 bushels. How much greater was the product in New York than in Iowa?

59. The value of live stock in the United States, June 1, 1890, was $2,208,767,573. June 1, 1880, it was $1,500,384,707. Find the increase in ten years.

60. Two vessels 4,872 miles apart start to meet each other. When one of them has gone 1,686 miles and the other 1,597 miles, how many miles are they apart?

61. How far apart would they have been at that time if they had sailed in opposite directions?

62. From what number must 713 be taken twice to leave 16,347?

63. To what number must 874 be added twice to make 12,316?

MULTIPLICATION.

55. *1.* How many doves are 2 doves and 2 doves and 2 doves?

2. How many are 3 pencils and 3 pencils and 3 pencils and 3 pencils? —— what?

3. How many are 4 plums and 4 plums and 4 plums and 4 plums and 4 plums? —— what?

4. How many are $7 and $7 and $7 and $7 and $7 and $7? —— what?

5. How many are 6 and 6 and 6 and 6? —— what?

6. How do the numbers to be added in each problem above compare in size?

7. How many equal numbers are to be added in each problem?

8. Find the cost of 8 pounds of beef at 9 cents a pound.

9 cents
8 times
72 cents

See illustration in the margin. This method of finding the sum of equal numbers is called *Multiplication.*

Why must the 72 mean 72 *cents?*

Multiplication is finding the sum of as many equal numbers as the number of units in another number.

The Multiplicand *is one of the equal numbers to be added.*

The Multiplier *shows how many of the equal numbers are to be added.*

The Product *is the sum or result.*

Which number is the multiplicand in each of the preceding problems? Which number is the multiplier? Which is the product or sum?

The multiplicand *may be either a concrete or an abstract number.*

The multiplier *is an abstract number.*

The multiplicand and the product *must be of the same kind in every respect.*

The sign of multiplication is an oblique cross, $\times$. This sign is read *times* when the multiplier precedes it. It is read *multiplied by* when the multiplier follows it. Thus: $4 \times \$7 = \$28$, is read 4 times \$7 equal \$28; $\$7 \times 4 = \$28$, is read \$7 multiplied by 4 equal \$28.

The multiplicand and multiplier are *factors* (*makers*) *of the product.*

The product is a *multiple of either of its factors.*

The product of two or more factors is the same, whatever their order may be. Thus: 4 times 3 are 12, and 3 times 4 are 12.

In the margin you see 4 columns of 3 dots each. How many times 3 dots are there? You can also see 3 rows of 4 dots each. How many times 4 dots are there? Then 4 times 3 dots is the same as 3 times 4 dots; in either case the result is 12 times 1 dot.

56. Multiplication Table.

Multiplication is possible only when the multiplication table is so thoroughly committed to memory that its results may be readily recalled. There can be no mul-

tiplication without using remembered results. It is *necessary* that the table should be learned as far as "nine times nine"; but it is very convenient to know it farther — at least as far as "twelve times twelve."

2 times 1=2	3 times 1=3	4 times 1=4
2 times 2=4	3 times 3=9	4 times 4=16
2 times 3=6	3 times 4=12	4 times 5=20
2 times 4=8	3 times 5=15	4 times 6=24
2 times 5=10	3 times 6=18	4 times 7=28
2 times 6=12	3 times 7=21	4 times 8=32
2 times 7=14	3 times 8=24	4 times 9=36
2 times 8=16	3 times 9=27	4 times 10=40
2 times 9=18	3 times 10=30	4 times 11=44
2 times 10=20	3 times 11=33	4 times 12=48
2 times 11=22	3 times 12=36	
2 times 12=24		

5 times 1=5	6 times 1=6	7 times 1=7
5 times 5=25	6 times 6=36	7 times 7=49
5 times 6=30	6 times 7=42	7 times 8=56
5 times 7=35	6 times 8=48	7 times 9=63
5 times 8=40	6 times 9=54	7 times 10=70
5 times 9=45	6 times 10=60	7 times 11=77
5 times 10=50	6 times 11=66	7 times 12=84
5 times 11=55	6 times 12=72	
5 times 12=60		

8 times 1=8	9 times 1=9	10 times 1=10
8 times 8=64	9 times 9=81	10 times 10=100
8 times 9=72	9 times 10=90	10 times 11=110
8 times 10=80	9 times 11=99	10 times 12=120
8 times 11=88	9 times 12=108	
8 times 12=96		

11 times 1=11	12 times 1=12
11 times 11=121	12 times 12=144
11 times 12=132	

DRILL EXERCISES.

57. The two orders in which the factors of the multiplication table may occur are promiscuously given below. Practice them daily until you can name the product of each set of factors the instant your eye rests upon the figures:

6× 2=	10× 4=	9× 2=	6× 3=
3× 7=	5× 6=	2×11=	12× 8=
5× 5=	2×12=	4× 6=	10×11=
6× 4=	3× 5=	9× 5=	8× 3=
7× 8=	4× 9=	11× 4=	3× 2=
9× 9=	8× 8=	12× 7=	10× 5=
5×10=	11× 1=	10× 9=	9× 4=
8× 4=	4× 5=	6×11=	7×12=
7×11=	7× 4=	2× 4=	5×11=
6× 8=	5×12=	10× 2=	12× 2=
9× 3=	8×10=	6× 5=	7× 5=
4× 7=	12× 9=	3× 3=	4×10=
11× 2=	9× 7=	4×11=	2× 5=
3×12=	2× 7=	6× 6=	6× 9=
8× 5=	3× 4=	11×10=	12× 6=
7× 7=	3×11=	2× 6=	10× 9=
10×10=	5× 8=	4× 8=	4× 3=
1× 8=	10× 7=	11× 9=	5× 7=
5× 9=	6×12=	2× 3=	12× 9=
8× 2=	2× 9=	3×10=	9× 8=
4×12=	3× 8=	11×11=	11× 7=
5× 3=	8×12=	8× 9=	5× 2=
9× 6=	7×10=	10×12=	12×11=
11× 8=	11× 9=	8×11=	6× 7=
2× 2=	8× 6=	12× 4=	10× 3=
4× 4=	12×10=	11× 6=	7× 2=
7× 6=	1×12=	2×10=	12×12=
3× 9=	7× 9=	4× 2=	8× 7=
2× 8=	11× 5=	11× 3=	12× 5=
7× 3=	10× 6=	10× 8=	3× 6=
6×10=	12× 3=	5× 4=	11×12=

6	9	10	6	3	5	2	12	10
3	2	4	2	7	6	11	8	11
—	—	—	—	—	—	—	—	—
8	10	9	7	5	12	7	4	3
3	5	4	12	11	2	5	10	6
—	—	—	—	—	—	—	—	—
6	12	10	4	5	12	9	11	5
9	6	9	3	7	9	8	7	5
—	—	—	—	—	—	—	—	—
6	7	9	5	8	7	6	9	4
4	8	9	10	4	11	8	3	7
—	—	—	—	—	—	—	—	—
11	3	8	7	10	5	8	4	12
2	12	5	7	10	9	2	12	5
—	—	—	—	—	—	—	—	—
9	11	4	7	3	8	7	6	12
6	8	4	6	9	7	3	10	3
—	—	—	—	—	—	—	—	—
10	11	7	12	8	11	12	8	3
6	5	9	10	6	9	12	12	8
—	—	—	—	—	—	—	—	—
2	6	10	5	3	3	9	12	8
9	12	7	8	11	4	7	9	10
—	—	—	—	—	—	—	—	—
5	7	4	8	4	3	2	12	10
12	4	5	8	9	5	12	7	9
—	—	—	—	—	—	—	—	—
4	9	11	6	10	6	3	4	6
6	5	4	11	2	5	3	11	6
—	—	—	—	—	—	—	—	—
11	2	4	11	12	6	11	8	10
10	6	8	9	11	7	11	9	12
—	—	—	—	—	—	—	—	—
8	12	11	7	4	11	10	5	11
11	4	6	10	2	3	8	4	12
—	—	—	—	—	—	—	—	—

EXERCISES.

58. Of what number are:

1. 2 and 4 the factors? 7 and 4? 9 and 4? 8 and 4? 4 and 4? 4 and 6? 4 and 11? 4 and 5?

2. 7 and 5 the factors? 9 and 5? 5 and 8? 5 and 3? 6 and 5? 12 and 5?

3. 9 and 6? 10 and 6? 6 and 3? 6 and 6? 6 and 8?

4. 6 and 7? 7 and 3? 7 and 7? 12 and 7? 7 and 9?

5. 8 and 4? 8 and 6? 8 and 9? 8 and 8? 8 and 5?

6. 3 and 9? 9 and 9? 12 and 9? 9 and 7? 10 and 9?

7. 11 and 3? 6 and 11? 11 and 11? 12 and 11? 11 and 9?

8. 3 and 12? 12 and 6? 8 and 12? 12 and 12? 12 and 9?

EXERCISES.

59. *1.* 3 times what number make 12, 21, 18, 9, 36, 24, 27?

2. 4 times what number make 16, 32, 36, 28, 40, 48, 24?

3. 5 times what number make 15, 35, 20, 30, 25, 45, 60, 40?

4. 6 times what number make 18, 36, 66, 72, 48, 42, 54?

5. 7 times what number make 14, 49, 35, 56, 70, 63, 28?

6. 8 times what number make 40, 72, 24, 64, 80, 96, 48?

7. 9 times what number make 54, 27, 81, 72, 99, 45, 63, 108?

8. 11 times what number make 33, 88, 110, 77, 55, 121, 132?

9. 12 times what number make 48, 72, 144, 84, 60, 96, 120?

ORAL EXERCISES.

60. *1.* How many horses will consume as many bushels of oats in 1 day as 6 horses consume in 3 days?

2. There are 6 feet in a fathom. How many feet are there in 5 fathoms?

3. A farmer sold 4 sheep at $8 apiece. How many dollars did he receive for them?

4. At 6 cents a quart, what will 7 quarts of milk cost? What will 9 quarts cost?

5. What must I pay for 9 cords of wood at $5 a cord?

6. If a peck of oats weighs 8 pounds, how many pounds will 4 pecks, or a bushel, weigh?

7. If 7 men can dig a ditch in 9 days, how many days will it take 1 man to dig it?

8. At 8 cents a quart, what will 8 quarts of cherries cost? What will 6 quarts cost?

9. If a freight train averages 8 miles an hour, how many miles will it run in 9 hours?

10. If a man sleeps 8 hours every night, how many hours will he sleep in 1 week?

11. If a family uses 9 pounds of flour a week, how many pounds will it use in 9 weeks?

12. At an average of 12 problems on a page, how many problems will there be on 7 pages?

13. 10 dimes make a dollar. How many dimes make 10 dollars?

14. A spider has 8 legs. How many legs have 12 spiders?

15. At $11 a ton, how many dollars will 12 tons of cannel coal cost?

16. How many trees are there in an orchard of 12 rows, 12 trees in a row?

17. What will be the cost of 6 cans of peas at 13 cents a can?

18. When beefsteak is 15 cents a pound, what will 5 pounds cost?

19. There are 4 houses in a row, and each of them has 16 windows. How many windows have all of them?

20. If a vessel sails at the rate of 14 miles an hour, how many miles will it sail in 7 hours?

21. If we spell 18 minutes each day, how many minutes do we spell in 5 days? (Find 5 times 10 and then 5 times 8, and add the results.)

22. What is the cost of 20 pounds of beef at 12 cents a pound?

23. Mr. Small gets 25 cents an hour for his work. How much does he get for 4 hours' work?

24. A boy went to school 21 days a month for 6 months. How many days did he go to school?

25. There are 16 ounces in a pound. How many ounces are there in 6 pounds?

26. A newsboy sold 23 papers at 3 cents apiece. How many cents did he receive for them?

27. How many feet have 30 cows? (10 times 3 times 4 feet.) 50 cows? 70 cows?

28. What is the cost of 5 pounds of butter at 24 cents a pound? (5 times 20 cents + 5 times 4 cents.)

29. The pupils of a certain class in arithmetic solved 20 problems a day for 5 days. How many problems did they solve in all?

30. What will 6 arithmetics cost at 42 cents apiece? (6 times 40 + 6 times 2.)

31. There are 60 minutes in an hour and a girl is in school 6 hours a day. How many minutes is she in school each day? In 2 days?

32. What will 12 suits of clothes cost at $30 a suit?

33. A family uses 3 quarts of milk a day. How many quarts will it use in 31 days?

34. In 1 bushel of corn there are 56 pounds. How many pounds are there in 8 bushels?

35. What will 7 sewing machines cost at $50 apiece?

36. If there are 3 bushels of apples in 1 barrel, how many bushels are there in 84 barrels?

EXERCISES.

61. *1.* Multiply 359 by 7.

359
7
2513

DIRECTIONS: Write the multiplier under the ones of the multiplicand. Why? Draw a line beneath. Why? Begin with the ones of the multiplicand to multiply. Why?

EXPLANATION: 7 times 9 ones are 63 ones, or 6 tens and 3 ones. Write the figure for the three ones in the product, under the ones of the multiplicand. Why? 7 times 5 tens are 35 tens. 35 tens and the 6 tens obtained by multiplying 9 ones by 7 are 41 tens, or 4 hundreds and 1 ten. Write the figure for the 1 ten in the product under the tens in the multiplicand. Why? 7 times 3 hundreds are 21 hundreds. 21 hundreds and 4 hundreds are 25 hundreds. Write the figures for twenty-five hundreds in the product.

Multiply:

2.	496 by 2	*14.*	827 by 6	*26.*	7408 by 9
3.	354 by 4	*15.*	309 by 4	*27.*	6039 by 7
4.	583 by 7	*16.*	932 by 7	*28.*	8400 by 5
5.	634 by 8	*17.*	664 by 5	*29.*	4009 by 8
6.	728 by 6	*18.*	721 by 9	*30.*	3894 by 3
7.	379 by 9	*19.*	497 by 8	*31.*	47136 by 4
8.	813 by 3	*20.*	568 by 3	*32.*	78565 by 8
9.	508 by 7	*21.*	879 by 4	*33.*	57683 by 7
10.	479 by 6	*22.*	4736 by 5	*34.*	94002 by 4
11.	730 by 9	*23.*	6485 by 7	*35.*	59023 by 3
12.	875 by 5	*24.*	8317 by 8	*36.*	79654 by 9
13.	596 by 8	*25.*	5672 by 6	*37.*	90708 by 6

EXERCISES.

62. *1.* Multiply 36 by 20.

EXPLANATION: 20 is 10 times 2. Hence, 20 times 36 is 10 times 2 times 36. 2 times 36 are 72. 10 times 2 times 36 is 10 times 72, which is found by moving the figures for seventy-two one place toward the left, according to Principles I and II, p. 15. Hence, 20 times 36 are 720.

Multiply:

2. 68 by 20; by 30; by 50; by 60; by 90.

3. 485 by 20; by 40; by 70; by 80; by 90.

4. 7894 by 20; by 30; by 60; by 70; by 80.

5. Multiply 28 by 200.

EXPLANATION: 200 is 100 times 2. Hence, 200 times 28 is 100 times 2 times 28. 2 times 28 are 56. 100 times 2 times 28 is 100 times 56, which is found by moving the figures for fifty-six two places toward the left. Hence, 200 times 28 are 5,600.

Multiply:

6. 47 by 200; by 300; by 500; by 600; by 800.

7. 563 by 200; by 400; by 700; by 800; by 900.

8. 648 by 300; by 500; by 600; by 700; by 900.

9. 54 by 2000; by 3000; by 6000; by 8000.

10. 378 by 4000; by 5000; by 7000; by 9000.

11. 8569 by 3000; by 4000; by 6000; by 8000.

12. 4290 by 5000; by 7000; by 8000; by 4000.

13. 9300 by 3000; by 9000; by 6000; by 8000.

EXERCISES.

63. *1.* Find the product of 48 multiplied by 32.

48
32
96=2 times 48.
144 =10 times 3 times 48.
1536=32 times 48.

DIRECTIONS: *1.* Write the multiplier under the multiplicand.
2. Find 2 times 48.
3. Find 10 times 3 times 48.
4. Find the sum of these products.

Find the product of:

2.	47×23	*9.*	50×49	*16.*	596×94
3.	39×18	*10.*	86×57	*17.*	485×68
4.	24×21	*11.*	97×74	*18.*	714×70
5.	53×37	*12.*	89×86	*19.*	906×89
6.	76×28	*13.*	98×98	*20.*	6435×90
7.	65×34	*14.*	227×87	*21.*	3786×64
8.	79×45	*15.*	348×76	*22.*	8042×87

WRITTEN EXERCISES.

64. *1.* If 49 pickets are required for a rod of fence, how many pickets will be needed to fence a garden requiring 34 rods of fence?

2. A clock which strikes the hours only, strikes 156 times a day. How many times will it strike in 93 days?

3. Sound travels at the rate of 1,120 feet a second. How far will it travel in 75 seconds?

4. A field of 640 acres made an average yield of 78 bushels of corn to the acre. How many bushels of corn did the field produce?

5. How many miles has a conductor traveled who has gone over the road between Chicago and Bloomington, a distance of 126 miles, once a day for 365 days?

6. A merchant bought 57 hogsheads of molasses, each hogshead containing 63 gallons. How many gallons of molasses did he buy?

7. How many trees are there in an orchard of 165 rows and 127 trees in a row?

8. A freight train consists of 28 cars, and each car contains 136 casks of lime, weighing 240 pounds each. How many pounds of lime are there in the whole cargo?

9. Another train of 26 cars is loaded with salt. Each car contains 98 barrels of 5 bushels each. How many bushels of salt in the whole train?

10. In 1890 Oklahoma produced 425 bales of cotton. What was it worth at 8 cents a pound, if the bales averaged 527 pounds each?

11. If a mill can turn out 896 yards of cloth a day, how many yards can it turn out in a year of 313 working days?

12. There are 320 rods in a mile. The Pennsylvania Canal is 358 miles long. How many rods long is it?

13. What sum of money will be required to pay a regiment of 987 men for a year's services, at $18 a month for each man?

14. Suppose a book to contain 456 pages, 45 lines on a page, and 48 letters in each line. How many letters would there be in the book?

15. If 1,637 barrels of flour feed the inhabitants of a certain city 1 day, how many barrels would be required to feed them 365 days?

16. How many yards of muslin in 57 bales, each bale containing 26 pieces, and each piece 63 yards?

17. A square mile is 640 acres. How many acres are there in the State of Delaware, the area of which is 2,050 square miles?

18. There are 5,280 feet in a mile. The distance between Philadelphia and Baltimore is 95 miles. How many feet of wire are there in the 9 telegraph lines connecting the two cities?

19. A regiment of 985 men have provisions for 62 days, allowing each man 18 ounces a day. How many ounces have they?

20. If 135 tons of rails are required to build a mile of railroad, how many tons will be required to build a road from Pittsburg to Chicago, a distance of 468 miles?

21. If a railroad train runs 28 miles in an hour, how many miles will it run in the month of January, running 20 hours a day?

22. A locomotive wheel revolves 264 times in going a mile. How many revolutions will it make between St. Louis and New Orleans, a distance of 1,092 miles?

23. Light travels about 186,000 miles per second. A ray of light leaving the sun reaches the earth in 498 seconds. How many miles is the earth from the sun?

24. If the earth moves around the sun at the rate of 68,400 miles an hour, how far does it move in a month of 31 days?

25. In 1890 Louisiana reported 193,694 acres of cane, from which were made an average of 1,509 pounds of sugar and 74 gallons of molasses per acre. How many pounds of sugar and how many gallons of molasses did Louisiana produce?

26. In 1890 Ohio reported 794,833 milch cows. The average dairy products were about 412 gallons of milk, 95 pounds of butter, and 21 ounces of cheese per cow. How many gallons of milk, pounds of butter, and ounces of cheese were produced in Ohio?

27. In 1890 Pennsylvania had 39,339 manufacturing establishments, which averaged 33,854 dollars' worth of products. What was the entire value of the manufactured products of the State?

28. The population of the District of Columbia in 1890 was 230,392. The valuation of real and personal property per capita was $1,491. What was the whole valuation of the property of the District?

29. In a codfish 9,334,000 eggs have been found. If from each egg a fish was hatched, how many fishes were hatched from the eggs of 358 such codfish?

30. If the area of a State is 58,325 square miles, and the population averages 317 persons to each mile, what would be the total population?

EXERCISES.

65.

1.	475×218	*22.*	7243×3206
2.	647×423	*23.*	5307×4075
3.	596×354	*24.*	8429×6800
4.	768×538	*25.*	4736×5009
5.	960×647	*26.*	9007×6324
6.	872×478	*27.*	8063×3009
7.	693×580	*28.*	7005×6008
8.	429×382	*29.*	6493×2809
9.	724×674	*30.*	39648×7025
10.	529×407	*31.*	57326×3465
11.	940×709	*32.*	49003×6007
12.	842×689	*33.*	70058×34062
13.	4763×568	*34.*	92005×30076
14.	3845×396	*35.*	68304×40752
15.	5607×681	*36.*	85073×44263
16.	7317×407	*37.*	60084×80326
17.	2176×720	*38.*	76317×68019
18.	4058×2476	*39.*	675004×71005
19.	6819×4260	*40.*	806200×50708
20.	5912×3804	*41.*	739468×471689
21.	7091×5016	*42.*	940384×283014

DIVISION.

66. *1.* 4 is one factor of 8; what is the other?

2. 3 is one factor of 9; what is the other?

3. 2 is one factor of 12; what is the other?

4. 4 is one factor of 16; what is the other?

5. 6 is one factor of 30; what is the other?

6. 5 is one factor of 35; what is the other?

7. 8 is one factor of 48; what is the other?

8. 7 is one factor of 49; what is the other?

9. 10 is one factor of 40; what is the other?

10. 9 is one factor of 72; what is the other?

11. 12 is one factor of 108; what is the other?

12. 11 is one factor of 132; what is the other?

When the factors are given to find the product, what is the work called?

When the product and one factor are given to find the other factor, the work is called *Division.*

The Dividend is *the number to be divided* (or the Product).

The Divisor is *the given factor.*

The Quotient is *the required factor.*

The sign of division is ÷. It is read *divided by.*

Thus, $54 \div 6 = 9$ is read: 54 divided by 6 equal 9.

Division demands a knowledge of the multiplication table quite as much as multiplication itself; for every step in dividing requires that we find the other factor which, taken with the divisor, will produce the largest multiple in that part of the dividend with which we are dealing. Division is just as impossible without using the remembered results of the multiplication table as is multiplication.

DRILL EXERCISES.

67. Name the quotients of the following instantly:

$12 \div 3$	$36 \div 6$	$99 \div 11$	$20 \div 2$	$12 \div 12$
$16 \div 4$	$55 \div 5$	$64 \div 8$	$24 \div 8$	$88 \div 11$
$12 \div 6$	$24 \div 3$	$48 \div 4$	$11 \div 11$	$18 \div 3$
$25 \div 5$	$44 \div 4$	$30 \div 3$	$108 \div 12$	$24 \div 4$
$18 \div 9$	$100 \div 10$	$16 \div 4$	$90 \div 10$	$48 \div 6$
$40 \div 10$	$96 \div 8$	$44 \div 11$	$72 \div 9$	$120 \div 10$
$32 \div 8$	$28 \div 4$	$132 \div 12$	$99 \div 9$	$96 \div 12$
$54 \div 6$	$12 \div 2$	$9 \div 9$	$36 \div 3$	$40 \div 8$
$45 \div 9$	$36 \div 12$	$16 \div 8$	$40 \div 4$	$16 \div 2$
$60 \div 12$	$81 \div 9$	$21 \div 3$	$56 \div 8$	$21 \div 3$
$28 \div 7$	$108 \div 9$	$30 \div 6$	$60 \div 10$	$54 \div 9$
$36 \div 4$	$48 \div 8$	$63 \div 9$	$48 \div 12$	$121 \div 11$
$77 \div 11$	$60 \div 5$	$110 \div 10$	$90 \div 9$	$24 \div 12$
$72 \div 8$	$42 \div 7$	$66 \div 11$	$22 \div 2$	$84 \div 7$
$27 \div 3$	$30 \div 10$	$24 \div 6$	$20 \div 5$	$32 \div 4$
$35 \div 5$	$45 \div 5$	$88 \div 8$	$9 \div 3$	$15 \div 3$
$18 \div 2$	$132 \div 11$	$50 \div 5$	$10 \div 2$	$56 \div 7$
$70 \div 10$	$24 \div 2$	$72 \div 6$	$36 \div 9$	$72 \div 12$
$63 \div 7$	$70 \div 7$	$49 \div 7$	$40 \div 5$	$27 \div 9$
$84 \div 12$	$120 \div 12$	$20 \div 4$	$14 \div 7$	$110 \div 11$
$18 \div 6$	$8 \div 8$	$35 \div 7$	$42 \div 6$	$144 \div 12$

68. The divisor for each line of dividends in the following exercises is written at the left. Name the quotients rapidly:

1.	3)18	24	33	12	21	9	15	36	27	30
2.	4)16	28	12	36	24	48	20	32	44	40
3.	5)20	35	15	40	25	60	45	55	30	50
4.	6)24	48	18	36	54	72	60	42	30	66
5.	7)42	21	35	84	77	28	56	49	63	14
6.	8)32	48	64	56	96	24	40	72	80	16
7.	9)27	72	54	9	36	63	81	45	108	99
8.	12)36	60	108	48	120	84	144	96	132	

In the following exercises name the quotients and the remainders:

9.	3)10	22	16	13	25	32	37	20	29	34
10.	4)14	11	27	35	25	18	38	50	42	44
11.	5)12	23	31	27	45	38	62	56	43	54
12.	6)15	19	38	27	50	75	67	56	44	62

13. 7)22	30	18	50	65	75	84	38	46	59
14. 8)33	67	28	76	100	85	17	45	59	51
15. 9)38	50	109	97	32	25	59	66	74	89
16. 10)44	26	78	111	65	37	83	99	122	104
17. 11)57	34	90	112	48	72	61	84	105	132
18. 12)40	87	29	68	52	110	145	100	76	130

69. Division and Partition Defined.

1. A boy averages 6 miles an hour on his bicycle. How many hours will it take him to go 48 miles?

2. Henry paid 63 cents for berries at 9 cents a quart. How many quarts did he buy?

3. An orchard has 72 trees, and there are 8 trees in a row. How many rows are there?

4. A boy goes 48 miles in 8 hours on his bicycle. How many miles does he average per hour?

5. Henry paid 63 cents for 7 quarts of berries. How many cents a quart did he pay?

6. In an orchard are 72 trees in 9 equal rows. How many trees are there in a row?

From these problems it will be seen that either factor may be the given one. In the first three the given factor corresponds to the multiplicand in the

opposite process of multiplication; it is of the same kind as the dividend. The quotient is an "abstract" number showing how many "times" the dividend contains the divisor. In the last three problems the given factor corresponds to the multiplier; it is an "abstract" number, showing what *part* of the dividend is to be found. The quotient is of the same kind as the dividend of which it is a part. Hence, there are two distinct cases in division when the dividend is a concrete number. The first is division proper, or, Division *is finding how many times one number is contained in another.*

The second case is sometimes called *Partition.*

Partition is finding one of the parts when a given number is separated into a given number of equal parts.

If both given numbers are abstract, the problem may be considered as one of division proper, or of partition, indifferently; but an explanation of the process should correspond to one or the other supposition.

The remainder is always of the same kind as the dividend.

ORAL EXERCISES.

70. Tell whether the following problems are division proper or partition:

1. At 6 cents a quart, how many quarts of milk will 42 cents buy?

SOLUTION: At 6 cents a quart, 42 cents will buy as many quarts of milk as there are times 6 cents in 42 cents, which is 7 times; hence, 7 quarts can be bought.

2. Hugh paid 35 cents for 5 cakes of maple sugar. What was the price per cake?

SOLUTION: If 5 cakes cost 35 cents, 1 cake costs one fifth of 35 cents, or 7 cents.

3. Lucy's hens lay 54 eggs a week, an average of 6 eggs for each hen. How many hens has Lucy?

4. In a class-room are 45 desks, and 9 desks in a row. How many rows are there?

5. Harry's grandfather, who is 64 years old, is 8 times as old as Harry. How old is Harry?

6. At the rate of 12 miles an hour, how many hours will a freight train require to run 36 miles?

7. If a man has 48 horses, how many 6-horse teams can he form with them?

8. The dividend is 63 and the divisor is 7. What is the quotient?

9. What number multiplied by 5 will give 40 for a product? What number will give 60?

10. At 10 cents a pound, how many pounds of fish can be bought for 90 cents?

11. During his vacation a boy worked 9 weeks and received $36. At the same rate what would he have received for 1 week?

12. A blacksmith shod 66 horses in 6 days. How many did he average a day?

13. If 1 bottle of ink costs 8 cents, how many bottles can be bought with 56 cents?

14. When eggs are selling at 12 cents a dozen, how many dozens will 60 cents buy?

15. If a man can dig 84 feet of trench in 7 days, how many feet can he dig in 1 day?

16. If 9 persons ride in a coach, how many coaches will be required to carry 72 persons?

17. How many weeks are there in 49 days?

18. A man paid $72 for 12 head of sheep. What was the price per head?

19. A man had 96 chestnuts in a bag. He divided them equally among 8 school children. How many chestnuts did each child receive?

20. How many weeks will it take a man to earn $81 if he earns $9 a week?

21. How many feet are there in 120 inches?

22. A farmer gathered 99 bushels of apples from 9 trees. What was the average number of bushels gathered from each of the trees?

23. 8 nails are required to fasten a horseshoe. How many horseshoes will 80 nails fasten?

24. If a horse eats 88 quarts of oats in 11 days, how many quarts does he average a day?

25. In 12 hours 108 horses crossed a bridge. What was the average number an hour?

26. Ellen paid 96 cents for ribbon at 12 cents a yard. How many yards did she buy?

27. Mr. Owen paid 144 cents for an advertisement of 12 lines. What was the rate per line?

28. If coal costs $7 per ton, how many tons can I buy for $84?

WRITTEN EXERCISES.

71. *1.* Divide 9,832 by 4.

```
4)9832(2458
  8
  18
  16
  23
  20
  32
  32
```

EXPLANATION: Write the divisor at the left of the dividend, with a curved line between, as in the margin. Four is found in nine two times; write 2 in the quotient, at the right of the dividend, separated by a curved line. Write two times four under the 9 and subtract, writing the remainder 1 below. This 9 that we have used, however, is 1,000 times nine, hence the 2 in the quotient is 1,000 times two, and the figure 2 must be regarded as standing in the thousands' place. The remainder also is 1,000, or 10 hundreds, which, with the 8 hundreds, gives 18 hundreds for a new dividend. Four is found in 18 four times; but as this 18 expresses hundreds, we write the 4 in hundreds' place in the quotient. Write four times four under the 18 and subtract, writing the remainder 2 below. This remainder is 200, or 20 tens, which, with the 3 tens, make 23 tens for a new dividend. Four is found in 23 five times, but as the 23 expresses tens, we write the 5 in tens' place in the quotient. Write five times four under the 23 and subtract. The remainder is 3 tens, or 30 ones, which, with the 2 ones, make 32 for a new dividend. Four is found in 32 eight times. Write 8 in ones' place in the quotient, multiply and subtract as before, and nothing remains. Hence, the quotient is 2,458.

This explanation proceeds on the supposition that we are to find how many times 4 in 9,832, or that it is a case of division proper. If we consider the problem to be to find one fourth of 9,832, the explanation would be as follows:

One fourth of 9 thousands is 2 thousands; we write the quotient 2 in thousands' place, and write 4 times 2 thousand under the 9 thousands. Subtracting, we have 1 thousand remaining, which, with the 8 hundreds, makes 18 hundreds. One fourth of 18 hundreds is 4 hundreds, and so we write 4 in hundreds' place in the quotient. Four times 4 hundreds is 16 hundreds, which we subtract from the 18 hundreds, leaving 2 hundreds, or 20 tens. These, with the 3 tens, make 23 tens for a new dividend. One fourth of 23 tens is 5 tens, so we write 5 in tens' place in the quo-

tient. Four times 5 tens is 20 tens. Subtracting this, we have 3 remaining, equal to 32 units with the 2 units added. One fourth of 32 units is 8 units. We write 8 in units' place, and take 4 times 8 from 32, leaving no remainder. Hence, one fourth of 9,832 is 2,458.

4)9832
2458

The process written out in full, as we have now done, is called Long Division. But when the divisor is small, the writing of the partial products and the successive remainders may be omitted and the process shortened. In such a case it is more convenient to write the quotient under the dividend, as above. Thus, the first quotient figure 2 is written in thousands' place as before; the 1 thousand remaining is joined, in the mind, to the 8 hundreds, making 18 hundreds, and the new quotient figure 4 is found and written in hundreds' place. The new product 16 is subtracted, in the mind, from the 18, and the remaining 2 hundreds is joined to the 3 tens, making 23 tens for a new dividend. The new quotient figure 5 is found and written in the proper place. The new product 20 is subtracted, in the mind, from 23, giving a remainder of 3 tens, which is joined to the 2 ones, making 32 ones. The last quotient figure is found to be 8, with no remainder, and is written in the proper place.

This process is called Short Division. It differs from Long Division in no respect except that the multiplications and subtractions are performed without the aid of figures.

The factors are 7 and 9. What is the product?

The product is 63, and one of the factors is 7. What is the other factor?

The product is 63, and one of the factors is 9. What is the other factor?

We can prove the work in multiplication by dividing the product by either factor. If the result is the other factor, the multiplication is correct.

To prove work in division, find the product of the divisor and the quotient. If their product gives the dividend, the work is correct. Should there be a final remainder, it must be added to this product.

(It is important to impress upon the pupil the relation of multiplication and division. In multiplication the factors are given to find the product; in division the product and one factor are given to find the other factor.)

ORAL EXERCISES.

72. *1.* When butter is selling at 25 cents a pound, how many pounds can be bought for 50 cents?

2. A woman bought a sewing machine for $56, and paid for it at the rate of $4 a month. How many months did it take her to pay for it?

3. When melons are 15 cents each, how many melons will 45 cents buy?

4. There are 16 ounces in a pound. How many pounds are there in 48 ounces?

5. If 20 pounds of corn-meal cost 60 cents, what is the price per pound?

6. How many weeks will it take a boy to save $75, if he saves $5 a week?

7. What number multiplied by 3 will give 39 for a product?

8. If maple syrup costs 20 cents a pint, how many pints will 80 cents buy?

9. I paid 70 cents for 2 pounds of coffee. What was the price per pound?

10. A conductor charged me 63 cents for a ride of 21 miles. What was the rate per mile?

11. There are 24 hours in a day. How many days are there in 72 hours?

12. A farmer has 96 acres in 4 fields of equal size. How many acres are there in each field?

13. A lady paid 90 cents for dolls at 30 cents apiece. How many dolls did she buy?

14. Ellen's grandmother is 78 years old, which is 6 times Ellen's age. How old is Ellen?

15. What number multiplied by 4 will give 64 for a product? Multiplied by 8?

16. At 5 cents a pound, how many pounds of sugar can be bought for 1 dollar?

17. A man received $75 for 25 days' work. How many dollars did he receive a day?

18. Four pecks make a bushel. How many bushels are there in 92 pecks?

19. A man paid $90 for cows at $45 apiece. How many cows did he buy?

20. A grocer received 120 cents for 4 pounds of butter. What was the price per pound?

21. A blacksmith uses 8 nails to fasten one shoe. In one day he used 128 nails. How many hoofs did he shoe? How many horses?

22. After school Frank gets 9 cents an hour for raking leaves in the park. How many hours will it take him to earn 162 cents?

23. If 7 acres of land which I purchased cost $280, what was the price per acre?

24. An express train made the distance of 270 miles in 6 hours. What was the average rate per hour?

25. A woman paid 150 cents for gingham at 25 cents a yard. How many yards did she buy?

26. A grocer paid $192 for flour at $8 a barrel. How many barrels did he buy?

27. I paid 285 cents for a railroad ticket at 3 cents a mile. How many miles did I ride?

28. What number multiplied by 9 will give 216 for a product?

29. If the average expense in visiting the World's Fair at Chicago was $7 a day, how many days could a person visit it for $154?

WRITTEN EXERCISES.

73. *30.* If it takes 5 bushels of wheat to make a barrel of flour, how many barrels will 94,735 bushels make? (By short division.)

31. The circumference of a wheel of a bicycle is 7 feet. How many revolutions will it make in going 18,480 feet (3½ miles)?

32. A railroad worth $346,275 is owned in equal shares by 9 persons. What is each person's share?

33. A county contains 184,320 acres of land lying in 8 equal townships. How many acres are there in each township?

34. A father divided $40,500 equally among his 6 children. How many dollars did each receive?

35. In 1890 New Hampshire had 12 establishments for the manufacture of paper. They paid their employes $233,268. What was the average number of dollars paid by each establishment?

36. A bushel of corn weighs 56 pounds. How many bushels are there in 36,400 pounds? (Long division.)

37. A man bought land at $87 an acre, paying $31,755 for it. How many acres did he buy?

38. The dividend is 38,157, and the divisor is 79. What is the quotient?

39. The quotient is 98, and the dividend is 9,604. What is the divisor?

40. 27 times a certain number is 12,636. What is the number?

41. How many farms of 156 acres each can be sold from a tract of land containing 5,928 acres?

42. 196 pounds of flour make a barrel. How many barrels will 17,640 pounds make?

43. There are 231 cubic inches in a gallon. How many gallons of vinegar does a tank containing 1,116,885 cubic inches hold?

44. A railroad company pays $154,504 for constructing 89 cars. What was the price per car?

45. The annual receipts of a manufacturing company were $230,055. What were the average daily receipts for the 313 working days?

46. A farm of 736 acres was sold for $55,936. What was the price received per acre?

47. If a garrison consume 674 pounds of bread in a day, how many days will 246,010 pounds feed them?

48. Mount Everest is 29,062 feet high. How many miles high is it, and how many feet over, there being 5,280 feet in a mile?

49. The area of Massachusetts is 8,315 square miles, and in 1890 its population was 2,238,943. How many persons were there to the square mile, and how many persons over?

50. If 768 shares of railroad stock are valued at $75,264, what is the value of each share?

51. Alaska contains 531,420 square miles, and New Jersey, 7,815 square miles. How many States the size of New Jersey might be made out of Alaska?

52. In 1890 there were in Connecticut, 26,350 farms, containing a total of 2,253,432 acres. What was the average number of acres in a farm, and how many acres over?

53. The earth's orbit is 580,000,000 miles in length. The earth travels over it in about 365 days. How many miles does it travel each day?

54. How many schooners, each carrying 7,900 bushels of wheat, will be required to carry 679,400 bushels?

55. In 1890 Nevada reported 9,273 milch cows, 2,532,052 gallons of milk, and 489,657 pounds of butter. What was the average number of gallons of milk per cow, and how many gallons over? What was the average number of pounds of butter per cow, and how many pounds over?

56. In 1890 Louisiana produced 75,645,433 pounds of rice on 84,377 acres. What was the average number of pounds produced per acre?

57. In 1889 the number of persons employed in the fisheries in the United States was 163,348. The value of the products was $44,277,514. What was the average value of the fish taken by each person, and how many dollars over?

EXERCISES.

74. Divide by short division, and prove the work in each case:

1. 735 by 3
2. 952 by 4
3. 768 by 6
4. 992 by 8
5. 3365 by 5
6. 6380 by 10
7. 7506 by 9
8. 2989 by 7
9. 4592 by 8
10. 4807 by 11
11. 2964 by 12
12. 12414 by 3
13. 67428 by 4
14. 19376 by 7
15. 32795 by 5
16. 43281 by 9
17. 38496 by 8
18. 74856 by 12
19. 57893 by 11
20. 41690 by 10
21. 374864 by 4
22. 629274 by 3
23. 947860 by 5
24. 158910 by 6
25. 647269 by 7
26. 598472 by 8
27. 960732 by 9
28. 103961 by 11
29. 113820 by 12
30. 716841 by 9

75. What are the quotients and remainders of:

1. 1768÷13
2. 1984÷14
3. 2147÷15
4. 2672÷16
5. 2928÷17
6. 4147÷18
7. 8797÷19
8. 6872÷24
9. 8519÷27
10. 9672÷31
11. 28490÷35
12. 18476÷42
13. 59314÷47
14. 39176÷54
15. 93658÷59
16. 47376÷63
17. 39587÷68
18. 62852÷76
19. 29072÷79
20. 71060÷85
21. 31592÷88
22. 47752÷94
23. 78473÷97
24. 35084÷49

25. 34506÷71	*34.* 228169÷392	*43.* 2159450÷2465
26. 240128÷64	*35.* 503652÷564	*44.* 3558324÷3851
27. 326112÷86	*36.* 467606÷614	*45.* 5047923÷5631
28. 697756÷38	*37.* 705776÷728	*46.* 8013798÷8934
29. 269982÷53	*38.* 508662÷693	*47.* 4868424÷7128
30. 525374÷82	*39.* 835716÷754	*48.* 8473069÷9506
31. 644220÷90	*40.* 892696÷839	*49.* 6973056÷6945
32. 351288÷136	*41.* 835644÷874	*50.* 9711873÷4839
33. 315008÷428	*42.* 902260÷916	*51.* 7540512÷9618

Prove your answers.

76. Divide:

1. 732 by 4	*12.* 4807 by 11	*23.* 383094 by 6
2. 973 by 7	*13.* 22148 by 7	*24.* 408079 by 7
3. 2841 by 3	*14.* 22836 by 4	*25.* 653832 by 9
4. 865 by 5	*15.* 43134 by 6	*26.* 214345 by 5
5. 1233 by 9	*16.* 51024 by 8	*27.* 668768 by 8
6. 1962 by 6	*17.* 32184 by 9	*28.* 378490 by 10
7. 1460 by 10	*18.* 27490 by 5	*29.* 138780 by 4
8. 3808 by 8	*19.* 63492 by 11	*30.* 742918 by 11
9. 2536 by 4	*20.* 11428 by 12	*31.* 765096 by 12
10. 3017 by 7	*21.* 58625 by 7	*32.* 548338 by 7
11. 2964 by 12	*22.* 31482 by 9	*33.* 838323 by 9

Prove each answer.

PROMISCUOUS PROBLEMS.

77. *1.* A man gave his older daughter $875, his younger daughter $185 less, and his son $765 more than he gave to both daughters. How many dollars did he give to all?

2. A farmer raised 365 bushels of wheat one year, twice as many bushels the next year, and the third year three times as many bushels as in both the two preceding years. How many bushels did he raise in three years?

3. A wholesale merchant sold a stock of goods to a country dealer for $8,435 and lost $1,956 by the transaction. How many dollars would he have gained if he had sold it for $12,317?

4. A farmer sold 16 loads of potatoes, each load containing 48 bushels. If he has 257 bushels left, how many bushels did he raise?

5. Multiply the sum of 276 and 347 by three times their difference?

6. A speculator bought 276 acres of wild land at $7 an acre, paid $157 in taxes and then sold it at $9 an acre. How many dollars did he make by the transaction? (How many dollars on each acre would he have gained had there been no expense?)

7. Two men are 967 miles apart. If they travel toward each other, one at the rate of 48 miles a day and the other at the rate of 57 miles a day, how many miles apart will they be at the end of 6 days?

8. A man sold a mill for $17,360 and lost $1,675. How many dollars would he have received for it if he had gained $1,675?

9. If the dividend is $5,200, and the quotient $89 and the remainder $38, what is the divisor?

10. The difference between two numbers is 7,569,483, and the subtrahend is 379,468. Find the minuend.

11. The smaller of two numbers is contained 18 times in 1,566, and the larger number is 69 times the smaller. Find the larger number.

12. A speculator sold 76 head of cattle for $2,888, and lost $267. At what price per head should he have sold them to gain $265?

13. If 45 acres produce 2,520 bushels of corn, how many bushels does 28 acres produce at the same rate?

14. At a certain election the whole number of votes cast for two opposing candidates was 12,672. The successful candidate received 216 majority. How many votes did each receive?

15. A merchant buys 168 barrels of flour for $672. How many barrels more can he buy for $984 at the same price?

16. A man bought 13 horses at $92 a head, and 15 at $84 a head. He sold them all at $90 a head. How many dollars did he gain?

17. A man sold land at $46 an acre, receiving $4,508 for it. How much would he have received for it if he had sold it at $58 an acre?

18. The divisor is 88, the quotient $248, and the remainder $66. What is the dividend?

19. The telegraph poles along a railroad are set 264 feet apart. How many poles will be required for a line from Chicago to Bloomington, a distance of 126 miles, there being 5,280 feet in a mile?

20. A park is 48 rods long and 32 rods wide. How many times must a boy go around it on his bicycle to travel 45 miles, there being 320 rods in a mile? How many times must he go around the park to travel 1 mile?

21. A garrison of 10,000 men have provisions to last 6 weeks. If 2,500 men are killed in a battle, how many weeks will the provisions last the remainder?

22. The product of two numbers is 40,796. One of the numbers is 124. What is the other?

23. Mr. Wilson sold a quantity of wood for $2,492, that cost $1,424, thus gaining $3 a cord. How many cords were there, and what was the cost per cord?

24. The circumference of each fore wheel of a carriage is 8 feet, and of each hind wheel 12 feet. How many more revolutions will the fore wheels make than the hind wheels in going 38 miles? (See problem 19.)

25. The year 1896 being leap year, contained 366 days. How many hours did it contain? If I had read 2 hours each day, at the rate of 16 pages an hour, how many books of 248 pages each could I have read during the year, and how many pages over?

FACTORS AND FACTORING.

78. Prime and Composite Numbers.

1. What two factors make 22, 25, 36, 49, 56?

2. What number do the factors 2 and 2 and 2 make? 2 and 2 and 3? 2 and 3 and 3? 2 and 2 and 5? 2 and 2 and 2 and 3? 3 and 3 and 3? 2 and 3 and 5? 2 and 2 and 3 and 3?

3. What two factors make 3, 5, 7, 11, 13?

A Prime number is *one that can not be made by the multiplication of any other integral number than 1,* as 5, 7, 13, 19, 23, etc.

A Composite number is *one that can be made by multiplying some other integral number than 1,* as 6, 9, 10, 14, 15, etc.

Numbers which have no common integral factor except 1 are *prime to each other,* as 8 and 9.

An even number is a number that contains 2 as a factor, as 4, 6, 8, etc.

An odd number is one that does not contain 2 as a factor, as 3, 5, 7, etc.

79. Principles.

I. *A factor of any number is also a factor of any multiple of that number.*

PROOF: If a number contains a factor, it is some number of times that factor. Therefore, to make a multiple of that number we take a certain number of times that factor a certain number of times. Hence, the multiple must be a certain number of times that factor.

ILLUSTRATION: 6 is 3 twos; hence, any multiple of 6 must be some number of times 3 twos, or it must contain 2 as one of its factors.

II. *A common factor of any two numbers must be a factor of their sum or their difference.*

PROOF: If two numbers have a common factor, each of them must be some number of times that factor. Hence, to make their sum, we put a number of times the common factor with some number of times the same factor. Therefore, the result must be some number of times that factor. Or, to find their difference, we take a number of times the common factor from some number of times the same factor. Therefore, the difference must be a certain number of times that factor.

ILLUSTRATION: 15 and 21 have the common factor 3; 15 is 5 threes, and 21 is 7 threes. To make their sum we put 5 threes and 7 threes together, which gives us 12 threes. To find their difference we take 5 threes from 7 threes; the result must be 2 threes.

NOTE: In order to have a clear knowledge of numbers, it is necessary that these two principles should be thoroughly understood and appreciated.

ORAL EXERCISES.

80. *1.* What are all the even numbers from 10 to 45?

2. What are all the odd numbers from 10 to 45?

3. What are all the prime numbers from 10 to 45?

4. What are all the composite numbers from 10 to 45?

5. What two prime factors make 21, 25, 35, 49, 55?

6. What two composite factors make 16, 24, 36, 54, 81?

7. What are all the prime factors of 18, 20, 24, 30?

8. Name two numbers that are prime to 6; to 8; to 9; to 11.

9. What prime factor is common to 14 and 16? To 15 and 18? To 21 and 28? To 25 and 30? To 33 and 55? To 7 and 13?

10. What prime factors are common to 6 and to 5 times 6? Why is it so? To 9 and to 2 times 9? To 10 and to 7 times 10? To 14 and to 5 times 14?

11. What factor is common to 8 and 10 and to their sum? Why is it so? To 6 and 9 and to their sum? To 10 and 15 and to their sum? To 21 and 35 and to their sum?

12. What factor is common to 9 and 15 and to their difference? Why is it so? To 10 and 25 and to their difference? To 14 and 35 and to their difference? To 16 and 44 and to their difference?

81. Tests of Divisibility.

There are several very practical tests that can be applied to numbers in order to determine whether or not they contain certain factors.

1. If the units of a number contain the factor 2, the whole number contains that factor.

2. The test for 5 is the same as the test for 2.

3. If the sum of the shape-values of the figures expressing any number contains the factor 9, the number expressed contains that factor. Thus, 473,967 is divisible by 9, because the sum of 4, 7, 3, 9, 6, and 7 is 36, which is a multiple of 9.

4. The test for 3 is the same as the test for 9.

5. If, in the expression of any number, the sum of the shape-values of the figures in the odd places differs from the sum of the shape-values of the figures in the even places by nothing, or by a multiple of 11, the whole number is a multiple of 11.

In the number 926,580,413 the odd orders are occupied by the figures 9, 6, 8, 4, 3, the sum of whose shape-values is 30. The even orders are occupied by the figures 2, 5, 0, 1, the sum of whose shape-values is 8. The difference of these sums is 22, which is a multiple of 11. Hence, 926,580,413 is a multiple of 11.

6. If the two right-hand figures in the expression of any number express a multiple of 4, the whole number is a multiple of 4.

Thus, 57,376 is a multiple of 4, because 76 is a multiple of 4.

7. The test for 25 is the same as that for 4.

8. The hundreds of any number are divisible by 25 and by 4.

9. If, in the expression of any number, the three right-hand figures express a multiple of 8, the whole number is a multiple of 8.

Thus, 57,192 is a multiple of 8, because 192 is a multiple of 8.

10. The thousands of any number are divisible by 8.

11. Any *even* number that is a multiple of an odd number is a multiple of twice that odd number. The even number 3,582 is a multiple of 3. Hence, it is a multiple of 6, or two times 3. Why must this be so?

EXERCISES.

82. Find, according to the preceding tests, whether or not the following numbers are multiples of 2, 3, 4, 5, 6, 8, 9, or 11:

1.	106524	*6.*	868464	*11.*	816345
2.	131238	*7.*	345015	*12.*	597384
3.	212520	*8.*	756249	*13.*	748440
4.	736560	*9.*	452848	*14.*	349272
5.	462088	*10.*	963471	*15.*	698544

83. Resolving Numbers into their Prime Factors.

1. What are all the prime factors of 4? 6? 7? 8? 9? 10? 11? 12? 13? 14? 15?

2. Write the prime factors of the numbers from 16 to 50 after the following form: $16=2\times2\times2\times2$.

3. Find the prime factors of 397,089.

$397089=3\times3\times3\times7\times11\times191=3^3\times7\times11\times191.$

44121
14707
2101
191

EXPLANATION: Write the number to be resolved with the sign of equality at the right. The test shows that the number does not contain the factor 2. It

does contain 9, or two factors 3. Write two 3's after the sign of equality, with the sign of multiplication between them. Take out the factor 9 from the number and 44,121 is left. The test shows that there is one 3 more, but not two. Take it out and write it after the other factors. 14,707 remains. The test shows that 5 is not a factor. By actual trial it is found that 14,707 contains the factor 7. Take it out and write it with the other factors. 2,101 remains. This does not contain another 7. The test shows the presence of 11 as a factor. Take it out and write it with the other factors. 191 is left. This does not contain another 11, nor does it contain 13, as is found by trial. The square of 17, the next prime number, is 289, which is a greater number than 191. Hence, 191 is prime. Write it with the other factors. Hence, the prime factors of 397,089 are 3^3, 7, 11, 191.

84. Rule for Resolving Numbers into their Prime Factors.

1. Write the number to be resolved with the sign of equality at the right of it; test the presence of every prime factor in order, taking out such as are successively found, and writing them as factors at the right of the sign of equality; in each case write the remaining factor under the number; when the last remaining factor so written is found to be a prime number, then write it with the other factors.

Do not cease testing for any factor till *all* are taken out of the number.

If more than one factor of any kind is found, the number of such factors may be indicated by a small figure at the right, and just above the factor, as 3^3 in the second writing of the factors in the preceding problem; it means three factors 3. Such a little figure is called an *Exponent.*

2. When we have tried unsuccessfully to factor a number by all the prime numbers in order until the square of the next one will exceed the number we are testing, we may be sure that that number is prime.

EXERCISES.

85. *1.* Learn the prime numbers from 1 to 100.

2. Write the prime factors of the other numbers from 51 to 100.

3. Remember the squares of all the numbers from 13 to 25, inclusive, $13^2=169$, etc.

4. Remember all cubes as far as 9^3, at least.

Find the prime factors of the following:

5.	*6.*	*7.*	*8.*
102	221	715	11165
105	242	845	12496
108	277	931	17199
120	298	1034	34587
126	331	1146	40579
128	343	1209	68068
143	363	1290	79576
150	441	1326	81796
162	449	4818	100000
197	570	7854	176400
200	625	9139	245025
217	700	9537	797433

LEAST COMMON MULTIPLE.

86. *1.* Of what numbers is 15 a multiple? 18? 20? 24? 30?

2. Any number is a multiple of any one of its factors.

3. Name two multiples of both 3 and 4; 3 and 5; 6 and 9; 3, 4, and 6; 2, 6, and 9; 5, 6, and 10; 4, 7, and 14. Are these all the multiples they can have?

4. A common multiple of several numbers is *a number which will contain each of them.*

5. What is the least number that is a multiple of 3 and 4? Of 3 and 5? Of 6 and 9? Of 3, 5, and 10? Of 6, 9, and 18? What is such a multiple called?

6. The least common multiple of several numbers is *the least number that will contain each of them.*

7. Name two multiples of 6 and 10. Have 6 and 10 any prime factors not found in these multiples? Name two multiples of 4, 6, and 9. Have they any prime factors not found in the multiples?

8. A common multiple of several numbers *contains all the prime factors found in each of them.*

9. What is the least common multiple of 6 and 10? Has it any prime factors not found in 6 and 10? What is the least common multiple of 6, 10, and 9? Has it any prime factors not found in 6, 10, and 9?

10. The least common multiple of several numbers *contains no prime factors except those found in the numbers.*

11. Find the least common multiple (L. C. M.) of 50, 40, 36, and 25.

$50 = 2 \times 5^2$
$40 = 2^3 \times 5$
$36 = 2^2 \times 3^2$
25

L. C. M. $= 2 \times 5^2 \times 2^2 \times 3^2 = 1800$

EXPLANATION: Resolve the numbers 50, 40, and 36 into their prime factors. Do not resolve 25 into its prime factors; as it is a factor of 50, it will be a factor of any multiple of 50, by Principle I. Write L. C. M., placing after it the sign of

equality. The L. C. M. must be a multiple of 50; hence, it must contain the prime factors of 50, which are 2 and two 5's. Write these factors after the sign of equality. The L. C. M. must be a multiple of 40 also. It must, therefore, contain the prime factors of 40, which are three 2's and 5. The 5 and one 2 have already been written as factors of the L. C. M., so write two 2's more as factors of the L. C. M. The L. C. M. must be a multiple of 36 also. Therefore, it must contain the prime factors of 36, which are two 2's and two 3's. Two 2's are already among the factors of the L. C. M.; hence, write only the two 3's as its factors. There is now indicated a number which will contain 50, 40, or 36. Because it contains the prime factors of these numbers, it is their common multiple; and because it contains no prime factors but those found in the numbers, it is their L. C. M.

The product of these factors is 1,800, the required L. C. M. (In finding this product, observe that $2^3 \times 5^2 = 100$.)

87. Rule for Finding the Least Common Multiple of Several Numbers.

Factor the several numbers; then, to make their least common multiple, take all the prime factors found in each of them; multiply together these prime factors only.

EXERCISES.

88. Find the L. C. M. of the following:

1.	28, 21, 14.	*7.*	60, 48, 36.
2.	30, 24, 18.	*8.*	78, 66, 63.
3.	36, 15, 9.	*9.*	135, 81, 45.
4.	54, 27, 18.	*10.*	110, 88, 66.
5.	48, 42, 36.	*11.*	49, 21, 12, 7.
6.	56, 33, 21.	*12.*	98, 28, 14, 4.

13. 60, 45, 30, 15.

14. 65, 39, 26, 13.

15. 40, 36, 27, 18.

16. 180, 54, 42, 24.

17. 144, 100, 96, 72.

18. 216, 96, 84, 64.

19. 49, 42, 40, 35, 28.

20. 66, 55, 44, 33, 22.

21. 80, 66, 48, 32, 16.

22. 140, 135, 70, 35, 15.

23. 280, 154, 140, 126, 98.

24. 128, 96, 64, 14, 9, 5.

25. What is the least number of apples that can be equally distributed among 8, 12, 16, or 20 boys?

26. What is the least sum of money that can be expended for sheep, cows, or horses at $9, $39, and $65 each, respectively?

27. What is the shortest piece of cord that can be cut exactly into pieces either 10, 12, 15, 16, or 18 feet long?

28. What are the fewest words that will make an integral number of lines across the page, whether the lines average 8, 9, or 10 words? How many lines would there be in each case?

GREATEST COMMON DIVISOR.

89. *1.* Name all the divisors (factors) of 6, 8, 9, 10, 12, 14, 15, 16, 20, 24, 25.

2. Name a number that is a divisor of 15 and 35; 28 and 49; 36 and 63; 33 and 88; 9, 15, and 21; 14, 21, and 35; 15, 20, and 25. What is such a number called?

3. A common divisor of several numbers is *a number that is a divisor of each of them.*

4. Name all the common divisors of 12 and 18; 10 and 20; 15 and 30; 24 and 36; 6, 12, and 18; 16, 24, and 32; 18, 27, and 36; 24, 36, and 48. Which is the greatest divisor in each case? What is such a number called?

5. The greatest common divisor of several numbers is *the greatest number that is a divisor of each of them.*

6. Name all the prime factors of 42. What is the product of any two of them? Is this product a divisor of 42?

7. Name all the prime factors of 60. What is the product of any three of them? Is this product a divisor of 60?

8. Any number is *a multiple of the product of any of its prime factors.*

9. Name all the common prime factors of 24 and 36. What is the product of any two of them? Is this product a divisor of 24 and 36?

10. Name all the common prime factors of 30 and 60. What is the product of any two of them? Is this product a divisor of 30 and 60?

11. A common divisor of any two numbers is *one of their common prime factors or a product of two or more of them.*

12. What is the product of all the common prime factors of 24 and 36? Is this product a divisor of 24 and 36? Have they a larger common divisor?

13. What is the product of all the common prime factors of 30 and 60? Is this product a divisor of 30 and 60? Have they a larger common divisor?

14. The greatest common divisor of several numbers *is the product of all the prime factors common to those numbers.*

15. Find the greatest common divisor (G. C. D.) of 140, 112, and 98.

$140=2^2\times5\times7$
$112=2^4\times7$
$98=2\times7^2$

G. C. D. $=2\times7=14$

EXPLANATION: Resolve the numbers into their prime factors. Write G. C. D. with the sign of equality after it. After the sign, write the prime factors common to those numbers. They are 2 and 7, and their product 14 is the greatest common divisor.

EXERCISES.

90. Find the G. C. D. of:

1. 24, 28, 40.
2. 25, 45, 60.
3. 36, 27, 54.
4. 72, 42, 30.
5. 32, 88, 56.
6. 49, 35, 63.
7. 33, 55, 99.
8. 60, 84, 108.
9. 28, 42, 70.
10. 21, 42, 63.
11. 40, 75, 100.
12. 36, 81, 135.
13. 72, 126, 216.
14. 234, 78, 468.
15. 315, 810.
16. 270, 360.
17. 216, 408.
18. 399, 266.
19. 462, 770.
20. 663, 442.
21. 1050, 495.
22. 1210, 726.
23. 114, 190, 266.
24. 186, 124, 248.
25. 216, 105, 405.
26. 320, 432, 368.
27. 294, 252, 210.
28. 290, 435, 232.

29.	648, 216, 144.	*35.*	80, 200, 120, 160.
30.	500, 225, 75.	*36.*	225, 375, 150, 450.
31.	210, 84, 126.	*37.*	198, 165, 231, 132.
32.	529, 782, 1127.	*38.*	208, 156, 130, 182.
33.	66, 78, 102, 114.	*39.*	399, 627, 513, 342.
34.	49, 98, 147, 196.	*40.*	1242, 690, 966, 414.

41. A fruit-grower has 105 trees of one kind, 90 of another kind, 60 of another kind, and 45 of another kind. He wants to plant them at an equal distance apart, in rows of equal length. What is the largest number he can put in a row? How many rows will there be in each case?

42. A man has one lot 96 feet long and 48 feet wide, and another lot 144 feet long and 64 feet wide. He wishes to inclose them with a fence of boards of equal length, and the longest that can be used without cutting. What must be the length of the boards? How many boards will it take in each case if he makes the fence 5 boards high?

43. A farmer has 231 bushels of rye, 273 bushels of wheat, and 315 bushels of oats which he wishes to put into the least number of bins containing the same number of bushels without mixing the grain. How many bushels must each bin hold? How many bins must there be?

44. Three men having \$434, \$465, and \$620, respectively, bought cows at the highest price per head that allowed each man to invest all his money. How many dollars did they pay per head? How many cows did each buy?

FRACTIONS.

91. *1.* What is a primary unit? What is a relative unit? (Review Sections 2, 3, and 4, pp. 9 and 10.)

2. Write a figure 1 to stand for a pint. Draw a short horizontal line beneath it. Under the line write the figure which tells how many pints make a quart. How many ones does the writing stand for? Is the 1 a primary or a relative unit? How many of this kind make a larger one of another kind? What is the primary unit to which the 1 is related? What part of a quart is the 1 pint?

3. Write a figure 2 to stand for feet. Beneath it, with a line between, write the figure which tells how many feet make a yard. How many ones does this writing stand for? Are they relative or primary ones? How many of this kind make a larger one of another kind? What is the primary unit to which they are related? What part of a yard are the 2 feet?

4. Fold a sheet of paper into two equal parts. Now fold each part into two equal parts. Write a 3 to stand for three of the equal parts. In the proper place write the figure which tells into how many equal parts the whole sheet is folded. How many ones have you expressed by the figures? What kind? What is the primary unit? What part of the whole sheet are the 3 parts expressed?

5. Write a 7 to stand for dimes, and beneath it write the figures which tell how many dimes make a dollar. What is the primary unit? How many relative units? What figure tells you? What part of the primary unit have you expressed? How do you know?

6. Write 15 to stand for hours, and in the proper place write the figures which tell how many hours make a day. How many units have you expressed? What kind? What is the primary unit? What part of the primary unit does the writing express?

7. What are such expressions as these called? (Fractions.)

8. A written fraction is *an expression for one or more relative units smaller than some primary unit to which they are related.*

9. The units of a fraction may be called *fractional* units because they may be considered as parts of their related primary unit; but, as *units*, they are just as whole and complete as any others.

NOTE: A number composed of fractional units does not differ in any essential particular from the same number composed of any other kind of units. For instance, seven is the same number, possessing the same properties and subject to the same uses, whether it be seven apples, seven hundreds, seven thirds, or seven tenths.

10. In the fractions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{10}$, and $\frac{15}{24}$, what are the figures which tell the number of units in each case? What are they called? What are the figures which tell how many of the relative units in each case make the primary unit? What are they called?

11. The numerator (numberer) is *the figure or figures which tell the number of fractional units.*

12. The denominator (namer) is *the figure or figures which tell how many relative units of this kind are required to make the primary unit to which they are related.*

13. The numerator and denominator are *the terms of the fraction.*

14. A fractional expression may also signify an *unexecuted division;* the numerator expresses the dividend, and the denominator expresses the divisor. Thus, $\frac{9}{3}=9\div3$, or $\frac{1}{3}$ of 9; $\frac{19}{6}=19\div6$, or $\frac{1}{6}$ of 19, etc.

EXERCISES.

92. Read the following fractions. In each case tell:
First. How many relative units are expressed.
Second. How many of these relative units make the primary unit.

$\frac{3}{7}$ $\frac{4}{10}$ $\frac{7}{8}$ $\frac{1}{12}$ $\frac{5}{9}$ $\frac{6}{14}$ $\frac{9}{20}$

$\frac{12}{25}$ $\frac{19}{31}$ $\frac{17}{59}$ $\frac{62}{75}$ $\frac{71}{100}$ $\frac{99}{186}$ $\frac{800}{835}$

$\frac{311}{312}$ $\frac{376}{400}$ $\frac{402}{410}$ $\frac{13}{413}$ $\frac{476}{501}$ $\frac{500}{600}$

Express in figures:

1. Four ninths. (How many? Four. ——what? Ninths. Therefore, write 9 under the 4 — thus, $\frac{4}{9}$.)

2. Two fifteenths; eleven twelfths; twelve nineteenths; one twentieth. (How many? ——what?)

3. Ten thirty-seconds; fourteen fifty-sevenths; six elevenths; twenty-one twenty-fifths.

4. Seventeen seventy-thirds; eight ninety-firsts; thirty-three eighty-sixths; fifty one hundred seventeenths.

5. Seventy-seven one hundred forty-eighths; thirty-five one hundred eighty-thirds.

6. One hundred twenty-seven two hundredths; one hundred sixty-three two hundred twelfths.

7. One hundred eight two hundred fortieths; two hundred sixty two hundred seventy-fourths.

8. Two hundred ninety-seven three hundred thirds; three hundred tenths.

9. Three hundred thirteen three hundred eighteenths; three hundred thirty-one three hundred fiftieths.

10. Three hundred seventy-nine four hundred ninths; forty-six four hundred twenty-sevenths.

11. Four hundred thirty-nine four hundred sixty-firsts; one hundred eleven four hundred ninetieths.

12. Five hundred one five hundred fifths; five hundred fifty-seven hundredths.

93. Fractions Classified.

1. With respect to *value,* fractions are either proper or improper.

2. A proper fraction is one which expresses a number of relative units fewer than the number required to make the primary unit, as $\frac{4}{7}$, $\frac{9}{13}$. Its value is less than one.

3. An improper fraction is one which expresses a number of relative units equal to, or greater than, the number required to make the primary unit, as $\frac{8}{8}$, $\frac{12}{6}$. Its value is one or more than one.

4. With respect to *form,* fractions are simple, complex, or compound.

5. A simple fraction is one that has a single number for each of its terms, as $\frac{4}{7}$, $\frac{8}{3}$.

6. A complex fraction is one that has a fraction in one or both of its terms, as $\frac{2\frac{1}{2}}{7}$, $\frac{4}{6\frac{1}{2}}$, $\frac{3\frac{1}{2}}{5\frac{1}{2}}$.

7. A compound fraction is a fraction of a fraction, as $\frac{2}{3}$ of $\frac{7}{8}$.

8. A mixed number is a whole number and a fraction joined together; it is two numbers, as $4\frac{6}{7}$, which is read, "four and six sevenths."

94. Principles.

I. *If both terms of a fraction are multiplied by the same number, its value is not changed;* for, while the number of units is increased, their size is decreased proportionately. (How do you know that the size is decreased?)

ILLUSTRATION: Multiplying both terms of the fraction $\frac{1}{3}$ by 3 gives $\frac{3}{9}$. There are three times as many units in $\frac{3}{9}$ as in $\frac{1}{3}$, but the units are only one third as large.

II. *If both terms of a fraction are divided by the same number, the value is not changed;* for, while the number of parts is diminished, their size is increased proportionately.

ILLUSTRATION: Dividing both terms of the fraction $\frac{2}{8}$ by 2 gives $\frac{1}{4}$. There are only one half as many units in $\frac{1}{4}$ as in $\frac{2}{8}$, but each unit is two times as large. (Why is one fourth twice as large as one eighth?)

REDUCTION OF FRACTIONS.

95. Reduction in arithmetic is *changing the form of a number without changing its value.* The form is changed by changing the kind of measuring unit. Of course, then, to preserve the value unchanged, the number of units must be changed proportionately, according to the principles stated on the preceding page.

96. To Change a Fraction to its Lowest Terms.

A fraction is in its lowest terms when the terms contain no common prime factor except one. The process consists in removing all the common prime factors from both numerator and denominator. Is the value of the fraction changed? Why not?

ORAL EXERCISES.

97. Reduce the following fractions to their lowest terms:

1. $\frac{10}{15}$, $\frac{18}{18}$, $\frac{16}{20}$, $\frac{11}{22}$, $\frac{18}{36}$, $\frac{18}{27}$, $\frac{15}{30}$, $\frac{28}{35}$, $\frac{24}{36}$, $\frac{30}{40}$.

2. $\frac{7}{42}$, $\frac{15}{45}$, $\frac{35}{45}$, $\frac{17}{51}$, $\frac{36}{54}$, $\frac{33}{55}$, $\frac{48}{60}$, $\frac{31}{62}$, $\frac{27}{63}$, $\frac{16}{64}$.

3. $\frac{39}{65}$, $\frac{51}{68}$, $\frac{49}{70}$, $\frac{60}{72}$, $\frac{50}{75}$, $\frac{64}{80}$, $\frac{48}{86}$, $\frac{80}{90}$, $\frac{19}{95}$, $\frac{75}{100}$.

WRITTEN EXERCISES.

98. *1.* Reduce $\frac{2835}{3402}$ to its lowest terms.

$2835 = 3^2 \times 3^2 \times 5 \times 7$
315
35

$3402 = 2 \times 3^2 \times 3^2 \times 3 \times 7$
1701
189
21

EXPLANATION: It will be seen by the work in the margin that the common prime factors are four 3's and a 7. Removing these from both terms, there remains a 5 only in the numerator, and 2×3 in the denominator; hence, $\frac{2835}{3402} = \frac{5}{6}$.

2.	$\frac{232}{576}$	7.	$\frac{243}{324}$	12.	$\frac{164}{492}$	17.	$\frac{495}{1452}$
3.	$\frac{225}{270}$	8.	$\frac{330}{462}$	13.	$\frac{300}{450}$	18.	$\frac{910}{1785}$
4.	$\frac{280}{308}$	9.	$\frac{338}{676}$	14.	$\frac{286}{442}$	19.	$\frac{1710}{2280}$
5.	$\frac{289}{340}$	10.	$\frac{364}{455}$	15.	$\frac{279}{434}$	20.	$\frac{3465}{4095}$
6.	$\frac{217}{357}$	11.	$\frac{224}{488}$	16.	$\frac{441}{882}$	21.	$\frac{4500}{6300}$

99. To Reduce an Improper Fraction to an Integer or Mixed Number.

Since the numerator expresses the number of relative units and the denominator shows how many of these make the primary unit or integer, there will be as many integers in any improper fraction as the given number of relative units is times the number required to make the primary unit or integer.

ORAL EXERCISES.

100. Reduce the following to integers or mixed numbers:

1. Reduce $\frac{28}{5}$ to a mixed number.

EXPLANATION: $\frac{28}{5}$ expresses twenty-eight relative units, five of which make a primary unit or integer. Then in $\frac{28}{5}$ there are as many integers as there are 5's in 28. There are five 5's in 28, and 3 relative units remaining. Hence, $\frac{28}{5}=5\frac{3}{5}$.

2. $\frac{24}{3}$, $\frac{15}{4}$, $\frac{56}{7}$, $\frac{50}{8}$, $\frac{72}{9}$, $\frac{45}{9}$, $\frac{121}{11}$, $\frac{26}{13}$, $\frac{87}{10}$, $\frac{100}{12}$.

3. $\frac{29}{7}$, $\frac{57}{5}$, $\frac{45}{3}$, $\frac{72}{8}$, $\frac{40}{20}$, $\frac{100}{11}$, $\frac{83}{9}$, $\frac{30}{15}$, $\frac{66}{22}$, $\frac{39}{13}$.

4. $\frac{108}{12}$, $\frac{28}{14}$, $\frac{51}{3}$, $\frac{96}{6}$, $\frac{75}{5}$, $\frac{55}{9}$, $\frac{63}{8}$, $\frac{38}{19}$, $\frac{80}{20}$, $\frac{100}{25}$.

5. $\frac{110}{10}$, $\frac{45}{15}$, $\frac{112}{8}$, $\frac{91}{7}$, $\frac{80}{6}$, $\frac{92}{4}$, $\frac{67}{3}$, $\frac{46}{23}$, $\frac{96}{16}$, $\frac{65}{13}$.

6. $\frac{70}{6}$, $\frac{93}{10}$, $\frac{135}{9}$, $\frac{75}{25}$, $\frac{100}{20}$, $\frac{60}{15}$, $\frac{128}{8}$, $\frac{78}{7}$, $\frac{96}{3}$, $\frac{117}{9}$.

7. $\frac{48}{16}$, $\frac{126}{9}$, $\frac{120}{12}$, $\frac{140}{20}$, $\frac{64}{16}$, $\frac{90}{15}$, $\frac{125}{25}$, $\frac{100}{4}$, $\frac{91}{13}$, $\frac{30}{18}$.

WRITTEN EXERCISES.

101. Change to whole or mixed numbers:

8. $\frac{588}{24}$	*12.* $\frac{423}{25}$	*16.* $\frac{637}{63}$	*20.* $\frac{6913}{65}$
9. $\frac{931}{15}$	*13.* $\frac{1512}{81}$	*17.* $\frac{644}{69}$	*21.* $\frac{2144}{67}$
10. $\frac{407}{13}$	*14.* $\frac{4879}{63}$	*18.* $\frac{342256}{116}$	*22.* $\frac{647821}{375}$
11. $\frac{3001}{19}$	*15.* $\frac{6713}{42}$	*19.* $\frac{67883}{874}$	*23.* $\frac{119771}{413}$

102. To Change an Integer or a Mixed Number to an Improper Fraction.

The proposed denominator shows how many of the required relative units there are in one primary unit. If this number is multiplied by the given number of primary units, the result is the required numerator.

1. Change 5 to eighths. $1=\frac{8}{8}$; $5=5$ times $\frac{8}{8}$, or $\frac{40}{8}$.

2. Change $4\frac{3}{7}$ to sevenths. Here we have two numbers of different denominations to add, for $4\frac{3}{7}$ is the same as $4+\frac{3}{7}$. The numbers can be made alike by changing the 4 to sevenths. The result is $\frac{28}{7}+\frac{3}{7}=\frac{31}{7}$.

ORAL EXERCISES.

103. Change:

1. 5 to fourths; 3 to sevenths; 4 to twelfths.

2. 6 to fifths; 4 to ninths; 7 to sixths.

3. 8 to tenths; 10 to fourths; 12 to thirds.

4. 13 to halves; 11 to ninths; 8 to eighths.

5. 14 to fifths; 16 to thirds; 9 to twelfths.

6. 12 to elevenths; 15 to sixths; 16 to fifths.

104. Change the following mixed numbers to improper fractions:

1. $2\frac{1}{3}$, $3\frac{1}{4}$, $5\frac{2}{6}$, $8\frac{3}{5}$, $4\frac{5}{7}$, $7\frac{6}{8}$, $9\frac{3}{4}$.

2. $12\frac{1}{2}$, $8\frac{5}{6}$, $10\frac{4}{5}$, $9\frac{5}{12}$, $11\frac{7}{10}$, $13\frac{2}{3}$, $7\frac{6}{11}$.

3. $9\frac{6}{7}$, $15\frac{3}{4}$, $11\frac{7}{12}$, $20\frac{2}{4}$, $33\frac{1}{3}$, $12\frac{4}{8}$, $9\frac{3}{5}$.

4. $16\frac{1}{4}$, $25\frac{1}{5}$, $22\frac{2}{3}$, $30\frac{5}{6}$, $20\frac{5}{8}$, $15\frac{4}{7}$, $12\frac{11}{12}$.

WRITTEN EXERCISES.

105. Change:

5. $14\frac{7}{15}$. *9.* $25\frac{14}{25}$. *13.* $96\frac{1}{25}$. *17.* $68\frac{18}{19}$.

6. $17\frac{9}{15}$. *10.* $18\frac{5}{37}$. *14.* $74\frac{19}{41}$. *18.* $80\frac{11}{60}$.

7. $24\frac{11}{15}$. *11.* $42\frac{31}{33}$. *15.* $59\frac{7}{11}$. *19.* $93\frac{23}{24}$.

8. $37\frac{4}{15}$. *12.* $57\frac{5}{9}$. *16.* $168\frac{9}{75}$. *20.* $256\frac{18}{35}$.

106. To Change Fractions to Equivalent Fractions having a Common Denominator.

1. Change $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, $\frac{7}{14}$, $\frac{8}{16}$, and $\frac{9}{18}$ to halves. How did you do it? Did you change the value of any of the fractions? Why not?

2. Can you change $\frac{1}{2}$ back to 6ths? To 8ths? To 10ths? To 12ths? To 14ths? To 16ths? To 18ths? How is it done? Does it change the value of the fraction? Why not?

3. Change $\frac{6}{9}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{12}{18}$, $\frac{14}{21}$, and $\frac{16}{24}$ to thirds.

4. Now change $\frac{2}{3}$ back to 9ths; to 12ths; to 15ths; to 18ths; to 21sts; to 24ths. How did you do it?

5. Change $\frac{6}{8}$, $\frac{9}{12}$, $\frac{12}{16}$, $\frac{15}{20}$, $\frac{18}{24}$, and $\frac{21}{28}$ to fourths. How did you do it?

6. Now change $\frac{3}{4}$ back to 8ths; to 12ths; to 16ths; to 20ths; to 24ths; to 28ths. How did you do it?

7. Change $\frac{2}{5}$ to 10ths; to 15ths; to 20ths; to 25ths.

8. Change $\frac{2}{6}$ and $\frac{5}{6}$ to 12ths; to 18ths; to 24ths; to 30ths.

9. Change $\frac{1}{3}$ and $\frac{2}{3}$ to 6ths; to 12ths; to 18ths; to 24ths.

10. Change $\frac{3}{4}$ and $\frac{5}{6}$ to 12ths; to 24ths; to 36ths.

11. Change $\frac{2}{3}$ and $\frac{1}{5}$ to 15ths; to 30ths; to 45ths.

12. Change $\frac{1}{2}$, $\frac{5}{6}$, and $\frac{8}{9}$ to 18ths; $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{7}{10}$ to 20ths.

Fractions which have the same denominators are called *similar fractions.*

Fractions which have not the same denominator are called *dissimilar fractions.*

The denominator of similar fractions is called a common denominator.

A common denominator of two or more fractions is any common multiple of their denominators.

The least common denominator of two or more fractions is the least common multiple of their denominators. (How is the least common multiple of several numbers found? See p. 97.)

EXERCISES.

107. *1.* Change $\frac{7}{15}$, $\frac{3}{10}$, $\frac{4}{21}$, and $\frac{5}{14}$ to equivalent fractions having the least common denominator.

EXPLANATION: The Least Common Multiple of the denominators$=2\times3\times5\times7=210$. The denominator 15 of the first fraction contains the factors 3 and 5 only. Put with them the other factors 2 and 7. Why? Put these same factors with the numerator 7. Why? The first fraction then becomes $\frac{98}{210}$. Proceeding

in the same manner with the other fractions, we find that the factors to be put with the numerators are 3 and 7, 2 and 5, 3 and 5, respectively, making the new fractions $\frac{63}{210}$, $\frac{40}{210}$, and $\frac{75}{210}$, respectively.

Change the following to equivalent fractions having the least common denominator:

2. $\frac{3}{5}$ and $\frac{4}{7}$, $\frac{2}{5}$ and $\frac{1}{6}$, $\frac{5}{6}$ and $\frac{9}{12}$, $\frac{7}{10}$ and $\frac{9}{15}$.

3. $\frac{1}{3}$ and $\frac{6}{7}$, $\frac{3}{5}$ and $\frac{9}{10}$, $\frac{5}{8}$ and $\frac{6}{10}$, $\frac{9}{12}$ and $\frac{8}{15}$.

4. $\frac{3}{4}$ and $\frac{4}{5}$, $\frac{2}{3}$ and $\frac{3}{7}$, $\frac{5}{9}$ and $\frac{7}{12}$, $\frac{10}{12}$ and $\frac{3}{15}$.

5. $\frac{3}{4}$ and $\frac{5}{6}$, $\frac{3}{6}$ and $\frac{8}{10}$, $\frac{9}{10}$ and $\frac{8}{12}$, $\frac{5}{12}$ and $\frac{10}{18}$.

6. $\frac{2}{4}$ and $\frac{3}{5}$, $\frac{5}{6}$ and $\frac{7}{9}$, $\frac{8}{10}$ and $\frac{5}{14}$, $\frac{7}{8}$ and $\frac{9}{12}$.

7. $\frac{1}{4}$ and $\frac{7}{10}$, $\frac{3}{6}$ and $\frac{5}{8}$, $\frac{6}{10}$ and $\frac{4}{15}$, $\frac{3}{12}$ and $\frac{9}{16}$.

8. $\frac{3}{5}$, $\frac{1}{4}$, and $\frac{5}{6}$; $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{3}$; $\frac{3}{5}$, $\frac{1}{2}$, and $\frac{5}{6}$.

9. $\frac{3}{5}$, $\frac{3}{4}$, and $\frac{5}{8}$; $\frac{1}{4}$, $\frac{4}{9}$, and $\frac{3}{5}$; $\frac{3}{4}$, $\frac{1}{5}$, and $\frac{5}{8}$.

10. $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$; $\frac{3}{4}$, $\frac{2}{6}$, and $\frac{7}{10}$; $\frac{2}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{5}{6}$.

WRITTEN EXERCISES.

108. Change:

11. $\frac{2}{3}$, $\frac{7}{12}$, $\frac{11}{18}$, and $\frac{10}{18}$.

12. $\frac{5}{9}$, $\frac{7}{8}$, $\frac{13}{16}$, and $\frac{1}{6}$.

13. $\frac{6}{7}$, $\frac{9}{14}$, $\frac{8}{10}$, and $\frac{1}{3}$.

14. $\frac{9}{12}$, $\frac{7}{8}$, $\frac{13}{16}$, and $\frac{20}{24}$.

15. $\frac{5}{6}$, $\frac{13}{15}$, $\frac{14}{20}$, and $\frac{55}{60}$.

16. $\frac{10}{14}$, $\frac{15}{21}$, $\frac{20}{28}$, and $\frac{33}{35}$.

17. $\frac{8}{13}$, $\frac{3}{5}$, $\frac{13}{39}$, and $\frac{18}{26}$.

18. $\frac{42}{50}$, $\frac{15}{20}$, $\frac{18}{25}$, and $\frac{72}{100}$.

19. $\frac{56}{84}$, $\frac{145}{168}$, $\frac{85}{60}$, and $\frac{16}{28}$.

20. $\frac{135}{165}$, $\frac{216}{270}$, and $\frac{42}{55}$.

21. $\frac{60}{72}$, $\frac{28}{40}$, $\frac{41}{45}$, and $\frac{25}{36}$.

22. $\frac{275}{315}$, $\frac{57}{63}$, and $\frac{85}{210}$.

23. $\frac{64}{70}$, $\frac{28}{63}$, $\frac{85}{90}$, $\frac{27}{35}$, and $\frac{42}{45}$.

24. $\frac{115}{161}$, $\frac{28}{42}$, $\frac{7}{33}$, $\frac{400}{483}$, and $\frac{180}{138}$.

25. $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$, $\frac{11}{13}$, and $\frac{9}{65}$.

26. $\frac{239}{451}$, $\frac{26}{43}$, and $\frac{313}{473}$.

27. Into what parts of the same size may $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{10}{12}$ of an apple be divided? How many such parts would there be in each case?

DECIMAL FRACTIONS.

109. *1.* Define fraction. Numerator. Denominator.

2. Express the following in figures: One tenth; one hundredth; one thousandth; one ten-thousandth; two tenths; two hundredths; two thousandths; two ten-thousandths.

3. Review Sections 16-27, pp. 13-17.

4. What is the use of the decimal point? For what is the zero used? What is the effect upon the value of a figure when it is moved one place to the left?

5. Moving a figure one place toward the left makes its value ten times what it was before. Moving it two places toward the left makes its value one hundred times what it was before, and so on.

6. Express the following numbers in figures in a column, using the decimal point: One thousand; one hundred; one ten; one. The value of the 1 standing in hundreds' place is what part of the value of the 1 standing in thousands' place? The value of the 1 in tens' place is what part of the value of the 1 in hundreds' place? The value of the 1 in ones' place is what part of the 1 in tens' place? What, then, is the effect upon the value of the figure when it is moved one place to the right?

7. Moving a figure one place to the right makes its value one tenth of what it was before. This is just as true on the right-hand side of the decimal point as it is on the left-hand side. Moving the 1 to the first place at the right of the decimal point, thus, .1, makes its value one tenth of 1.; moving it to the second place, thus, .01, makes its value one tenth of .1; moving it to the third place, thus, .001, makes its value one tenth of .01, and so on.

8. If the value of .1 is one tenth of 1., how shall we read .1? Why? If the value of .01 is one tenth of .1, how many like .01 are required to make 1.? How, then, shall we read .01? If the value of .001 is one tenth of .01, how many like .001 are required to make 1.? How, then, shall we read .001? How shall we read .0001? Why?

9. A figure in the first place at the right of the point signifies tenths, because it signifies relative units of which it takes ten to make one primary unit. A figure in the second place at the right of the point signifies hundredths, because it signifies relative units, of which one hundred are required to make one primary unit. In the same way, the name of the third place at the right is thousandths, of the fourth place is ten-thousandths, etc.

10. In how many ways, then, can one tenth be expressed in figures? One hundredth? One thousandth?

11. Express in both these ways, two tenths; two hundredths; two thousandths; two ten-thousandths; two hundred-thousandths; seven tenths; seven hundredths; seven thousandths; seven ten-thousandths; seven hundred-thousandths.

What are such fractions called?

A Decimal Fraction is one whose denominator is expressed by 1 with zeros attached, or it is one whose denominator may be omitted and the fraction written by the decimal system of notation.

NOTATION AND NUMERATION OF DECIMALS.

110. The names of the orders at the right of the decimal point are:

First order, tenths.
Second order, hundredths.
Third order, thousandths.
Fourth order, ten-thousandths.
Fifth order, hundred-thousandths.
Sixth order, millionths.
Seventh order, ten-millionths.
Eighth order, hundred-millionths.
Ninth order, billionths, and so on.

There is a correspondence between the names of the orders at the right of the decimal point and those at the left. The fourth order at the left is the order of thousands, but the third order at the right is the order of thousandths. In the same way the corresponding name at the right is always one place nearer the decimal point than the name at the left to which it corresponds.

Great care must be taken with the compound names of the orders at the right of the point. The fifth order at the left is that of tens of thousands, or ten-thousands; but the fourth order at the right is not tens of thousandths, but tenths of thousandths, or ten-thousandths. Tens of thousandths are hundredths.

The law of the zero is the same on both sides of the point; its sole use is to "fill places that would otherwise be vacant between some figure and the decimal point." Placed at the right of a decimal fraction, a zero has no significance whatever. It is often written there when we wish to consider the decimal as some number of units of a lower denomination, as when we wish to think of 5 tenths as 50 hundredths. But it serves no real purpose; that is, .5 and .50 are equivalent expressions.

EXERCISES.

111. Read the following decimals, and in each case tell:

First. How many relative units are given.

Second. How many of them make a primary unit.

1. .4, .06, .11, .005, .014, .226, .107.

2. .0009, .0023, .0318, .2435, .2308, .5007.

3. .00003, .00047, .00451, .07012, .57008.

4. .000008, .000064, .009173, .708605.

5. .0000004, .0000086, .0004819, .0376002.

6. .4076089, .6000097, .00000006, .00041085.

7. .35800604, .90900909, .000467189.

8. 3003003003, .00000008008.

If a number is written on both sides of the decimal point, it may be read as one number or as two numbers; for instance, 1.7 may be read as one and seven tenths, or as seventeen tenths. But the same might be said of

figures at the left of the point; 17 may be regarded as one ten and seven ones, or as seventeen ones. There is nothing at the decimal point which makes any break in the system any more than at any other place.

EXERCISES.

112. Read the following as one number, and then as two numbers:

1. 2.3, 14.6, 308.9, 5691.8, 4.05, 7.38, 10.03.

2. 471.48, 5003.91, 5.007, 18.018, 436.705.

3. 3713.058, 5.0005, 12.0509, 286.4013.

4. 301.0301, 8.00028, 33.04075, 66.66666.

5. 100.1, 79.09, 111.111, 13,4053, 9.797979.

Before there can be any facility in writing decimals, the names and the orders at the right of the decimal point must be thoroughly familiar. Hence, the pupil should learn the first nine orders at least, so that, when the name is given, its number can be named instantly; or, when the number is given, its name will come without effort.

WRITTEN EXERCISES.

113. Express in figures:

1. Eight tenths; six hundredths; sixty-three hundredths; sixty-three tenths. (In each case ask yourself, How many? What?)

2. Nine thousandths; thirty-nine thousandths; four hundred three thousandths.

3. Three ten-thousandths; fifteen ten-thousandths; five hundred seven ten-thousandths.

4. Two hundred forty-one ten-thousandths. (Write 241 in answer to How many? Then put the decimal point so that 1 shall be in ten-thousandths' place, in answer to What?)

5. Four thousand eleven ten-thousandths; eighteen hundred thousandths; eighteen hundred-thousandths.

6. Seven hundred four hundred-thousandths; one thousand twenty-seven hundred-thousandths.

7. Sixty thousand five hundred-thousandths; five millionths; four hundred nine millionths.

8. One thousand sixteen millionths; seventy-two ten-millionths; four thousand twelve ten-millionths.

9. Nineteen hundred-millionths; six hundred four hundred-millionths.

10. Four hundred five hundredths; twenty-one and twenty-three hundredths. (21.23 are how many hundredths?)

11. Seven hundred eighty-three tenths; eight thousand three hundred fifty-six hundredths.

12. Three hundred two and ninety-two thousandths; forty-five thousand nine hundred eight thousandths.

13. Eight hundred fourteen and fifty-seven ten-thousandths; six thousand and ninety-eight hundred-thousandths.

14. Three thousand thirty-nine billionths; eighty-six and one hundred eleven ten-thousandths.

REDUCTION OF DECIMALS.

114. To Change a Common Fraction to an Equivalent Decimal.

Consider the fraction as an expression of a division, or rather of a partition, and perform the work it indicates.

EXERCISES.

115. *1.* Change $\frac{7}{8}$ to an equivalent decimal.

8)7.000
.875

EXPLANATION: $\frac{7}{8}=\frac{1}{8}$ of 7. Since $\frac{1}{8}$ of 7 does not give an integer, change 7 to tenths. We find $\frac{1}{8}$ of 7.0=.8, with a remainder of .6. Change .6 to hundredths. $\frac{1}{8}$ of .60=.07, with a remainder of .04. Change .04 to thousandths. $\frac{1}{8}$ of .040=.005. Hence, $\frac{7}{8}$=.875. (See form in the margin.)

In the successive steps of changing ones to tenths, tenths to hundredths, etc., the prime factors 2 and 5 were introduced into the dividend each time. Hence, if the denominator of the original fraction in its lowest terms contains any other prime factor than 2 and 5, the division can never terminate. In every such case the operation will give a mixed decimal, or a repetend.

ILLUSTRATIONS: $\frac{2}{3}=.6\frac{2}{3}=.66\frac{2}{3}=.\dot{6}$. $\frac{2}{7}=.2\frac{8}{7}=.28\frac{4}{7}=.\dot{2}85714\dot{2}$.

NOTE: A dot over a figure, as over the 6, signifies that it is to be repeated indefinitely. When two dots are used, as in the value of $\frac{2}{7}$, the group of figures is to be repeated indefinitely. These are *repetends*.

116. Reduce the following to decimals:

NOTE: Be sure each fraction is in its lowest terms before beginning the reduction.

1. $\frac{1}{4}$, $\frac{2}{5}$, $\frac{3}{8}$, $\frac{1}{20}$.

2. $\frac{3}{16}$, $\frac{2}{25}$, $\frac{7}{40}$, $\frac{6}{50}$.

3. $\frac{3}{4}$, $\frac{9}{20}$, $\frac{12}{25}$, $\frac{13}{40}$.

4. $\frac{4}{5}$, $\frac{5}{8}$, $\frac{23}{50}$, $\frac{5}{16}$.

5. $\frac{21}{40}$, $\frac{3}{64}$, $\frac{27}{50}$, $\frac{33}{50}$.

6. $\frac{11}{16}$, $\frac{1}{32}$, $\frac{24}{125}$, $\frac{99}{200}$.

7. $\frac{4}{16}$, $\frac{15}{64}$, $\frac{21}{40}$, $\frac{15}{25}$.

8. $\frac{18}{32}$, $\frac{101}{125}$, $\frac{87}{128}$, $37\frac{1}{2}$.

9. $\frac{54}{60}$, $\frac{80}{96}$, $\frac{77}{250}$, $8\frac{3}{5}$.

10. $\frac{75}{80}$, $\frac{336}{625}$, $\frac{503}{800}$, $25\frac{1}{20}$.

11. $18\frac{3}{4}$, $.7\frac{1}{2}$, $.12\frac{1}{8}$, $.4\frac{1}{25}$.

12. $\frac{4}{9}$, $\frac{9}{11}$, $\frac{7}{15}$, $\frac{23}{27}$, $\frac{14}{65}$.

13. $\frac{8}{12}$, $\frac{7}{25}$, $\frac{54}{900}$, $14\frac{2}{5}$.

14. $\frac{6}{72}$, $\frac{208}{1040}$, $17\frac{10}{25}$, $37\frac{1}{8}$.

Why are the results in number 12 mixed decimals?

117. To Change a Decimal to its Equivalent Common Fraction.

Remove the decimal point, write the proper denominator, and change the fraction to its lowest terms.

ILLUSTRATIONS: $.625=\frac{625}{1000}=\frac{5}{8}$; $.850=\frac{850}{1000}=\frac{17}{20}$; $.33\frac{1}{3}=\frac{33\frac{1}{3}}{100}=\frac{1}{3}$; $.866\frac{2}{3}=\frac{866\frac{2}{3}}{1000}=\frac{2600}{3000}=\frac{13}{15}$.

EXERCISES.

118. Change to common fractions the following decimals:

1. .6, .45, .175, .043.

2. .08, .025, .75, .0108.

3. .0039, .64, .00015.

4. .625, .0009, .37625.

5. .56, .768, .2805.

6. .06375, .01, .0112.

7. 4.15, 9.07, 5.012.

8. $.3\frac{1}{3}$, .0044, 14.1.

9. .003, $.62\frac{1}{2}$, $.13\frac{3}{5}$.

10. 39.38, .000301.

11. 1.0001, 100.47.

12. $.4\frac{4}{5}$, .96875, .1001.

13. .00625, .3216.

14. .000125, .5125.

15. 5.216, 3.4375, .54.

16. $.07\frac{1}{2}$, $.078\frac{1}{2}$, $2.21\frac{3}{5}$.

NOTE: In mixed decimals do nothing with the integral part.

DENOMINATE NUMBERS.

119. *1.* A Denominate Number is *any concrete number.*

2. A Simple Number is *one in which ten units of one denomination uniformly make one of the next higher.* Thus, all numbers in Federal Money or the Metric System are simple.

3. A Compound Number is *one in which the ratio of increase is variable, or is some number other than ten.* Thus, 5 bu. 3 pks. 7 qts. is *a compound denominate number.* All mixed numbers in common fractions are compound numbers, as $15\frac{3}{8}$.

MEASURES OF VALUE.

120. Federal Money.

Money is a measure of value.

Federal, or United States, Money is the money established by the Congress of the United States. The *unit* of measure is the dollar.

TABLE.

10 mills (m.) - - - - - - - -	=1 cent (ct.)
10 cents - - - - - - - - -	=1 dime (d.)
10 dimes - - - - - - - - -	=1 dollar ($)
10 dollars - - - - - - - -	=1 eagle (E.)

1. In business, both coin and paper are used as money.

2. *Coin* is metal with certain characters stamped on it for the purpose of indicating its value, and of making it legally current as money. The coins are: *Gold*—The double-eagle, eagle, half-eagle, and quarter-eagle. *Silver*—Dollar, half-dollar, quarter-dollar, and dime. *Nickel*—The five-cent piece. *Bronze*—The one-cent piece.

3. The weight of the gold dollar is 25.8 grains Troy; of the silver dollar, 412.5 grains Troy.

4. Gold and silver coins are hardened and made more durable by an alloy of copper. The gold coins contain 9 parts of gold and 1 part of copper; the silver coins contain 9 parts of silver and 1 part of copper. The five-cent piece contains 1 part nickel and 3 parts copper; and the one-cent piece, 19 parts copper and 1 part tin and zinc.

5. Paper money consists of Treasury notes, United States notes, National bank notes, and gold, silver, and currency certificates. All these are really promises to pay money, but they are used the same as money.

EXERCISES.

121. Read the following decimally; then as dollars, cents, and mills:

1. \$14.50; \$8; \$.45; \$5.04; \$11.111; \$.37½.

2. \$80.765; \$1.01; \$100.001; \$.66⅔.

3. \$150.055; \$.444⅘; \$97.40; \$13.105.

4. \$4.001⅛; \$175.00; \$.62½; \$.003¾.

Express the following in figures:

5. Five dollars fifty cents; twenty-five cents.

6. Seventeen and thirty-five hundredths dollars; twenty dollars twelve cents five mills.

7. Eighty-five dollars ten cents six mills; forty-eight mills.

All the operations in Federal Money are performed just like similar operations with other simple numbers.

122. English Money.

English, or Sterling, Money is the money of the British Empire.

The *unit* is the pound sterling (sovereign). Its nominal value is $4.8665 in Federal Money.

TABLE.

4 farthings (far.) - - - - -	=1 penny (d.)
12 pence - - - - - - - - -	=1 shilling (s.)
20 shillings - - - - - - - -	=1 pound (£)

123. Reduction.

What is reduction? (See Section 95, p. 108.) When the new unit is *smaller* than the given one, the change is called "Reduction Descending." When the new unit is *larger* than the given one, the change is called "Reduction Ascending."

ORAL EXERCISES.

124. *1.* How many farthings are there in 6d.? In 9d.? In 12d.?

2. How many pence are there in 5s.? 7s.? 9s.? 10s.? 12s.? 15s.?

3. How many shillings are there in £2? £3? £5? £8? £11?

4. How many farthings in 4d. 2 far.? 7d. 1 far.? 11d. 3 far.?

5. How many pence in 3s. 6d.? 6s. 10d.? 8s. 5d.?

6. How many shillings in £1 15s.? £4 7s.? £9 10s.?

7. How many pence in 20 far.? 28 far.? 37 far.? 42 far.? 50 far.?

8. How many shillings in 24d.? 36d.? 50d.? 75d.? 132d.?

9. How many pounds in 25s.? 47s.? 70s.? 93s.? 140s.?

10. How many *shillings* in 48d.? 96d.? 144d.? 175d.? 288d.?

WRITTEN EXERCISES.

125. *11.* Reduce £6 13s. 9d. 2 far. to farthings.

EXPLANATION: £6=6 times 20s.=120s.
120s.+13s.=133s.
133s.=133 times 12d.=1596d.
1596d.+9d.=1605d.
1605d.=1605 times 4 far.=6420 far.
6420 far.+2 far.=6422 far.
Hence, £6 13s. 9d. 2 far.=6422 far.

Reduce to farthings:

12. £2 5s. 7d. 1 far.

13. £5 16s. 3 far.

14. £4 11d. 2 far.

15. £10 10s. 10d.

16. £8 8d.

17. 9d. 3 far.

18. £14 6d.

19. £17 17s. 11d. 3 far.

EXERCISES.

126. *1.* Reduce 3,581 far. to pounds.

4 far.)3581 far.
895 times, 1 far. remaining.

12d.)895d.
74 times, 7d. remaining.

20s.)74s.
3 times, 14s. remaining.

Hence, 3,581 far. = £3 14s. 7d. 1 far.

EXPLANATION: Since 4 far. make a penny, there are as many pence in 3,581 far. as there are *times* 4 far. in 3,581 far., which is 895 *times*, with 1 far. remaining. In 895d. there are as many shillings as there are *times* 12d. in 895d., which is 74 *times*, with 7d. remaining. In 74s. there are as many pounds as there are *times* 20s. in 74s., which is 3 *times*, with 14s. remaining. Hence, 3,581 far. = £3 14s. 7d. 1 far.

Reduce to higher denominations:

2.	3579 far.	*4.*	6169d.	*6.*	39818s.
3.	4328d.	*5.*	26242 far.	*7.*	13200d.

127. Canada Money.

Canada currency is decimal. The denominations and table are the same as for Federal Money.

The *gold coins* used in Canada are the British sovereign, worth $4.8665, and the half-sovereign.

128. French Money.

The currency of France is decimal, and the *unit* is the *Franc*, worth $.193.

129. German Money.

The currency of Germany is decimal, the *unit* being the *Mark*, worth $.2385.

MEASURES OF EXTENSION.

130. Long Measure.

Long, or Linear, Measure is used in measuring lines and distances. The *unit* is the yard.

TABLE.

12	inches (in.)	=1 foot (ft.)
3	feet	=1 yard (yd.)
5½	yards, or 16½ feet,	=1 rod (rd.)
40	rods	=1 furlong (fur.)
8	furlongs, or 320 rods,	=1 mile (mi.)

1 mile=1,760 yards, or 5,280 feet.

NOTE: Any quantity must be measured by a unit of the same kind; hence, length is measured by a unit of length, surface by a unit of surface, etc.

ORAL EXERCISES.

131. *1.* How many inches are there in 3 ft.? In 8 ft.? In 12 ft.? In 5 ft. 3 in.? In 7 ft. 6 in.?

2. How many feet in 6 yds.? In 9 yds.? In 13 yds.? In 15 yds.? In 16 yds.? In 20 yds.? In 25 yds.? In 7 yds. 1 ft.? In 12 yds. 2 ft.?

3. How many yards in 2 rds.? In 3 rds.? In 4 rds.? In 7 rds.? In 6 rds. 2 yds.? In 8 rds. 4 yds.?

4. How many feet in 2 rds.? In 3 rds.? In 4 rds.?

5. How many rods in 2 mi.? In 3 mi.? In 4 mi.?

6. How many feet are there in 48 in.? In 60 in.? In 75 in.? In 99 in.? In 116 in.?

7. How many yards are there in 15 ft.? In 24 ft.? In 31 ft.? In 50 ft.?

8. How many rods are there in 11 yds.? In $16\frac{1}{2}$ yds.? In 22 yds.? In 33 yds.? In $38\frac{1}{2}$ yds.? In 46 yds.?

9. How many inches in $\frac{1}{2}$ ft.? In $\frac{1}{3}$ ft.? In $\frac{2}{3}$ ft.? In $\frac{1}{4}$ ft.? In $\frac{3}{4}$ ft.? In $\frac{1}{6}$ ft.? In $\frac{5}{6}$ ft.?

WRITTEN EXERCISES.

132. *10.* Reduce 6 rds. 5 yds. 2 ft. to feet.

EXPLANATION: 6 rds. = 6 times $5\frac{1}{2}$ yds. = 33 yds.
33 yds. + 5 yds. = 38 yds.
38 yds. = 38 times 3 ft. = 114 ft.
114 ft. + 2 ft. = 116 ft.
Hence, 6 rds. 5 yds. 2 ft. = 116 ft.

Reduce:

11. 18 yds. 9 in. to inches.

12. 26 yds. to inches.

13. 4 rds. 3 yds. 2 ft. to feet.

14. 6 rds. 2 ft. 7 in. to inches.

15. 7 mi. 49 rds. to rods.

16. 10 mi. 289 rds. to rods.

17. 16 rds. 3 yds. to feet.

18. 22 rds. 5 yds. to inches.

19. Reduce 213 in. to higher denominations.

12 in.)213 in.
17 times, 9 in. remaining.

3 ft.)17 ft.
5 times, 2 ft. remaining.

Hence, 213 in.=5 yds. 2 ft. 9 in.

EXPLANATION: Since 12 in. make a foot, there are as many feet in 213 in. as there are *times* 12 in. in 213 in., which is 17 *times*, with 9 in. remaining. Since 3 ft. make a yard, there are as many yards in 17 ft. as there are *times* 3 ft. in 17 ft., which is 5 *times*, with 2 ft. remaining. Hence, 213 in.=5 yds. 2 ft. 9 in.

Reduce to higher denominations:

20.	217 in.	*23.*	1725 in.	*26.*	10750 rds.
21.	494 in.	*24.*	4218 rds.	*27.*	28637 rds.
22.	1354 in.	*25.*	8767 rds.	*28.*	40000 rds.

133. Surveyors' Long Measure.

The *unit* in *land surveying* is the *Gunter's chain*, which consists of 100 links, and is 4 rods (66 ft.) long.

TABLE.

7.92 inches - - - - - - - - -	=1 link (l.)
25 links - - - - - - - - -	=1 rod (rd.)
4 rods, or 100 links, - - - -	=1 Gunter's chain (ch.)
80 chains (ch.) - - - - - - -	=1 mile (mi.)

EXERCISES.

134. Reduce:

1. 35 l. to inches.

2. 4 rds. 22 l. to inches.

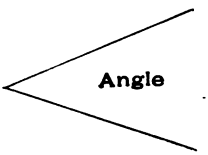
3. 49 ch. 4 l. to links.

4. 7 mi. 79 ch. to links.

5. 12 mi. 99 l. to links.
6. 968 l. to rods.
7. 6,347 l. to chains.
8. 48,086 l. to miles.
9. 112,300 l. to miles.
10. 280,000 l. to miles.

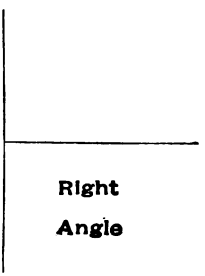
MEASURES OF SURFACE.

135. Definitions.



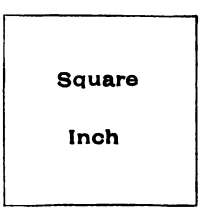
1. A surface has *length* and *breadth* only.

2. An angle is *the difference in direction* of two lines that meet.



3. If one line intersects another so as to make *the two angles on the same side equal,* each is a right angle. A right angle is sometimes called a "square corner." An acute angle is *smaller* than a right angle. An obtuse angle is *larger* than a right angle.

4. A rectangle is *a plane or flat surface having four straight sides and four right angles.*



5. A square is *a rectangle whose four sides are all equal.*

6. A square inch is a square, *each side* of which is an inch in length. What is a square foot? A square yard?

7. The *unit* of surface is a square one unit in length on each side.

8. The *area* of a surface is the number of square units it contains.

EXERCISES.

136. *1.* Draw a square that is 2 in. on each side. Divide it into square inches. How many does it contain? What is its area? What is the unit of measure? Which is more, 2 sq. in. or 2 inches square? How much more? Show it by a drawing.

2. Draw a 3-in. square. Divide it into square inches. How many does it contain? Show by a drawing how many square inches a 4-in. square contains; a 6-in. square.

3. How many square feet does a 2-ft. square contain? A 3-ft. square? A 5-ft. square?

4. Draw a square foot. Divide it into square inches. How many does it contain?

5. Draw a square yard. Divide it into square feet. How many does it contain?

137. Square Measure.

The measure of surface is called *square* measure.

TABLE.

144	square inches (sq. in.)	=1 square foot (sq. ft.)
9	square feet - - -	=1 square yard (sq. yd.)
30¼	square yards - - -	=1 square rod (sq. rd.)
160	square rods - - -	=1 acre (A.)
640	acres, or 1 section, -	=1 square mile (sq. mi.)

WRITTEN EXERCISES.

138. *1.* Reduce 25 sq. yds. 8 sq. ft. 120 sq. in. to square inches.

EXPLANATION:

25 sq. yds. = 25 times 9 sq. ft. = 225 sq. ft.
225 sq. ft. + 8 sq. ft. = 233 sq. ft.
233 sq. ft. = 233 times 144 sq. in. = 33552 sq. in.
33552 sq. in. + 120 sq. in. = 33672 sq. in.
Hence, 25 sq. yds. 8 sq. ft. 120 sq. in. = 33672 sq. in.

Reduce:

2. 7 sq. ft. 7 sq. in. to square inches.

3. 18 sq. yds. 65 sq. in. to square inches.

4. 4 sq. rds. 21 sq. yds. 3 sq. ft. to square inches.

5. 8 sq. rds. 135 sq. in. to square inches.

6. 29 sq. yds. 129 sq. in. to square inches.

7. 625 A. 40 sq. rds. to square rods.

8. 4 sq. mi. 160 A. to square rods.

9. 7 sec. 120 sq. rds. to square rods.

10. 10 sec. 320 A. 15 sq. rds. to square rods.

11. 13 sq. mi. 140 sq. rds. to square rods.

139. *12.* Reduce 204,800 sq. rds. to higher denominations.

160 sq. rds.)204800 sq. rds.
1280 times.

640 A.)1280 A.
2 times.

Hence, 204,800 sq. rds. = 2 sq. mi., or 2 sec.

Reduce to higher denominations:

13.	7632 sq. in.	*18.*	37584 sq. in.
14.	9216 sq. in.	*19.*	76940 sq. rds.
15.	5320 sq. rds.	*20.*	332800 sq. rds.
16.	25600 sq. rds.	*21.*	823279 sq. rds.
17.	16848 sq. in.	*22.*	917834 sq. rds.

140. Area of Rectangular Surfaces.

1. Draw a rectangle 4 in. long and 1 in. wide. Divide it into square inches. What is the area? What is the unit of area?

2. Draw a rectangle 3 in. long and 2 in. wide. Divide it into square inches. What is the area of the rectangle?

3. Draw a rectangle 4 in. long and 3 in. wide. Divide it into square inches. How many strips 4 in. long and 1 in. wide are there in the rectangle? How many square inches are there in 1 strip? How many times 4 sq. in. are there in 3 strips 4 in. long and 1 in. wide? What, then, is the area of a rectangle 4 in. long and 3 in. wide? How is it found?

4. Draw a rectangle 6 in. long and 5 in. wide. How many strips 6 in. long and 1 in. wide are there in the rectangle? How many square inches are there in 1 strip? How many times 6 sq. in. are there in 5 such strips? What, then, is the area of a rectangle 6 in. long and 5 in. wide? How found?

ORAL EXERCISES.

141. *1.* What is the area of the floor of a porch 10 ft. long and 6 ft. wide?

EXPLANATION: Since the floor is 10 ft. long, a strip 1 ft. wide will contain 10 sq. ft. And since the floor is 6 ft. wide, there are 6 such strips, or 6 times 10 sq. ft., which is 60 sq. ft. Hence, a floor 10 ft. long and 6 ft. wide contains 60 sq. ft.

2. The page of a book is 7 in. long and 5 in. wide. What is its area? Make a drawing to represent the area of the page.

3. What is the area of a blackboard 9 ft. long and 4 ft. wide? What is the unit of measure?

4. What is the area of a piece of tin 9 in. long and 8 in. wide?

5. How many square feet are there in a square of oil-cloth 5 ft. on each side?

6. How many square feet are there in a rectangular walk 15 ft. long and 4 ft. wide?

7. How many square yards of plastering are there in the ceiling of a room 6 yds. long and 4 yds. wide? What is the unit of area?

8. The glass over a picture is 16 in. long and 10 in. wide. What area does it cover?

9. A garden is 15 rds. long and 9 rds. wide. How many units in its surface? What is the unit?

10. If a slate is 8 in. long and 6 in. wide, how many square inches are there in the surface?

11. A sheet of paper is 8 in. long and contains 48 sq. in. How wide is it?

EXPLANATION: A sheet of paper 8 in. long and 1 in. wide contains 8 sq. in. A sheet of paper 8 in. long to contain 48 sq. in. must be as many *times* 1 in. wide as there are *times* 8 sq. in. in 48 sq. in., which is 6 *times*. Hence, the sheet must be 6 times 1 in. wide, or 6 in. wide, to contain 48 sq. in.

12. The surface of a blackboard is 48 sq. ft., and it is 4 ft. wide. How long is it?

13. The area of a hall floor is 108 sq. yds. If it is 12 yds. long, how wide is it?

14. How wide must a garden be that is 16 rds. long to contain an acre?

WRITTEN EXERCISES.

142. *15.* The length of a field is 72 rds. and its width is 20 rds. How many acres are there in the field?

16. Find the number of square yards in a house-lot 84 ft. front and 102 ft. deep.

17. What would be the cost, at 50 cts. a square yard, of plastering the four walls of a room 18 ft. long, 15 ft. wide, and 9 ft. high?

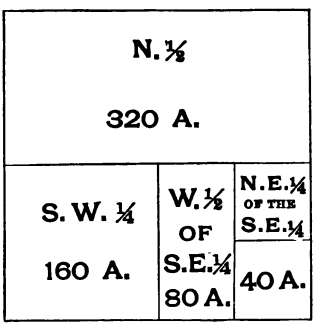
18. What is the value of a farm 128 rds. long and 90 rds. wide, at $65 an acre?

19. Mr. Williams built a tight-board fence 45 yds. long and 2 yds. high. What did the boards cost at $2 a hundred square feet?

20. What will be the cost of the bricks for a pavement 200 ft. long and 9 ft. wide, at $9 a thousand, if it takes 8 bricks for each square foot?

21. A field that contains 80 acres is 128 rds. long. How wide is it?

143. Divisions of the Section of Land.



DIVISIONS OF THE SECTION OF LAND.

1. Draw a 2-in. square. Let it represent a section of land. How many acres does it represent? Mark the right-hand side "east," the left-hand side "west," the upper "north," and the lower "south." Divide it equally by an east-and-west line, and divide the south half as in the margin. One of these smaller squares is what part of the large square? How many acres does each of these squares represent? Which square represents the S. E. ¼ of the section? Which represents the S. W. ¼? Study the divisions of the S. E. ¼.

EXERCISES.

144. Draw a square and mark each of the following tracts, and tell how many acres it contains:

1. E. ½ of the section; N. W. ¼.

2. S. ½ of the N. W. ¼.

3. N. E. ¼ of the N. W. ¼.

4. N. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$.

5. N. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$.

6. N. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$.

7. N. W. $\frac{1}{4}$ of the S. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$.

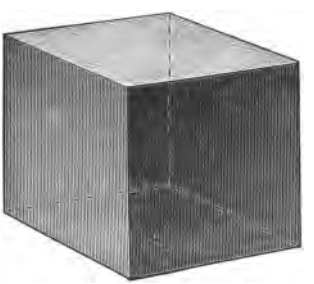
8. E. $\frac{1}{2}$ of the N. W. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$.

9. E. $\frac{1}{2}$ of the N. $\frac{1}{2}$ of the N. E. $\frac{1}{4}$ of the S. E. $\frac{1}{4}$.

NOTE: The teacher should explain the United States system of surveying public lands, making clear the following facts: What a township is; the number of sections a township usually contains; how the sections are numbered; and how the townships are described.

MEASURES OF VOLUME.

RECTANGULAR SOLID.



CUBIC INCH.

145. Definitions.

1. A solid has three dimensions — *length, breadth,* and *thickness.*

2. A rectangular solid is a *solid bounded by six rectangular surfaces.*

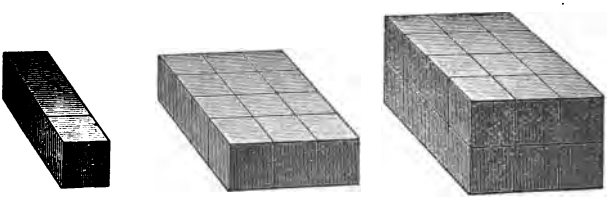
The surfaces of a solid are called *faces*, and the sides of the faces are called *edges*. How many edges has a rectangular solid? How many corners?

3. A cube is a *rectangular solid having square faces.*

4. A cubic inch is a *cube whose edges are each one inch long.* What is a cubic foot? A cubic yard?

5. The *unit* of solid measure is a cube whose edges are one unit in length.

6. The *volume,* or *contents,* of a solid is the number of units it contains.



EXERCISES.

146. *1.* Draw a square 1 in. on each side. Let this represent one face of a rectangular solid. If this solid is 1 in. thick, how many cubic inches does it contain?

2. Draw a rectangle 2 in. long and 1 in. wide. Let this represent one surface of a rectangular solid. If this solid is 1 in. thick, how many cubic inches does the solid contain? Make it 3 in. long. Now, how many cubic inches does the solid contain? How many if it were 4 in. long? 5 in. long? 7 in. long? 12 in. long?

3. Draw a rectangle 8 in. long and 1 in. wide. How many square inches does it contain? Let this rectangle represent one surface of a rectangular solid 1 in. thick. How many inches does it contain? What kind? Make the rectangle 2 in. wide. How many cubic inches does the solid contain if it is 1 in. thick? How many times 8 cu. in.? Make it 3 in. wide. How many cubic inches does it contain? How many times 8 cu. in.? How many would it contain if it were 4 in. wide? 5 in. wide? How many times 8 cu. in. in each case?

4. Draw a rectangle 6 in. long and 5 in. wide. How many square inches does it contain? Let the rectangle be the face of a rectangular solid 1 in. thick. How many cubic inches does it contain? If it were 2 in. thick how many cubic inches would it contain? How many times 30 cu. in., if it were 3 in. thick? If it were 5 in. thick? 6 in. thick? 9 in. thick? How many cubic inches in each case?

5. Draw a square 1 ft. on each side. Divide it into square inches. How many does it contain? Let it be one face of a rectangular solid 1 in. thick. How many cubic inches does the solid contain? If the solid were 2 in. thick, how many cubic inches would it contain? How many *times* 144 cu. in.? How many *times* 144 cu. in., if it were 3 in. thick? 5 in. thick? 8 in. thick? 12 in. thick? How many cubic inches? What is a solid 12 in. long, 12 in. wide, and 12 in. thick called? How many cubic inches in a cubic foot?

6. How many square feet in a surface 3 ft. long and 3 ft. wide? If the surface is a face of a solid 1 ft. thick, how many cubic feet in the solid? If the solid

were 3 ft. thick, how many cubic feet would it contain? How many times 9 cu. ft.? How many cubic feet in a cubic yard?

147. Cubic Measure.

The measure of volume is called *cubic* measure.

TABLE.

1728 cubic inches (cu. in.)=1 cubic foot (cu. ft.)
27 cubic feet - - - =1 cubic yard (cu. yd.)

EXERCISES.

148. *1.* How many cubic feet are there in 9 cu. yds. 18 cu. ft.?

EXPLANATION: 9 cu. yds.=9 times 27 cu. ft.=243 cu. ft.
243 cu. ft.+18 cu. ft.=261 cu. ft.
Hence, 9 cu. yds. 18 cu. ft.=261 cu. ft.

2. How many cubic feet are there in 9,876 cu. in.?

1728 cu. in.)9876 cu. in.
5 times, 1236 cu. in. remaining.

EXPLANATION: Since 1,728 cu. in. make 1 cu. ft., in 9,876 cu. in. there are as many cubic feet as there are times 1,728 cu. in. in 9,876 cu. in.; that is, 5 times and 1,236 cu. in. remaining. Hence, 9,876 cu. in.=5 cu. ft. 1,236 cu. in.

How many cubic inches are there in:

3. 23 cu. ft. 1,625 cu. in.?

4. 8 cu. yds. 13 cu. ft.?

5. 14 cu. yds.?

6. 18 cu. yds. 634 cu. in.?

7. 16 cu. yds. 26 cu. ft. 1,596 cu. in.?

8. 19 cu. ft. 789 cu. in.?

9. 75 cu. yds. 15 cu. ft.?

Change to higher denominations:

10.	31,104 cu. in.	*15.*	21,237 cu. ft.
11.	1,912 cu. ft.	*16.*	629,005 cu. in.
12.	27,743 cu. in.	*17.*	1,041,984 cu. in.
13.	317,010 cu. in.	*18.*	1,415,857 cu. in.
14.	20,945 cu. ft.	*19.*	1,886,976 cu. in.

EXERCISES.

149. *1.* What are the contents of a rectangular solid 6 ft. long, 4 ft. wide, and 3 ft. thick?

EXPLANATION: A solid 6 ft. long, 1 ft. wide, and 1 ft. thick contains 6 cu. ft. A solid 6 ft. long, 4 ft. wide, and 1 ft. thick contains 4 times 6 cu. ft., or 24 cu. ft. A solid 6 ft. long, 4 ft. wide, and 3 ft. thick contains 3 times 24 cu. ft., or 72 cu. ft.

2. What is the volume of a rectangular solid 9 ft. long, 5 ft. wide, and 4 ft. thick?

3. How many cubic yards of earth must be removed in digging a cellar 38 ft. long, 26 ft. wide, and 6 ft. deep?

4. How many cubic feet of water will a rectangular cistern hold that is 8 ft. square and 18 ft. deep?

5. How many gallons does such a cistern hold, there being 231 cu. in. in a gallon?

6. How many cubic feet of air are there in a room 18 ft. long, 16 ft. wide, and 9 ft. high?

7. How many cubic inches are there in a block of granite 6 ft. 4 in. long, 4 ft. 3 in. wide, and 2 ft. 10 in. thick? How many cubic feet?

8. How many 1-in. cubes could be sawed from a 3-ft. cube, if there were no waste in sawing?

9. A pile of bricks, solid, is 10 ft. long, 8 ft. wide, and 6 ft. high. How many bricks are there in the pile, a brick being 8 in. long, 4 in. wide, and 2 in. thick?

10. A rectangular block of marble which contains 72 cu. ft. is 6 ft. long and 4 ft. wide. How many feet thick is it? (A block 6 ft. long, 4 ft. wide, and 1 ft. thick contains 24 cu. ft.)

11. The end of a rectangular piece of timber which contains 768 cu. in. is 8 in. long and 6 in. wide. What is the length of the piece of timber?

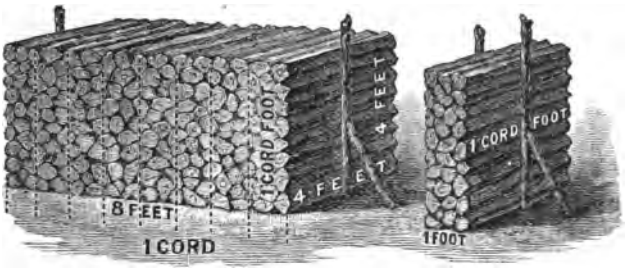
12. What is the length in feet of a piece of timber which contains 11,232 cu. in., the ends being rectangles 8 in. long and 6 in. wide?

13. From a cellar 36 ft. long and 21 ft. wide was taken 3,780 cu. ft. of earth. What is the depth of the cellar?

150. Wood Measure.

TABLE.

16 cubic feet - - - - - - - =1 cord foot (cd. ft.)
8 cord feet, or 128 cubic feet, =1 cord (cd.)



The typical form of a cord of wood is a compact pile 8 ft. long, 4 ft. wide, and 4 ft. high. In such a pile every foot in length contains 1 cd. ft.

EXERCISES.

151. *1.* How many cords are there in a pile of wood 24 ft. long, 8 ft. wide, and 6 ft. high? (Solve without figures; first suppose the pile to be 4 ft. wide and 4 ft. high.)

2. How many cord feet in a pile of wood 28 ft. long, 12 ft. wide, and 8 ft. high? How many cords? How many cord feet in 1 ft. of length?

3. A pile of wood containing 45 cd. is 60 ft. long and 16 ft. wide. How high is it?

4. What must be the length of a load of wood $5\frac{1}{3}$ ft. wide and 3 ft. high to contain 1 cd.?

5. How many cubic inches are there in 4 cd. and 16 cd. ft.?

6. How many cubic inches in a pile of wood 10 ft. long, 6 ft. wide, and 5 ft. high? How many cords?

7. At $5 a cord, what is the value of the wood that can be piled under a shed 48 ft. long, 24 ft. wide, and 12 ft. high?

152. Lumber Measure.

The *unit* of lumber measure is the board foot, which contains 144 cu. in. Hence, 1 cu. ft. contains 12 board feet.

Lumber that is less than an inch thick is counted an inch thick. If it is more than an inch thick, the excess is usually counted.

EXERCISES.

153. How many feet of lumber are there in:

1. 28 boards, 18 ft. long, 8 in. wide, and 1 in. thick?

2. 8 joists, 16 ft. long, 6 in. wide, and 2 in. thick?

3. 6 sills, 24 ft. long, 8 in. wide, and 6 in. thick?

4. 60 rafters, 2 in. by 4 in., 10 ft. long?

5. 36 boards, 10 in. wide and 20 ft. long?

6. 12 beams, 30 ft. long, 8 in. wide, and 10 in. thick?

7. Find the cost of the following bill of lumber, at $18 per M feet:

46 boards, 12 in. wide and 18 ft. long;
15 boards, 8 in. wide and 14 ft. long;
14 sills, 6 in. by 8 in., 16 ft. long;
10 corner posts, 4 in. by 4 in., 18 ft. long;
48 joists, 2 in. by 8 in., 16 ft. long;
72 rafters, 2 in. by 4 in., 12 ft. long;
60 studs, 2 in. by 4 in., 18 ft. long;
72 fence boards, 6 in. wide, 16 ft. long.

MEASURES OF CAPACITY.

154. Liquid Measure.

The standard *unit* of liquid measure is the gallon, which contains 231 cu. in.

TABLE.

4 gills (gi.) - - - - - - -	=1 pint (pt.)
2 pints - - - - - - - - -	=1 quart (qt.)
4 quarts - - - - - - - -	=1 gallon (gal.)

NOTE: In measuring the capacity of *cisterns, reservoirs*, etc., 31½ gals. make a barrel (bbl.), and 63 gals. a hogshead (hhd.); but in *commerce*, the barrel and the hogshead vary in capacity.

ORAL EXERCISES.

155. *1.* How many gills in 8 pts.? In 12 pts.? In 15 pts.? In 6 pts. 2 gi.? In 9 pts. 3 gi.?

2. How many pints in 9 qts.? In 14 qts.? In 16 qts.? In 7 qts. 1 pt.?

3. How many quarts in 10 gals.? In 16 gals.? In 7 gals. 2 qts.? In 9 gals. 3 qts.?

4. How many quarts in 18 pts.? In 23 pts.? In 30 pts.?

5. How many gallons in 24 qts.? In 30 qts.? In 36 qts.? In 45 qts.?

6. In 50 pts. of kerosene how many quarts? How many gallons?

7. What is a 6-gal. can of milk worth, at 5 cts. a quart?

8. What must I pay for $3\frac{1}{2}$ gals. of syrup, at 20 cts. a quart?

9. What part of a gallon are 2 qts.? 3 qts.? 3 pts.? 7 pts.?

10. What will 5 gals. 3 qts. of oil cost, at 8 cts. a quart?

11. What will 72 gi. of vinegar cost, at 7 cts. a quart?

WRITTEN EXERCISES.

156. *12.* How many pints in 5 gals. 2 qts.?

EXPLANATION: 5 gals. = 5 times 4 qts. = 20 qts.
20 qts. + 2 qts. = 22 qts.
22 qts. = 22 times 2 pts. = 44 pts.
Hence, 5 gals. 2 qts. = 44 pts.

13. Change 119 pts. to higher denominations.

2 pts.) 119 pts.
59 times, with 1 pt. remaining.
4 qts.) 59 qts.
14 times, with 3 qts. remaining.

EXPLANATION: Since 2 pts. make 1 qt., in 119 pts. there are as many quarts as there are *times* 2 pts. in 119 pts., which is 59 *times*, with 1 pt. remaining. Since 4 qts. make a gallon, in 59 qts. there are as many gallons as there are *times* 4 qts. in 59 qts., which is 14 times, with 3 qts. remaining. Hence, 119 pts. = 14 gals. 3 qts. 1 pt.

14. How many gills in 4 gals. 2 qts. 1 pt.? In 7 gals. 1 pt. 2 gi.? In 12 gals. 1 qt. 1 pt. 2 gi.? In 20 gals. 3 gi.?

15. Change to higher denominations, 320 gi.; 1,371 pts.; 2,785 pts.; 3,908 gi.

16. How many days will a barrel of oil containing 45 gals. last, if a pint is burned each day?

17. What are 504 pts. of molasses worth, at 40 cts. a gallon?

18. How many barrels, each containing 43 gals. 3 qts. 1 pt., can be filled from a tank containing 1,184 gals. 2 qts. 1 pt. of oil? (How many pints in 1 bbl.? How many pints in the tank?)

19. What is the cost of 192 gi. of alcohol, at $3 a gallon?

157. Dry Measure.

The standard *unit* of dry measure is the bushel, which is 18½ in. in diameter and 8 in. deep; it contains 2,150.42 cu. in.

TABLE.

2 pints (pt.) - - - - - - -	=1 quart (qt.)
8 quarts - - - - - - - -	=1 peck (pk.)
4 pecks - - - - - - - -	=1 bushel (bu.)

NOTE: The liquid quart contains 57¾ cu. in., and the dry quart, 67⅕ cu. in.

ORAL EXERCISES.

158. *1.* How many quarts in 7 pks.? In 9 pks.? In 12 pks.? In 5 pks. 3 qts.? In 8 pks. 5 qts.? In 11 pks. 7 qts.?

2. How many pecks in 40 qts.? In 50 qts.? In 56 qts.? In 68 qts.? In 78 qts.?

3. How many bushels in 33 pks.? In 43 pks.? In 64 pks.? In 80 pks.? In 100 pks.?

4. What is the cost of 6 qts. of beans, at 6 cts. a pint?

5. A boy picked 2½ bu. of cherries, at 15 cts. a peck. How much money did he receive for his work? What were the cherries worth, at 10 cts. a quart?

6. How many pies could be made from the cherries, if a pint be used to make 1 pie?

7. What part of a bushel is 1 pk.? 2 pks.? 3 pks.?

8. What part of a peck is 1 qt.? 3 qts.? 7 qts.?

9. What is 1 bu. 3 pks. of clover seed worth, at 8 cts. a quart?

10. What will 128 qts. of gooseberries cost, at 50 cts. a peck?

11. Find the cost of 480 qts. of peas, at $2 a bushel.

WRITTEN EXERCISES.

159. *12.* How many pints are there in 8 bu. 3 qts. 1 pt.? In 18 bu. 3 pks. 6 qts. 1 pt.?

13. Change to higher denominations: 3,042 pts.; 4,913 pts.; 4,582 qts.

14. Find the cost of 36 bu. of beans, at 25 cts. a quart?

15. What are 54,144 pts. of chestnuts worth, at $3 a bushel? (How many pints in a bushel?)

16. How many barrels, each holding 3 bu. 1 pk. 5 qts., are required to hold 183 bu. 3 pks. 6 qts. of potatoes? (How many quarts of potatoes? How many quarts in a barrel?)

17. A fruit-grower sold 2,496 qts. of plums in one season. How many bushels did he sell?

MEASURES OF WEIGHT.

160. Troy Weight.

Weight is *the measure of the earth's gravity.* It is determined by the use of scales, steelyards, or balances.

Troy weight is used in weighing gold, silver, and jewels.

The United States standard *unit* of Troy weight is the Troy pound, which contains 5,760 grains.

TABLE.

24 grains (gr.) - - - =1 pennyweight (pwt.)
20 pennyweights - - =1 ounce (oz.)
12 ounces - - - - - =1 pound (lb.)

ORAL EXERCISES.

161. *1.* How many grains are there in 2 pwt.? In 3 pwt.? In 4 pwt.? In $\frac{1}{2}$ pwt.? In $\frac{3}{4}$ pwt.? In $\frac{5}{6}$ pwt.? In $\frac{5}{8}$ pwt.?

2. How many pennyweights in 3 oz.? In 7 oz.? In 9 oz.? In 4 oz. 8 pwt.? In 5 oz. 16 pwt.?

3. How many pennyweights in $\frac{1}{4}$ oz.? In $\frac{3}{4}$ oz.? In $\frac{3}{5}$ oz.? In $\frac{4}{5}$ oz.? In $\frac{3}{10}$ oz.? In $\frac{9}{10}$ oz.?

4. How many ounces in 8 lbs.? In 9 lbs.? In 4 lbs. 6 oz.? In 7 lbs. 7 oz.? In $\frac{2}{3}$ lb.? In $\frac{5}{6}$ lb.? In $\frac{7}{12}$ lb.?

5. How many grains in 12 oz.? In 1 lb.?

6. How many pennyweights in 36 gr.? In 48 gr.? In 72 gr.?

7. What part of a pennyweight are 12 gr.? 8 gr.? 15 gr.? 16 gr.? 20 gr.?

8. How many ounces in 40 pwt.? In 50 pwt.? In 60 pwt.? In 75 pwt.? In 90 pwt.?

9. What part of an ounce are 10 pwt.? 12 pwt.? 15 pwt.? 18 pwt.?

10. How many pounds in 36 oz.? In 50 oz.? In 65 oz.? In 80 oz.?

WRITTEN EXERCISES.

162. *11.* Change 4 oz. 16 pwt. to grains.

EXPLANATION: 4 oz. = 4 times 20 pwt. = 80 pwt.
80 pwt. + 16 pwt. = 96 pwt.
96 pwt. = 96 times 24 gr. = 2304 gr.
Hence, 4 oz. 16 pwt. = 2304 gr.

12. Change 1,340 pwt. to higher denominations.

20 pwt.)1340 pwt.
67 times.
12 oz.)67 oz.
5 times, 7 oz. remaining.

EXPLANATION: Since 20 pwt. make 1 oz., there are as many ounces in 1,340 pwt. as there are *times* 20 pwt. in 1,340 pwt., or 67 *times*. Since 12 oz. make a pound, there are as many pounds in 67 oz. as there are *times* 12 oz. in 67 oz., or 5 *times*, with 7 oz. remaining. Hence, 1,340 pwt. = 5 lbs. 7 oz.

Reduce:

13. 9 oz. 15 pwt. to grains.

14. 6 lbs. 8 oz. to pennyweights.

15. 12 lbs. 13 gr. to grains.

16. 17 lbs. 17 oz. 17 pwt. to grains.

Reduce to higher denominations:

17. 456 gr. *19.* 4,820 gr. *21.* 132,480 gr.

18. 399 pwt. *20.* 2,418 pwt. *22.* 177,432 gr.

23. What is the value of 7 lbs. 9 oz. 16 pwt. of gold dust, at $.60 a pennyweight?

24. How many silver spoons, each weighing 2 oz. 8 pwt., can be made from a bar of silver weighing 7 lbs. 2 oz. 8 pwt.? (How many pennyweights of silver? How many in one spoon?)

163. Avoirdupois Weight.

Avoirdupois weight is used in weighing all coarse and heavy articles, such as grain, hay, groceries, coal, iron, etc.

The *unit* is the pound avoirdupois, which contains 7,000 Troy grains.

TABLE.

16 ounces (oz.) - - - -	=1 pound (lb.)
25 pounds - - - - - -	=1 quarter (qr.)
100 pounds - - - - - -	=1 hundredweight (cwt.)
2000 pounds - - - - - -	=1 ton (T.)

NOTE: In weighing coal and iron at the mines, and in levying duties at the United States Custom House, the long ton of 2,240 pounds is sometimes used.

164. Most of the States have adopted the following

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL.

Oats, - - - -	32 lbs.	Rye, - - - -	56 lbs.
Timothy seed, -	45 lbs.	Clover seed, - -	60 lbs.
Barley, - - -	48 lbs.	Potatoes, - - -	60 lbs.
Salt, - - - -	50 lbs.	Wheat, - - - -	60 lbs.
Corn, - - - -	56 lbs.	Coal, - - - -	80 lbs.

165. TABLE OF MISCELLANEOUS WEIGHTS.

14 lbs. of lead or iron - - - - -	=1 stone.
56 lbs. of butter - - - - - - -	=1 firkin.
100 lbs. of grain or flour - - - -	=1 cental.
100 lbs. of dried fish - - - - -	=1 quintal.
100 lbs. of nails - - - - - - - -	=1 keg.
196 lbs. of flour - - - - - - - -	=1 barrel.
200 lbs. of pork - - - - - - - -	=1 barrel.
280 lbs. of salt at N. Y. Works - -	=1 barrel.
240 lbs. of lime - - - - - - - -	=1 cask.

ORAL EXERCISES.

166. *1.* What is the cost of 1 lb. 4 oz. of canary seed, at 2 cts. an ounce?

2. What is the value of 4 oz. of sunflower seed, at 40 cts. a pound?

3. How much must I pay for 2 lbs. 8 oz. of butter, at 20 cts. a pound?

4. Find the cost of $\frac{3}{4}$ lb. of cinnamon, at 10 cts. an ounce.

5. What will be the cost of 4 lbs. 6 oz. of honey, at 2 cts. an ounce?

6. At 12 cts. a pound, what will 80 oz. of cheese cost?

7. How many ounces in 6 lbs. of sugar? In 6 lbs. of gold? Are the pounds the same? Are the ounces the same? Which ounce is the larger?

8. What will 12 oz. of coffee cost, at 36 cts. a pound?

9. Find the value of 72 oz. of pepper, at 20 cts. a pound.

10. At 8 cts. an ounce, what will $3\frac{3}{4}$ lbs. of licorice cost?

11. A teamster had a load of iron weighing 2 T. How many pounds in the load?

WRITTEN EXERCISES.

167. *12.* Reduce 26 lbs. 13 oz. to ounces.

13. How many pounds are there in 8,752 oz.? In 9,936 oz.? In 13,472 oz.?

14. Find the cost of 4 T. of nails, at 4 cts. a pound.

15. What is the value of 1,384 oz. of tea, at 80 cts. a pound?

16. What will 8 bbls. of pork cost, at 11 cts. a pound?

17. A dealer bought $\frac{1}{4}$ T. of millet seed for $20, and sold it for 6 cts. a pound. How much did he gain?

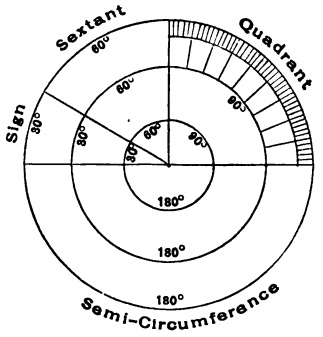
18. A farmer sold a load of corn weighing 2,688 lbs., and a load of wheat weighing 2,700 lbs. For the wheat he received 85 cts. a bushel, and for the corn 47 cts. How much did he receive for both the wheat and the corn?

19. Change 321,663 oz. to higher denominations.

20. How many ounces in 7 T. 1,895 lbs. 14 oz.?

CIRCULAR MEASURE.

168. Definitions.



DIVISIONS OF THE CIRCLE.

1. A circle is *a plane surface bounded by a curved line, all points of which are equally distant from a point within the plane, called the center.*

2. The *curved line* which bounds the circle is its circumference.

3. A diameter of a circle is *any straight line passing through the center* and having its *extremities* lying in the circumference.

4. A radius of a circle is *a straight line connecting the center with any point in the circumference.* It is one half of a diameter.

5. An arc is *any part of a circumference.*

6. Every circumference, great or small, may be divided into 360 equal parts. Each of these parts is called a degree. Every degree may be divided into 60 equal parts. Each of these parts is called a minute. Every minute may be divided into 60 equal parts. Each of these parts is called a second.

7. Circular measure is used to *measure* angles and arcs of circles. It is used in *surveying, navigation, astronomy,* and *geography;* it is used for *reckoning* latitude and longitude, *determining the location* of places and vessels, *measuring the motion* of the earth, and for *computing differences* of time.

TABLE.

60 seconds (″) - - - - - -	=1 minute (′).
60 minutes - - - - - - -	=1 degree (°).
360 degrees - - - - - - -	=1 circumference (cir.)

1. A semi-circumference is one half of a circumference, or 180°.

2. A quadrant is one fourth of a circumference, or 90°.

3. A sextant is one sixth of a circumference, or 60°.

4. A sign is one twelfth of a circumference, or 30°.

5. Since every circumference contains 360°, the length of a degree depends upon the size of the circle; thus, a degree of longitude at the equator is 69.16 statute miles; at 30° of latitude, 59.81; at 60° of latitude it is 34.58 miles; and at 90° it is nothing.

6. A minute of the earth's equator is called a geographical or nautical mile; it is about 1.16 common miles.

WRITTEN EXERCISES.

169. *1.* Reduce 7° 35″ to seconds.

EXPLANATION: 7°=7 times 60′=420′.
420′=420 times 60″=25200″.
25200″+35″=25235″.
Hence, 7° 35″=25235″.

2. How many seconds in 1 quadrant?

3. In 4 signs?

4. In a semi-circumference?

5. In 5 sextants?

6. In 26° 40′ 50″?

7. In 75° 55′?

8. How many degrees does the minute hand of a clock move over in 4 hrs.? In $\frac{1}{2}$ hr.?

9. How many degrees does the second hand of a watch move over in 7 min.? In $\frac{1}{4}$ of a minute?

10. Reduce 64,825″ to higher denominations.

60″)64825″
1080 times, with 25″ remaining.
60′)1080
18 times.

EXPLANATION: Since 60″ make a minute, in 64,825″ there are as many minutes as there are *times* 60″ in 64,825″, which is 1,080 *times*, with 25″ remaining. Since 60′ make a degree, in 1,080′ there are as many degrees as there are *times* 60′ in 1,080′, which is 18 *times*. Hence, 64,825″=18° 25″.

Reduce to higher denominations:

11. 46,824″. *13.* 53,684″.

12. 32,295′. *14.* 536,058″.

15. How many signs in 972,000′?

16. How many quadrants in 8,100′?

17. A boy in going around a circular park on his bicycle traveled 10,368,000″. How many times did he ride around the park?

18. If he traveled 6 mi., what was the length of a degree in feet?

MEASURES OF TIME.

170. The ordinary divisions of time are as follows:

TABLE.

60 seconds (sec.) - - - - -	=1 minute (min.)
60 minutes - - - - - - -	=1 hour (hr.)
24 hours - - - - - - - -	=1 day.
7 days - - - - - - - -	=1 week (wk.)
365 days (or 52 weeks 1 day)-	=1 common year (yr.)
366 days - - - - - - - -	=1 leap year.
100 years - - - - - - - -	=1 century (cen.)

The names of the months and the number of days in each are:

1.	January (Jan.) - - 31	*7.*	July - - - - - 31
2.	February (Feb.) 28 or 29	*8.*	August (Aug.) - - 31
3.	March - - - - - 31	*9.*	September (Sept.) - 30
4.	April (Apr.) - - - 30	*10.*	October (Oct.) - - 31
5.	May - - - - - - 31	*11.*	November (Nov.) - 30
6.	June - - - - - - 30	*12.*	December (Dec.) - 31

1. The *solar day* is the time the earth takes to turn upon its axis so as to bring the sun to the same meridian again.

2. The time in which the earth makes one revolution around the sun is 365 days 5 hrs. 48 min. 50 sec., nearly. This is the true year.

3. The common year thus loses 5 hrs. 48 min. 50 sec. ($\frac{1}{4}$ day, nearly), or one day, nearly, in four years. This defect is made up by adding one day to one year in four, thus making a year of 366 days. Such years are called leap years. But this adds about 44 min. 40 sec. too much. To correct this error, the closing year of each century is not counted as a leap year, excepting once in four centuries.

4. Every year that is exactly divisible by 4 is a leap year, except the centennial years, and every centennial year that is exactly divisible by 400 is a leap year.

5. The years are numbered from the beginning of the Christian era.

6. In most business transactions, 30 days are considered a month.

7. The *civil day* begins and ends at 12 o'clock midnight.

8. A. M. (ante-meridian) is used to denote the time between midnight and noon; M. (meridian) to denote noon; and P. M. (post-meridian) to denote the time between noon and midnight.

ORAL EXERCISES.

171. *1.* How many seconds in 10 min.? In 30 min.?

2. How many minutes in 4 hrs.? In 540 sec.?

3. How many hours in 5 days? In 720 min.?

4. How many days in 13 wks.? In 960 hrs.?

5. Which of the following years were leap years: 1835, 1856, 1800, 1886, 1860, 1600, 1870, 1872, 1890?

6. What part of a minute are 20 sec.? 30 sec.?

7. What part of an hour are 15 min.? 12 min.?

8. What part of a day are 9 hrs.? 20 hrs.?

9. How many years in a quarter of a century? In one half of a century?

10. How many days in Feb., 1882? 1896?

11. How many hours in the month of June?

12. How many hours from 9.00 A. M. to 6.00 P. M.?

13. How long is the night when the sun sets at 7.40 and rises at 4.25?

14. How many days from Jan. 31 to May 30, 1894? From May 10 to Sept. 18? From Aug. 17 to Nov. 12?

15. What century is this? In what century was 1492?

WRITTEN EXERCISES.

172. *16.* How many seconds in 365 days 5 hrs. 48 min. 50 sec.?

17. Reduce 5,683,762 min. to higher denominations.

18. How many minutes in the first three months of the year 1885?

19. Change 1,153,800 min. to higher denominations.

20. How many seconds in the 19th century?

21. How many common years in 220,752,000 sec.?

MISCELLANEOUS TABLES.

173. This table is often used in counting:

TABLE.

12 ones - - - - - -	=1 dozen (doz.)
12 dozen - - - - -	=1 gross.
12 gross - - - - -	=1 great gross (gt. gross).
20 ones - - - - - -	=1 score.

174. A stationers' table for quantities of paper:

TABLE.

24 sheets of paper - - - - -	=1 quire (qu.)
20 quires - - - - - - - - -	=1 ream (rm.)
2 reams - - - - - - - - -	=1 bundle (bun.)
5 bundles - - - - - - - -	=1 bale.

ORAL EXERCISES.

175. *1.* Name some articles that are sold by the dozen. By the gross.

2. How many pencils in a gross? In 3 score?

3. How many sheets of paper in a ream? In $\frac{1}{2}$ a ream? In $\frac{1}{4}$ a ream?

4. What will 72 sheets of paper cost, at 20 cts. a quire?

5. How many dozens in a great gross?

6. How many quires in 5 rms.? In 96 sheets?

7. How many reams in 25 bun.? In 100 qu.?

8. How many are 3 score and 10 yrs.?

9. I bought 3 gross of pens at $1.40 a gross, and sold them at 2 cts. apiece. How many cents did I gain?

WRITTEN EXERCISES.

176. *10.* What will 120 doz. of buttons cost, at 50 cts. a gross?

11. How many sheets of paper in 3 bales 3 bun. 1 rm. 16 qu. 20 sheets?

12. How many bales in 33,600 sheets?

13. How many weeks and days are there in 2 score and 7 yrs.?

14. How many rows of buttons, 12 in a row, are there in a great gross?

GENERAL RULE FOR ALL REDUCTIONS.

177. Definitions.

1. Reduction is *changing the form of a number without changing its value.* The form is changed by changing the *unit* of measurement. To preserve the value, of course, the number of units must be changed proportionally. When the change is made to a smaller unit, the process is called *Reduction Descending;* when it is made to a larger unit, the process is *Reduction Ascending.*

2. A ratio is *the part which one number is of another;* as, 3 is $\frac{3}{7}$ of 7 — that is, $\frac{3}{7}$ is the ratio of 3 to 7.

3. A reciprocal of a number is *that number, integral or fractional, by which it must be multiplied to give 1 for a product;* thus, 3 and $\frac{1}{3}$, $\frac{1}{3}$ and 9, $\frac{2}{7}$ and $\frac{7}{2}$, are reciprocals.

178. The Comprehensive Rule.

Compare the unit to which we change with the given unit; multiply the given number of units by the reciprocal of the ratio thus found

179. The Rule Illustrated.

1. Example of reduction descending: Change 3 bu. to quarts.

EXPLANATION:
3 bu.=4 times 3 pks.=12 pks.=8 times 12 qts.=96 qts.

FORMULA FOR EXPLANATION: "I am to change bushels to pecks; a peck is $\frac{1}{4}$ of a bushel, hence there must be 4 times as many pecks as bushels, or 4 times 3 pks. I am to change pecks to quarts; a quart is $\frac{1}{8}$ of a peck, hence there must be 8 times as many quarts as pecks, or 8 times 12 qts."

2. Example of reduction ascending: Change 571 in. to higher denominations.

EXPLANATION:
571 in.=$\frac{1}{12}$ of 571 ft.=47 ft., with $\frac{7}{12}$ ft., or 7 in., remaining.
47 ft.=$\frac{1}{3}$ of 47 yds.=15 yds., with $\frac{2}{3}$ yd., or 2 ft., remaining.
15 yds.=$\frac{2}{11}$ of 15 rds.=2 rds., with $\frac{8}{11}$ rd., or 4 yds., remaining.
Hence, 571 in.=2 rds. 4 yds. 2 ft. 7 in.

MODIFIED FORMULA FOR EXPLANATION: "I am to change inches to feet; a foot is 12 times an inch, hence there are $\frac{1}{12}$ as many feet as inches, or $\frac{1}{12}$ of 571 ft., etc. I am to change feet to yards; a yard is 3 times a foot, hence there are $\frac{1}{3}$ as many yards as feet, or $\frac{1}{3}$ of 47 yds., etc. I am to change yards to rods; a rod is $\frac{11}{2}$ of a yard, hence there are $\frac{2}{11}$ as many rods as yards, or $\frac{2}{11}$ of 15 rds., etc."

3. Change 4 yds. 2 ft. $5\frac{1}{4}$ in. to the fraction of a rod.

SOLUTION:

$5\frac{1}{4}$ in. $=\frac{21}{4}$ in. $=\frac{1}{11}$ of $\frac{21}{4}$ ft. $=\frac{7}{16}$ ft.
$2\frac{7}{16}$ ft. $=\frac{39}{16}$ ft. $=\frac{1}{4}$ of $\frac{39}{16}$ yd. $=\frac{13}{16}$ yd.
$4\frac{13}{16}$ yds. $=\frac{77}{16}$ yds. $=\frac{7}{11}$ of $\frac{77}{16}$ rds. $=\frac{7}{8}$ rd.
Hence, 4 yds. 2 ft. $5\frac{1}{4}$ in. $=\frac{7}{8}$ rd.

4. Change $\frac{5}{8}$ of a mile to lower denominations.

SOLUTION:

$\frac{5}{8}$ mi. $=8$ times $\frac{5}{8}$ fur. $=\frac{40}{8}$ fur. $=6\frac{2}{3}$ fur.
$\frac{2}{3}$ fur. $=40$ times $\frac{2}{3}$ rd. $=\frac{80}{3}$ rd. $=26\frac{2}{3}$ rds.
$\frac{2}{3}$ rd. $=\frac{1}{4}$ of $\frac{2}{3}$ yd. $=\frac{1}{4}$ yd. $=3\frac{1}{4}$ yds.
$\frac{2}{3}$ yd. $=3$ times $\frac{2}{3}$ ft. $=2$ ft.
Hence, $\frac{5}{8}$ mi. $=6$ fur. 26 rds. 3 yds. 2 ft.

5. 2 pks. 3 qts. = what decimal of a bushel?

SOLUTION:

3 qts. $=\frac{1}{8}$ of 3 pks. $=.375$ pk.
2.375 pks. $=\frac{1}{4}$ of 2.375 bu. $=.59375$ bu.
Hence, 2 pks. 3 qts. $=.59375$ bu.

6. .282 T. = what?

SOLUTION:

.282 T. $=20$ times .282 cwt. $=5.640$ cwt.
.64 cwt. $=4$ times .64 qr. $=2.56$ qrs.
.56 qr. $=25$ times .56 lb. $=14$ lbs.
Hence, .282 T. $=5$ cwt. 2 qrs. 14 lbs.

These examples will show that the rule and the formula of explanation will apply to any problem in reduction. Pupils of some maturity will have no trouble in mastering and understanding the process; but this method should not be given to pupils in the elementary stage.

Note that, in the solutions given above, all mixed numbers are changed to improper fractions, and that common factors in multipliers and divisors are canceled whenever it is possible.

EXERCISES.

180. Solve all these problems by the general rule:

1. Change 34 cd. to cubic feet.

2. How many bushels, etc., in 37,184 pts.?

3. Reduce $\frac{4}{5}$ of a pound Troy to lower denominations.

4. Change .1875 of a day to lower denominations.

5. 5 days 3 hrs. is what part of a week?

6. Express $\frac{7}{11}$ of a chain in lower denominations.

7. Express .625 gal. in lower denominations.

8. Express 8s. 10d. as the decimal of a £.

9. Find the value of $\frac{5}{8}$ of a degree.

10. Change 12 cu. ft. 486 cu. in. to the fraction of a cubic yard.

11. What decimal of a bushel are 1 pk. 5 qts. 1 pt.?

12. Find the value, in lower denominations, of $\frac{5}{32}$ T.

13. Express 1 qt. 1 pt. 1 gi. as the fraction of a gallon.

14. Express $\frac{3}{5}$ of a mile in rods, feet, and inches.

15. Change 25″ to the decimal of a degree.

16. How many acres in 1,611,720 sq. ft.?

REVIEW.

ADDITION.

181. *1.* Define number. Simple number.

2. Define unit. Primary unit. Relative unit.

3. What is the use of the decimal point? Of the zero?

4. What is the effect upon the value of a figure when it is moved one place toward the left? When it is moved one place toward the right?

5. What is a decimal fraction? What kind of numbers are decimal fractions? What kind of units do decimal fractions express?

6. Define addition. Sum. How are numbers written for adding? Why? Where begin to add? Why?

EXERCISES.

182. *1.* Find the sum of \$347, \$516, and \$789.

2. Find the sum of \$.347, \$.516, and \$.789.

How shall the numbers in the second problem be written for adding? Why? Where begin to add? Why? How does addition of decimals differ from addition of integral numbers?

Addition of decimals is the same as addition of integers. The only new consideration is the decimal point.

3. A boy earned $.625 on Monday, $.75 on Tuesday, $.6 on Wednesday, $.875 on Thursday, $.85 on Friday, and $.7 on Saturday. How many dollars did he earn during the week?

4. A man sold three loads of hay. For the first, he received $18.75; for the second, $13.90; and for the third, $16.45. How many dollars did he receive for the three loads?

5. What is the weight of five bars of silver weighing, respectively, .85 lb., .765 lb., .9 lb., .875 lb., and .78 lb.?

6. What were the annual expenses of a family that paid $236.35 for clothing, $396.78 for provisions, $200 for rent, $35.50 for the services of a physician, $63.875 for fuel, $23.88 for books, and $115.625 for all other expenses?

7. A vessel sailed, in six successive days, the following distances: 236.26 mi., 249.13 mi., 215.9 mi., 198.86 mi., 244 mi., 238.165 mi. How many miles did it sail in the six days?

8. How many cords of wood in four piles which contain, respectively, 5.437 cd., 3.8 cd., 4.75 cd., and 6.095 cd.?

9. How many rods of fence will inclose a field the four sides of which are, respectively, 44.25 rds., 32.865 rds., 42.9 rds., and 33.125 rds.?

10. A man bought three Jersey cows. For the first he paid $30.75; for the second, $60.35; and for the third, $48.65. How many dollars did he pay for the three?

11. A farmer sold five loads of oats. The first weighed 48.33 bu.; the second, 45.125 bu.; the third, 46.5 bu.; the fourth, 43.75 bu.; and the fifth, 46.625 bu. How many bushels did he sell?

12. A man has 37.9 A. in oats, 75.875 A. in corn, 60.33 A. in pasture, 45.125 A. in wheat, 9.1 A. in orchard, and 6.67 A. in lawn and yards. How many acres in his farm?

Find the sums of the following numbers:

13. $8.934, $.078, $4.73, $4.008, $.9, and $10.

14. 13.15 gals., 27.0014 gals., 9.9 gals., 16.0307 gals., .8176 gal.

15. 4.09°, 7.009°, 18.4°, .87°, .0014°, .10023°.

16. 10.201 T., .4387 T., .00347 T., 5 T., 1.8 T., .30197 T.

17. 1.35796, .800048, .7, .413, .719, 3.07, 4.4, .00009.

18. 8, .8, .08, .008, .00008, 8.0000008, .808, .0808.

19. 718, 71.8, 7.18, .718, .0718, .00718, .000718, .0000718.

183. Fractions.

1. Define fraction. Proper fraction. Improper fraction.

2. Define numerator. Denominator.

3. Define similar fractions. Dissimilar fractions.

4. Define mixed number. Compound number.

ORAL EXERCISES.

184. *1.* A lady paid $\$\frac{4}{10}$ for coffee, and $\$\frac{5}{10}$ for butter. How many tenths of a dollar did she pay for both?

2. William sawed $\frac{3}{8}$ of a cord of wood, and Hugh sawed $\frac{4}{8}$ of a cord. How many eighths of a cord did both boys saw?

3. A girl learned $\frac{2}{6}$ of a poem in 1 hr., and $\frac{3}{6}$ of it the second hour. How many sixths of the poem did she learn in 2 hrs.?

4. I bought $\frac{4}{12}$ of an acre of land from one man, and $\frac{5}{12}$ of an acre from another. How many twelfths of an acre did I buy?

5. A boy earned $\$\frac{3}{5}$ on Monday, $\$\frac{3}{5}$ on Tuesday, and $\$\frac{4}{5}$ on Wednesday. How many fifths of a dollar did he earn in the three days? How many dollars?

6. A grocer sold to one customer, $\frac{3}{4}$ of a gallon of vinegar; to another, $\frac{2}{4}$ of a gallon; to a third, $\frac{1}{4}$ of a gallon; to a fourth, $\frac{3}{4}$ of a gallon; and to a fifth, $\frac{3}{4}$ of a gallon. How many gallons did he sell to all?

7. A boy picked $\frac{5}{8}$ of a peck of cherries from one tree, $\frac{6}{8}$ of a peck from another, and $\frac{7}{8}$ of a peck from another. How many pecks did he pick from all the trees?

8. A farmer sold $4\frac{1}{2}$ T. of hay to one man, $3\frac{1}{2}$ T. to another, and $5\frac{1}{2}$ T. to another. How many tons did he sell to all?

9. A plumber worked $6\frac{4}{8}$ hrs. on Monday, $7\frac{3}{8}$ hrs. on Tuesday, and $8\frac{5}{8}$ hrs. on Wednesday. How many hours did he work in all?

10. A boy paid $2\frac{3}{4}$ for a hat, $3\frac{3}{4}$ for shoes, $\frac{3}{4}$ for a tie, and $1\frac{1}{4}$ for gloves. How many dollars did he pay for all?

11. Apples were gathered from seven apple trees as follows: $2\frac{7}{8}$ bbls., $3\frac{4}{5}$ bbls., $5\frac{3}{5}$ bbls., $4\frac{3}{5}$ bbls., $3\frac{3}{5}$ bbls., $5\frac{4}{5}$ bbls., and $6\frac{4}{5}$ bbls. How many barrels were gathered from all the trees?

12. A man bought steak on five successive days as follows: $2\frac{8}{16}$ lbs., $3\frac{9}{16}$ lbs., $3\frac{5}{16}$ lbs., $2\frac{10}{16}$ lbs., and $3\frac{4}{16}$ lbs. How many pounds did he buy in all?

13. Mary paid $\frac{3}{4}$ for ribbon, $\frac{1}{2}$ for lace, and $\frac{7}{8}$ for silk. How many dollars did she pay for all? (Make the units alike. Why? How?)

14. Henry studied his spelling lesson $\frac{1}{3}$ of an hour, his arithmetic lesson $\frac{5}{6}$ of an hour, his reading lesson $\frac{1}{2}$ of an hour, and his history lesson $\frac{3}{4}$ of an hour. How many hours did he study?

15. A man walked $3\frac{2}{3}$ mi. one hour, and $3\frac{1}{2}$ mi. the next hour. How many miles did he walk in 2 hrs.?

16. Frank hoed $\frac{1}{3}$ of a field in 1 day, and Warren hoed $\frac{2}{5}$ of it. What part of the field did both boys hoe in 1 day?

17. Ellen's mother gave her $\frac{1}{2}$; her father, $\frac{7}{10}$; and her brother, $\frac{2}{5}$. How many dollars were given her?

18. A butcher sold $3\frac{1}{2}$ lbs. of pork to one man, $2\frac{7}{8}$ lbs. to another, and $4\frac{3}{16}$ lbs. to another. How many pounds did he sell to all the men?

19. A man dug $\frac{1}{2}$ of a ditch in 1 day and $\frac{3}{7}$ of it the next day. What part of the ditch did he dig in 2 days?

20. A farmer plowed $2\frac{3}{4}$ A. one day, $3\frac{1}{4}$ A. the second day, and $2\frac{1}{2}$ A. the third day. How many acres did he plow in all?

21. A student read $\frac{1}{3}$ of a book one day, $\frac{1}{6}$ of it the second day, and $\frac{1}{9}$ of it the third day. What part of the book did he read in 3 days?

22. If a pound of coffee costs $\$\frac{2}{5}$; a pound of tea, $\$1\frac{1}{15}$; a pound of butter, $\$\frac{3}{10}$; and a pound of raisins, $\$\frac{1}{6}$; how many dollars must I pay for a pound of each?

23. A gardener planted $1\frac{1}{3}$ A. in cabbage, $\frac{1}{2}$ an acre in peas, $5\frac{3}{8}$ A. in potatoes, and $1\frac{1}{8}$ A. in beans. How many acres did he plant?

24. A drover sold $\frac{1}{7}$ of his flock of sheep to one man, $\frac{3}{14}$ of the flock to another, and $\frac{9}{28}$ of the flock to a third. What part of the flock did he sell?

25. Grace practiced her music $4\frac{5}{12}$ hrs. on Monday, $3\frac{4}{5}$ hrs. on Tuesday, $5\frac{3}{10}$ hrs. on Wednesday, and $4\frac{1}{6}$ hrs. on Thursday. How many hours in all did she practice?

26. Mr. Walker owns $\frac{2}{5}$ of a ship, and Mr. Brown owns $\frac{2}{7}$ of it. What part of the ship do both men own?

27. A lady bought $5\frac{1}{4}$ yds. of one kind of cloth, $3\frac{2}{3}$ yds. of another kind, $7\frac{1}{6}$ yds. of another kind, and $6\frac{5}{18}$ yds. of another kind. How many yards did she buy?

28. $16\frac{1}{4}$ pears, $3\frac{1}{8}$ pears, $\frac{3}{8}$ of a pear, $\frac{3}{20}$ of a pear, and $4\frac{1}{10}$ pears are how many pears?

WRITTEN EXERCISES.

185. In the following problems practice the making of integers:

29. $4\frac{3}{8}+\frac{7}{8}+2\frac{3}{4}$ are how many?

SHORT SOLUTION: Put one of the $\frac{3}{8}$ with the $\frac{7}{8}$, thus making an integer, and put the remaining $\frac{2}{8}$, or $\frac{1}{4}$, with $\frac{3}{4}$, making another integer. Now collect all the integers. The result is $4+1+2+1=8$.

30. Find the sum of $16\frac{2}{9}$, $\frac{14}{18}$, and $10\frac{7}{30}$.

SHORT SOLUTION: Put $\frac{3}{30}$, or $\frac{1}{18}$, with the $\frac{14}{18}$, thus making an integer. The remaining $\frac{15}{30}=\frac{1}{2}$; add $\frac{2}{9}$ and $\frac{1}{2}$. $\frac{2}{9}=\frac{4}{18}$, and $\frac{1}{2}=\frac{9}{18}$. $\frac{4}{18}+\frac{9}{18}=\frac{7}{18}$. The result is $16+1+10+\frac{7}{18}=27\frac{7}{18}$.

31. Add $7\frac{3}{4}$, $9\frac{5}{6}$, and $15\frac{7}{8}$.

SHORT SOLUTION: Put 2 of the $\frac{7}{8}$ with $7\frac{3}{4}$, thus making 8. We now have $\frac{5}{8}$ to add to $\frac{5}{6}$. $\frac{5}{8}=\frac{15}{24}$, and 4 of these $\frac{15}{24}$ are $\frac{5}{6}$, which, put with the $\frac{5}{6}$, make 1. The result is $8+9+1+15+\frac{11}{24}=33\frac{11}{24}$.

Find the sum of:

32. $17\frac{1}{2}$, $9\frac{2}{3}$, and $12\frac{5}{6}$.

33. $2\frac{5}{8}$, $\frac{15}{16}$, and $15\frac{1}{2}$.

34. $4\frac{2}{3}$, $1\frac{13}{15}$, and $27\frac{4}{5}$.

35. $\frac{5}{6}$, $38\frac{3}{4}$, and $15\frac{1}{3}$.

36. $4\frac{7}{10}$, $16\frac{1}{2}$, and $9\frac{4}{5}$.

37. $23\frac{5}{9}$, $17\frac{1}{6}$, and $10\frac{2}{3}$.

38. $42\frac{4}{7}$, $8\frac{1}{2}$, and $12\frac{11}{14}$.

39. $\frac{17}{20}$, $8\frac{3}{4}$, $\frac{9}{10}$, $64\frac{4}{5}$.

40. $85\frac{1}{4}$, $1\frac{5}{8}$, $3\frac{5}{6}$, $28\frac{3}{3}$.

41. $14\frac{4}{5}$, $5\frac{7}{15}$, $77\frac{5}{6}$, $\frac{19}{30}$.

42. $8\frac{13}{14}$, $2\frac{1}{2}$, $6\frac{2}{3}$, $59\frac{6}{7}$.

43. $4\frac{2}{3}$, $17\frac{5}{8}$, $1\frac{23}{24}$, $9\frac{1}{2}$.

44. $12\frac{4}{9}$, $\frac{25}{36}$, $72\frac{5}{6}$, $3\frac{11}{18}$.

45. $\frac{27}{28}$, $11\frac{5}{7}$, $6\frac{11}{16}$, $37\frac{9}{14}$.

186. Denominate Numbers.

EXERCISES.

1. Define denominate number. Compound number. Are the numbers expressed in the preceding exercise simple, mixed, or compound? What are they in this exercise?

2. Define addition. Sum. How are numbers written for adding? Why? Where begin to add? Why?

3. Find the sum of 32 bu. 2 pks. 6 qts. 1 pt.; 21 bu. 1 pk. 1 pt.; 16 bu. 3 pks. 7 qts.; and 10 bu. 5 qts. 1 pt.

bu.	pks.	qts.	pts.
32	2	6	1
21	1	0	1
16	3	7	0
10	0	5	1
81	0	3	1

EXPLANATION: Express units of the same kind in the same column, as in the margin. Draw a line beneath. Begin with the lowest or right-hand order. The sum of the pints is 3 pts., which equals 1 qt. 1 pt. Express 1 pt. under the column of pints. The sum of the quarts is 18 qts.; 18 qts. + 1 qt. = 19 qts., which equals 2 pks. 3 qts. Express 3 qts. under the column of quarts. The sum of the pecks is 6 pks. 6 pks. + 2 pks. = 8 pks., which equals 2 bu. Write 0 under the column of pecks. The sum of the bushels is 79 bu. 79 bu. + 2 bu. = 81 bu., which is written under the column of bushels.

4. A grocer received an invoice of four barrels of kerosene. The first contained 38 gals. 3 qts. 1 pt.; the second, 40 gals. 1 qt. 1 pt.; the third, 42 gals. 2 qts. 1 pt.; and the fourth, 39 gals. 3 qts. 1 pt. How many gallons were there in all four barrels together?

5. A lady purchased a silver teapot, weighing 1 lb. 11 oz. 9 pwt. 16 gr.; a sugar bowl, weighing 10 oz. 18 pwt. 6 gr.; a cream pitcher, weighing 6 oz. 15 pwt. 10

gr.; and a butter dish, weighing 1 lb. 2 oz. 12 pwt. What was the weight of all four pieces?

6. Mr. Baker owns three farms. The first contains 358 A. 60 sq. rds.; the second, 232 A. 85 sq. rds.; the third, 279 A. 15 sq. rds. How many acres of land does he own? How many sections of land would his farms make?

7. A merchant in England bought four rolls of cloth. For the first, he paid £16 12s. 8d.; for the second, £20 18s. 10d.; for the third, £9 8s. 3d.; and for the fourth, £13 10s. 11d. How many pounds, shillings, and pence did he pay for all?

8. A ship sailed 2° 25′ 30″ the first day; 3° 55′ 45″ the second day; 3° 12′ 15″ the third day; and 2° 50′ 30″ the fourth day. How many degrees and minutes did it sail in four days?

9. How many square yards of plastering in the ceilings of four rooms which contain 22 sq. yds. 4 sq. ft. 120 sq. in.; 18 sq. yds. 8 sq. ft. 84 sq. in.; 25 sq. yds. 6 sq. ft. 60 sq. in.; and 16 sq. yds. 7 sq. ft. 20 sq. in., respectively?

10. Find the sum of the following:

rds.	yds.	ft.	in.
12	5	0	9
0	4	2	10
9	3	2	8
50	4	2	6
74	2	1	3

SHORT SOLUTION: Put 2 of the 6 in. with the 10, and the remaining 4 with the 8; we have 2 ft., with the 9 in. remaining. But 1 of the 2 ft. with 6 of the 9 in. equals ½ a yard, which, put with 5 yds., make 1 rd. Now write the remaining 3 in. and also the remaining 1 ft., since 3 times 2 ft. give just 2 yds. Write these 2 yds. under the column of yards, for the sum of 4,

3, and 4 yds. is just 2 rds. This gives 3 rds. to be added to those already given, making 74 rds. Solve this problem by the same method you solved the preceding ones, and see how many more figures you need.

In the following problems practice the making of integers:

11. Find the sum of 85 bu. 3 pks. 7 qts. 1 pt.; 32 bu. 2 pks. 5 qts. 1 pt.; 76 bu. 1 pk. 6 qts. 1 pt.; and 112 bu. 3 pks. 1 qt. 1 pt.

12. What is the sum of 324 days 18 hrs. 26 min. 31 sec.; 48 days 7 hrs. 24 min. 45 sec.; 138 days 22 hrs. 58 min. 35 sec.; and 295 days 6 hrs. 40 min. 29 sec.?

13. Add 3 mi. 50 ch. 40 l.; 1 mi. 78 ch. 75 l.; 4 mi. 65 ch. 80 l.; and 7 mi. 45 ch. 25 l.

14. Add 4 T. 800 lbs. 9 oz.; 1 T. 1,998 lbs. 10 oz.; 3 T. 1,385 lbs. 7 oz.; and 2 T. 1,200 lbs. 12 oz.

15. What is the sum of 34 sq. rds. 18 sq. yds. 7 sq. ft. 63 sq. in.; 15 sq. rds. 12 sq. yds. 8 sq. ft. 94 sq. in.; 39 sq. rds. 21 sq. yds. 6 sq. ft. 87 sq. in.?

Find the sum of:

	cu. yds.	cu. ft.	cu. in.
16.	14	22	1568
	7	18	1600
	27	24	1296
	9	15	928

	gals.	qts.	pts.	gi.
18.	14	3	1	3
	10	0	1	2
	7	3	1	3
	18	2	1	1

	cd.	cd. ft.	cu. ft.
17.	18	6	10
	24	5	14
	13	7	13
	8	4	15

	mi.	rds.	yds.	ft.	in.
19.	2	240	4	2	10
	7	308	3	2	7
	1	180	5	2	6
	5	60	4	2	11

187. Addition Review.

EXERCISES.

1. A merchant deposited the following amounts in a bank: Monday, $837.76; Tuesday, $1,359.16; Wednesday, $1,138.85; Thursday, $1,403.08; Friday, $1,296.30; Saturday, $1,548. What was the sum of his deposits for the week?

2. A mechanic earned 26\frac{2}{3}$ in one week, 30\frac{5}{6}$ the second week, 28\frac{3}{4}$ the third week, and 35\frac{7}{10}$ the fourth week. What was the amount of his earnings?

3. Find the sum of 60 A. 100 sq. rds. 25 sq. yds.; 187 A. 95 sq. rds. 16$\frac{1}{2}$ sq. yds.; 78 A. 148 sq. rds. 20$\frac{3}{4}$ sq. yds.; and 110 A. 50 sq. rds. 7$\frac{2}{3}$ sq. yds.

4. Alice is 13$\frac{5}{6}$ yrs. old, Frances is 3$\frac{3}{8}$ yrs. older than Alice, and their father is 38$\frac{5}{12}$ yrs. older than Frances. What is the age of their father?

5. What is the sum of 7 wks. 6 days 20 hrs. 48$\frac{8}{15}$ min.; 12 wks. 4 days 12 hrs. 35$\frac{11}{12}$ min.; 3 wks. 5 days 15 hrs. 58$\frac{7}{10}$ min.; and 5 wks. 3 days 22 hrs. 12$\frac{3}{4}$ min.?

6. Find the sum of 3.138 T., 7.45 T., 18.038 T., .9875 of a ton, 5.3057 T., and 10.8 T.

7. Add 6 T. 675 lbs. 12$\frac{3}{4}$ oz.; 1 T. 1,876 lbs. 10$\frac{1}{8}$ oz.; 5 T. 1,588 lbs. 4$\frac{5}{8}$ oz.; and 2 T. 1,015 lbs. 15$\frac{13}{16}$ oz.

8. A merchant bought five pieces of cloth containing 33$\frac{1}{6}$, 28$\frac{7}{8}$, 30$\frac{11}{12}$, 34$\frac{3}{4}$, and 36$\frac{1}{2}$ yds., respectively. How many yards in all five pieces?

9. .4293+.7+7.946582+2.14+5.0017+9=?

10. A farmer raised from one field, 158 bu. 3 pks. 7 qts. of rye; from a second, 296 bu. 2 pks. 5 qts. of wheat; from a third, 413 bu. 1 pk. 6 qts. of oats; and from a fourth, 98 bu. 3 pks. 7 qts. of barley. How much grain did he raise in all?

11. How many cords in five piles of wood containing $16\frac{7}{16}$, $13\frac{1}{64}$, $12\frac{3}{4}$, $15\frac{25}{32}$, and $10\frac{1}{2}$ cd., respectively?

12. Find the sum of 8 cd. 6 cd. ft. 11 cu. ft.; 12 cd. 4 cd. ft. 15 cu. ft.; 9 cd. 7 cd. ft. 10 cu. ft.; 14 cd. 5 cd. ft. 13 cu. ft.; 10 cd. 2 cd. ft. 15 cu. ft.; and 5 cd. 6 cd. ft. 12 cu. ft.

13. A boy, in going from one city to another on his bicycle, traveled $42\frac{1}{2}$ mi. the first day; the next, $39\frac{9}{14}$ mi.; the third, $37\frac{5}{8}$ mi.; the fourth, $35\frac{3}{4}$ mi.; the fifth, $31\frac{1}{2}\frac{7}{8}$ mi. What was the distance between the two cities?

Add the following:

	sign.	°	′	″
14.	5	17	36	29
	7	25	41	21
	8	15	16	9
	3	12	8	10
	4	28	50	40

	cu. yds.	cu. ft.	cu. in.
15.	75	24	1129
	37	18	1645
	53	7	396
	16	26	1500
	97	14	230

16. A certain farm is divided into four fields, containing $36\frac{5}{8}$ A., $42\frac{3}{8}$ A., $24\frac{7}{10}$ A., and $56\frac{11}{40}$ A., respectively. How many acres are there in the farm?

Can you tell how the addition of fractions, mixed numbers, and compound denominate numbers differs from the addition of simple numbers and decimal fractions?

SUBTRACTION.

188. *1.* Define subtraction. Minuend. Subtrahend. Difference.

2. How are numbers written for subtraction? Why? Where begin to subtract? Why?

EXERCISES.

189. *1.* A lady who had $714, paid $485 for a piano. How many dollars had she left?

2. A boy earned $7.14 and paid $4.85 for a coat. How many dollars had he left? How does the working of this problem differ from that of Problem 1?

3. A man whose income was $4,162.60, spent $2,839.87. How much did he save?

4. A tub of butter weighed 48.367 lbs. The tub alone weighed 3.56 lbs. What was the weight of the butter?

5. A cistern that will hold 215.375 bbls. contains 178.75 bbls. How many barrels does it lack of being full?

6. A farmer raised 94.4375 bu. of flaxseed. He sold 76.3125 bu. How many bushels had he left for seed?

7. In 1880, Minnesota had 2,989.59 mi. of railroad; and in 1890, 5,379.5 mi. How many miles were built during the decade?

8. A man owning 638.125 A. of land, sold 175 A. How many acres had he left?

9. A teacher's salary for one year was $1,200. His expenses were $865.87. How many dollars did he save?

10. In 1889, the production of gold in Idaho was 95,983.4765 oz., and in Washington it was 9,005.7825 oz. How many more ounces were produced in Idaho than in Washington?

Find the value of:

11.	$13.485 − $9.798.	*23.*	15.46 − 5.7.
12.	$20.18 − $8.045.	*24.*	.46 − .0037.
13.	$87.005 − $49.72.	*25.*	.00719 − .00298.
14.	$65.108 − $50.	*26.*	10 − .4317.
15.	$10.15 − $6.15.	*27.*	11.1111 − .9999.
16.	$35.01 − $20.875.	*28.*	1 − .7777.
17.	$75 − $27.45.	*29.*	100 − .0001.
18.	$90 − $1.94.	*30.*	4.004 − .1001.
19.	$43 − $10.01.	*31.*	7 − .7.
20.	$100 − $12.375.	*32.*	91.10001 − 1.375.
21.	8.1 gals. − 3.14 gals.	*33.*	.0003 − .00003.
22.	3.16 ft. − .625 ft.	*34.*	23 − .0023.

190. Fractions.

ORAL EXERCISES.

1. James earned $$\frac{7}{8}$$ and spent $$\frac{3}{8}$$. How many eighths of a dollar had he left?

2. Mr. Roberts owned $$\frac{15}{16}$$ of an acre of land, and sold $$\frac{9}{16}$$ of an acre. How many sixteenths of an acre had he left?

3. It took Ruth $\frac{7}{10}$ of an hour to learn her arithmetic lesson, and $\frac{3}{10}$ of an hour to learn her history lesson. How much longer did it take her to prepare one lesson than the other? How long to prepare both?

4. A boy had $\frac{5}{8}$ of a bushel of oats. How much had he left after feeding his pony $\frac{1}{8}$ of a bushel?

5. Lucy's father gave her a dollar. She spent $\frac{2}{5}$ of it for thread and needles. What part of the dollar had she left?

6. $\frac{4}{5}$ of a pole is under water, and the rest is above water. What part of the pole is above water?

7. Walter has read $\frac{7}{15}$ of a certain book. What part of the book has he yet to read?

8. A man spends $\frac{9}{20}$ of his salary for board. What part of his salary is left?

9. How many pounds of beef will be left after $\frac{5}{8}$ of a pound has been cut from a piece weighing 4 lbs.?

10. A lady who paid $\$\frac{7}{10}$ for coffee, gave the clerk a five-dollar bill. How much should the clerk return?

11. I had $10, and paid $\$3\frac{1}{2}$ for a pair of shoes. How much had I left?

12. A farmer cut 15 T. of hay from two fields. From one he cut $6\frac{7}{20}$ T. How many tons did he cut from the other?

13. From a piece of lace containing $9\frac{5}{8}$ yds., a milliner used $4\frac{1}{8}$ yds. How many yards were left?

14. Andrew is $14\frac{11}{12}$ yrs. old. How old was he $4\frac{5}{12}$ yrs. ago?

15. Mr. Wallace brought to market $16\frac{1}{4}$ bu. of potatoes. He took $3\frac{3}{4}$ bu. home, having sold the rest. How many bushels did he sell?

16. How many dollars must be added to $\$12\frac{7}{10}$ to make $\$17\frac{3}{10}$?

17. Two boys together picked $18\frac{5}{12}$ crates of berries. How many crates did the second boy pick, if the first picked $9\frac{7}{12}$ crates?

18. Henry engaged to saw into stove-lengths, a pile of wood containing $14\frac{3}{16}$ cd. How many cords remained to be sawed after he had sawed $8\frac{9}{16}$ cd.?

19. Frank earned $\$\frac{4}{5}$, and Ben earned $\$\frac{1}{2}$. What part of a dollar did Frank earn more than Ben? (Make the units alike. Why? How?)

20. Lulu had $\frac{5}{8}$ of a yard of ribbon. After using $\frac{3}{4}$ of a yard, what part of a yard had she left?

21. If from $\frac{3}{5}$ of a pennyweight of gold $\frac{2}{5}$ of a pennyweight be used, what part of a pennyweight will be left?

22. A boy picked $18\frac{1}{8}$ pks. of cherries from two trees. From one tree he picked $10\frac{9}{16}$ pks. How many pecks did he pick from the other?

23. Julia added two numbers, and her answer was $20\frac{1}{2}$. One of the numbers was $11\frac{5}{7}$. What was the other?

24. A tub of butter, with the tub, weighed $25\frac{3}{16}$ lbs. The tub weighed $3\frac{3}{4}$ lbs. What was the weight of the butter?

25. Harold bought a calf for $\$7\frac{3}{5}$, and sold it for $\$9\frac{1}{4}$. How many dollars did he gain?

26. A man has $23\frac{1}{3}$ rds. of ditch to dig. After he has dug $12\frac{4}{6}$ rds., how many rods remain to be dug?

27. The distance between two towns is $25\frac{1}{8}$ mi. After a boy has gone $14\frac{1}{6}$ mi., how many miles farther has he to go?

28. What number must be added to $13\frac{1}{4}$ to make $26\frac{1}{7}$?

29. John learned the arithmetic lesson in $\frac{14}{15}$ of an hour, and George learned it in $\frac{4}{5}$ of an hour. Which learned it in the shorter time? What part of an hour shorter?

WRITTEN EXERCISES.

191. Find the value of:

30. $47\frac{5}{8}-16\frac{31}{32}$.

31. $71\frac{1}{5}-30\frac{6}{7}$.

32. $58\frac{1}{2}-13\frac{14}{15}$.

33. $93\frac{5}{9}-48\frac{3}{45}$.

34. $100\frac{5}{24}-60\frac{1}{36}$.

35. $85\frac{17}{22}-72\frac{8}{33}$.

36. $126\frac{3}{5}-99\frac{5}{17}$.

37. $173\frac{3}{5}-101\frac{7}{51}$.

38. $69\frac{7}{15}-34\frac{33}{55}$.

39. $314\frac{8}{9}-97\frac{7}{8}$.

40. $500\frac{7}{24}-300\frac{17}{36}$.

41. $619-286\frac{7}{25}$.

42. $478\frac{17}{18}-290$.

43. $100\frac{1}{4}-100\frac{1}{25}$.

192. Denominate Numbers.

EXERCISES.

1. From 25 bu. 2 pks. 5 qts., take 21 bu. 3 pks. 7 qts. 1 pt.

bu.	pks.	qts.	pt.
25	2	5	0
21	3	7	1
3	2	5	1

EXPLANATION: Write the subtrahend under the minuend, units of the same kind in the same column. Why? Begin with the lowest or right-hand denomina-

tion. Why? There are no pints in the minuend. Take 1 qt. from the 5 qts. 1 qt.=2 pts. 2 pts.−1 pt.=1 pt., which is expressed under the column of pints. 7 qts. can not be taken from 4 qts. Take 1 pk. from 2 pks. 1 pk.=8 qts. 8 qts.+4 qts.=12 qts. 12 qts.−7 qts.=5 qts., which is expressed in the remainder. 3 pks. can not be taken from 1 pk. Take 1 bu. from 25 bu. 1 bu.=4 pks. 4 pks.+1 pk.=5 pks. 5 pks.−3 pks.=2 pks., expressed in the remainder. 24 bu.−21 bu.=3 bu., expressed in the remainder. How does this differ from subtraction in simple numbers?

2. From a barrel of molasses containing 38 gals. 2 qts. 1 pt. 1 gi., a grocer sold in one day 12 gals. 3 qts. 1 pt. 3 gi. How many gallons, etc., remained in the barrel?

3. A man having a pile of wood containing 32 cd. 4 cd. ft. 10 cu. ft., used 14 cd. 6 cd. ft. 15 cu. ft. for fuel. How many cords, etc., were left?

4. Mr. Long, having £23 12s. 3d. ready money, paid a debt of £7 18s. 10d. 3 far. How much money had he left?

5. A lady bought 5 yds. of linen. How much had she left after using 3 yds. 2 ft. 9 in.?

6. A quantity of gold weighed 5 lbs. 7 oz. 11 pwt. before refining, and 3 lbs. 11 oz. 17 pwt. 21 gr. afterward. How much of the weight was lost by the process?

7. From 6 T. 1,100 lbs. 8 oz., take 2 T. 1,565 lbs. 15 oz.

8. From 872 cu. yds. 872 cu. in., take 300 cu. yds. 25 cu. ft. 1,287 cu. in.

9. What is the difference between 37° 47′ 28″ and 20° 54′ 45″?

10. Subtract 89 days 15 hrs. 54 min. 48 sec. from 248 days 9 hrs. 47 min. 18 sec.

11. How many acres, square rods, and square yards must be put with 16 A. 150 sq. rds. 14 sq. yds. to make a field that will contain 45 A. 60 sq. rds. 29 sq. yds.?

12. How much greater is 8 mi. 35 ch. 2 rds. 13 l. than 3 mi. 65 ch. 3 rds. 20 l.?

13. What is the time between 1.00 P. M., July 4, 1776, and 6.00 A. M., Jan. 8, 1865?

yr.	mo.	day.	hr.
1865	1	8	6
1776	7	4	13
88	6	3	17

SUGGESTIONS: The later date is the minuend; January is the first month and July is the seventh month of the year. 6.00 A. M. is the sixth hour of the day, and 1.00 P. M. is the thirteenth hour.

SHORTER SOLUTION:

1. Write no figures except the remainder.
2. Find all the whole years.
3. Find all the whole months.
4. Find all the whole days.
5. Find all the whole hours, if they are required.
6. Find all the whole minutes, if they are required.

ILLUSTRATION: From July 4, 1776, to July 4, 1864, is 88 yrs.; from July 4, 1864, to Jan. 4, 1865, is 6 mo.; from Jan. 4, 1865, to Jan. 7, is 3 days; from 1.00 P. M., Jan. 7, to 6.00 A. M., Jan. 8, is 17 hrs.

Solve the following without figures:

14. James A. Garfield was born Nov. 19, 1831, and died Sept. 19, 1881. How old was he? If he were living, how old would he be to-day?

15. The war of the American Revolution was begun April 19, 1775, and peace was restored Jan. 20, 1783. What length of time did the war continue?

16. What was the length of time from the landing of the Pilgrims, Dec. 22, 1620, to the Declaration of Independence, July 4, 1776?

17. Required, the time between the births of Benjamin Franklin, Jan. 17, 1706, and Horace Mann, May 4, 1796.

18. How old was Horace Mann when he died, Aug. 2, 1859?

19. How long has a note to run that is dated Jan. 4, 1895, and made payable Aug. 8, 1898?

20. What length of time will elapse from 40 min. past 3 o'clock P. M., Sept. 5, 1896, to 15 min. past 7 o'clock A. M., June 18, 1902?

193. Subtraction Review.

EXERCISES.

1. A farmer having $450\frac{3}{8}$ A. of land, sold $294\frac{5}{8}$ A. How many acres had he left?

2. What number must be added to .000465 to make .023?

3. From $\frac{1}{2}$ sec. of land, take 236 A. 120 sq. rds. $12\frac{1}{4}$ sq. yds.

4. A dealer bought a quantity of coal for $\$144\frac{5}{8}$, and sold it for $\$182\frac{1}{8}$. How much did he gain?

5. A speculator sold a quantity of wool for \$3,716.375, and gained \$319.4375. Find the cost of the wool.

6. Subtract 9 T. 1,437 lbs. 15 oz. from 13 T. 500 lbs. 10 oz.

7. The sum of two numbers is $97\frac{1}{15}$, and the smaller number is $39\frac{1}{4}$. What is the larger number?

8. How many pounds of sugar must be put with 196.875 lbs., to make a barrel of sugar weighing 300 lbs.?

9. From $26\frac{7}{24}$ take $25\frac{13}{15}$.

10. From one cellar was taken 1,672 cu. yds. 25 cu. ft. 1,495 cu. in., and from another 1,948 cu. yds. 14 cu. ft. 1,687 cu. in. How much more was taken from the second cellar than from the first?

11. 42.08—34.0574=?

12. $481\frac{18}{25}-397\frac{33}{50}$=?

13. How much greater is 7 rds. 5 yds. 2 ft. 5 in. than 3 rds. 4 yds. 2 ft. 9 in.?

14. From 94 cd. 5 cd. ft. 7 cu. ft. 69 cu. in. take 75 cd. 7 cd. ft. 12 cu. ft. 85 cu. in.

15. The difference between two numbers is $3\frac{9}{14}$. The minuend is $28\frac{6}{63}$. What is the subtrahend?

16. The minuend is 400 and the subtrahend is .00004. What is the difference?

17. $71\frac{7}{15}$ is $14\frac{5}{13}$ greater than what number?

18. What is the difference between 41 sq. rds. 27 sq. yds. 5 sq. ft. 100 sq. in. and 18 sq. rds. 18 sq. yds. 8 sq. ft. 128 sq. in.?

19. One ship is in north latitude 43° 43′ 43″ and another is in north latitude 60° 27′ 35″. How many degrees, minutes, and seconds of latitude are they apart?

20. What number must be added to $23\frac{5}{36}$ to give $62\frac{11}{24}$?

21. 17 lbs. 3 oz. 14 pwt. 18 gr. is how much greater than 9 lbs. 9 oz. 19 pwt. 19 gr.?

22. What is the difference between $\frac{3}{51}$ and $\frac{11}{68}$?

23. If we add 8 to both terms of $\frac{5}{7}$, do we increase or diminish the value of the fraction, and how much?

MULTIPLICATION.

194. *1.* Define multiplication. Multiplicand. Multiplier. Product.

2. Define number. Concrete number. Abstract number.

3. What kind of a number is the multiplicand? Multiplier? Product?

4. Define a decimal fraction. Mixed number. In how many ways may a mixed decimal be read? What are they?

5. What is the use of the decimal point? The zero?

6. What is the effect upon the value of a figure when it is moved one place to the right? Two places to the right? Three places to the right?

EXERCISES.

195. *1.* Multiply 27.5 by 28.

27.5
28
2200
550
770.0

EXPLANATION: We are asked to find 28 times 275 tenths. 28 times 275 is always 7,700; and as the multiplicand is tenths, so must the product be tenths; that is, 28 times 27.5=770.0=770.

2. Multiply .75 by .3.

.75
.075
3
.225

EXPLANATION: This problem asks us to find 3 tenths of 75 hundredths. $\frac{1}{10}$ of .75 is .075, found by moving the figures of the multiplicand one place toward the right. $\frac{3}{10}$ of .75 is 3 times .075, or .225.

3. Multiply 4.25 by 2.16.

4.25
.0425
216
.2550
.425
8.50
9.1800

EXPLANATION: We are to find 216 hundredths of 425 hundredths. $\frac{1}{100}$ of 4.25 is .0425, found by moving the figures of the multiplicand two places to the right. $\frac{216}{100}$ of 4.25 is 216 times .0425, or 9.18.

4. Multiply .15 by .3.

.15
.015
3
.045

EXPLANATION: In this problem we are to take 3 tenths of 15 hundredths. The process is the same as before; but in the product it is found necessary to use a zero between the figures and the decimal point.

In each of the examples just explained, what is the sum of the decimal places in both multiplicand and multiplier? How many decimal places in each product?

Make a rule for the number of decimal places in the product.

Find the cost of:

5. 24 lbs. of butter, at 23.5 cts. a pound.

6. 35 bbls. of flour, at $4.95 a barrel.

7. 258 sheep, at $8.65 apiece.

8. 218 qts. of berries, at $.095 a quart.

9. 18.6 T. of coal, at $7.5 a ton.

10. 138.6 bu. of wheat, at $.8725 a bushel.

11. 62.75 bu. of oats, at $.45 a bushel.

12. 13.375 cd. of wood, at $4.6 a cord.

13. A section of land, at $57.875 an acre.

14. 45.6 T. of hay, at $16.25 a ton.

15. A barrel of sugar, weighing 288 lbs., at $.04$\frac{7}{8}$ a pound. (There are only two decimal places in $.04$\frac{7}{8}$.)

16. .75 of a pound of beef, at $.08$\frac{1}{3}$ a pound.

17. .66$\frac{2}{3}$ of a dozen of bananas, at $.12 a dozen.

18. 18 oranges, at $.045 apiece.

19. 15 lbs. of nails, at $.04$\frac{2}{3}$ a pound.

20. A mileage book of 1,000 mi., at $.02 a mile.

21. 375.6 lbs. of coffee, at $.16$\frac{2}{3}$ a pound.

22. A man distributed a sum of money among 29 poor families, giving each $3.85. How much did he give to all?

23. If a vessel sails 14.375 mi. an hour, how many miles will it sail in 2 days?

24. There are 24.75 cu. ft. in a perch of masonry. How many cubic feet in 35.56 perches?

25. If an acre of land produces 128.25 bu. of potatoes, how many bushels will 40.5 A. produce at the same rate?

26. If a cubic foot of granite weighs 168.48 lbs., how much will a block weigh that is 4.5 ft. long, 3.75 ft. wide, and .8 of a foot thick?

27. When a bushel of corn is worth 2.6 bu. of oats, how many bushels of oats must be given for 38.5 bu. of corn?

28. A pound sterling is worth $4.8665. What are £28.8 worth?

29. A gallon of distilled water weighs 8.33888 lbs. What is the weight of 46.875 gals.?

30. The circumference of a circle is 3.1416 times its diameter. What is the circumference of a circle whose diameter is .0065 of a rod?

31. If the circumference of the wheel of a bicycle is 10.085 ft., how far will it go in making 485.0006 revolutions?

32. A mark, German money, is worth $.2385. What is .18 of a mark worth?

33. A pound of oak wood, when burned, yields .022 of a pound of ashes. How many pounds of ashes will 9.125 lbs. yield?

34. A bushel contains 2150.42 cu. in. How many cubic inches are there in a bin which holds 87.625 bu.?

Multiply:

35. 56.4 by 38.

36. 6.34 by .045.

37. 42.2 by 2.005.

38. 761 by .013.

39. .673 by 64.

40. .0505 by .001.

41. 1943 by .0086.

42. .0084 by .025.

43. 512.3 by 86.431.

44. 37.0804 by 4.605.

45. 419.764 by 3.0001.

46. 40607.0004 by .30604.

47. 46000 by .80006.

48. 647.601 by 3.0701.

49. .00312 by 35.

50. 341.072 by 395.16045.

51. .05182 by .00026.

52. 8.4002 by 2.46.

196. To Multiply a Fraction by an Integer.

ORAL EXERCISES.

1. 4 times $\frac{2}{3}$ of an orange are how many thirds of an orange? How many oranges?

2. 3 times $\frac{4}{5}$ are how many fifths of a dollar? How many dollars?

3. 5 times $\frac{3}{4}$ of a pound are how many fourths of a pound? How many pounds?

4. 7 times $\frac{5}{8}$ of a peck are how many eighths of a peck? How many pecks?

5. 12 times $\frac{5}{6}$ of a day are how many days?

6. 10 times $\frac{8}{9}$ of a yard are how many yards?

7. 8 times $\frac{9}{12}$ of a degree are how many degrees?

8. 9 times $\frac{11}{16}$ of an acre are how many acres?

9. A boy earned $\$3\frac{3}{5}$ a day. How many dollars did he earn in 6 days?

10. How many dollars will 10 lbs. of coffee cost, at $\$\frac{7}{20}$ a pound?

11. 8 boys picked $\frac{5}{6}$ of a crate of berries apiece. How many crates did they all pick?

12. If a man builds $\frac{9}{11}$ of a rod of fence a day, how many rods will he build in 10 days?

13. How many tons of coal will a furnace that consumes $\frac{2}{3}$ of a ton a day, consume during the month of January?

14. What will 20 arithmetics cost, at $\$\frac{5}{8}$ apiece?

15. If the current of a river is $\frac{9}{11}$ of a mile an hour, how many miles will a canoe float in 12 hrs.?

16. What will it cost to dig 10 rds. of drain, at $\$1\frac{3}{20}$ a rod?

17. A boy sold 20 lbs. of old iron, at $\frac{3}{10}$ of a cent a pound. How many cents did he receive for it?

18. If a coat requires $\frac{3}{4}$ of a dozen buttons, how many dozen buttons will be required for 15 coats?

19. If a family uses $\frac{3}{16}$ of a pound of butter a day, how many pounds will it use in 22 days?

20. One cubic foot contains $\frac{1}{2}$ of a bushel. How many bushels will a box hold that is 5 ft. long, 3 ft. wide, and 2 ft. high?

21. What must I pay for a gross of pencils, at $\$\frac{1}{24}$ apiece?

22. If $\frac{5}{8}$ of a peck of oats feed a horse 1 day, how many pecks will be required to feed it 2 wks.?

23. Find the cost of 15 pear trees, at $\$\frac{7}{12}$ apiece.

24. If Fred saws $\frac{11}{16}$ of a cord of wood a day, how many cords will he saw in 10 days?

25. The length of one side of a square is $\frac{5}{18}$ of a yard. How many yards in the length of the boundary of 2 such squares?

197. Short Solutions.

The solution of many problems of the nature of those just given may be shortened.

26. How many are 5 times $\frac{14}{16}$?

FIRST SOLUTION: 5 times $\frac{14}{16}=\frac{70}{16}=\frac{1}{8}=4\frac{3}{8}$.

SECOND SOLUTION: 5 times $\frac{14}{16}=\frac{14}{8}=4\frac{3}{8}$.

The result in the second case is obtained by dividing the denominator instead of multiplying the numerator; for, while the number of relative units is unchanged, their size is increased proportionately to the number used in dividing the denominator. This solution gives the answer in its lowest terms; it thus saves one reduction, and it avoids the use of larger numbers.

27. Multiply $\frac{7}{18}$ by 12.

In this example the denominator contains some of the prime factors of the multiplier, but not all. The result is obtained by taking from the multiplier and from the denominator all their common prime factors (which are 2 and 3), and multiplying the numerator by 2, the remaining factor of the multiplier. This gives the result $\frac{14}{3}=11\frac{1}{3}$ in its lowest terms.

Turn back to the problems already given, and see which of them can be solved by these short methods.

WRITTEN EXERCISES.

198. Multiply (use short method):

28. $\frac{89}{108}$ by 9.

29. $\frac{48}{67}$ by 19.

30. $\frac{71}{89}$ by 11.

31. $\frac{69}{120}$ by 36.

32. $\frac{47}{84}$ by 63.

33. $\frac{68}{75}$ by 45.

34. $\frac{93}{132}$ by 33.

35. $\frac{83}{271}$ by 81.

36. $\frac{13}{15}$ by 58.

37. $\frac{128}{360}$ by 60.

38. $\frac{108}{133}$ by 112.

39. $\frac{47}{256}$ by 144.

40. $\frac{25}{26}$ by 351.

41. $\frac{55}{44}$ by 473.

42. $\frac{13}{70}$ by 525.

43. $\frac{91}{462}$ by 546.

199. To Multiply an Integer by a Fraction.

ORAL EXERCISES.

This means taking a certain part, or a certain number of parts, of an integer.

1. What is $\frac{1}{3}$ of 21 cts.? $\frac{2}{3}$ of 21 cts.? $\frac{1}{4}$ of 28 cts.? $\frac{3}{4}$ of 28 cts.? $\frac{1}{5}$ of 30 cts.? $\frac{4}{5}$ of 30 cts.? $\frac{5}{6}$ of 48 cts.? $\frac{4}{7}$ of 56 cts.? $\frac{6}{9}$ of 63 cts.?

2. A boy paid 45 cts. for an arithmetic and $\frac{4}{9}$ as many cents for a slate. How many cents did he pay for the slate?

3. A boy earned $28 in one month, and paid $\frac{4}{7}$ of the money for a suit of clothes. How many dollars did the suit cost?

4. A farmer sold 36 bbls. of apples, and $\frac{5}{6}$ as many barrels of potatoes. How many barrels of potatoes did he sell?

5. What must I pay for $\frac{1}{4}$ of an acre of land, at $100 an acre? For $\frac{3}{4}$ of an acre?

6. A man paid $39 for a cow, and $\frac{8}{13}$ as many dollars for her calf. How many dollars did he pay for the calf?

7. Flora's age is $\frac{8}{25}$ of her mother's age. Her mother is 50 yrs. old. How old is Flora?

8. If a man travels 44 mi. a day, how many miles will he travel in $\frac{6}{11}$ of a day?

9. A man who had 56 A. to plow, plowed $\frac{3}{8}$ of it in 1 wk. How many acres did he plow that week?

10. There are 60 pupils in a school, $\frac{7}{12}$ of whom are girls. How many girls in the school? How many boys?

11. Jane has $\frac{8}{9}$ of 63 cts., and Lucy has $\frac{6}{7}$ of 63 cts. Which girl has the more cents? How many more?

12. A man who earns $54 a month pays $\frac{1}{3}$ of his salary for board. How many dollars does he pay a month for board?

13. An orchard contains 72 trees. $\frac{5}{12}$ of them are apple trees; $\frac{1}{4}$ of them are pear trees; and $\frac{2}{3}$ of them are cherry trees. How many trees are there of each kind?

14. A boy went from one city to another, a distance of 96 mi., on his bicycle. He went $\frac{3}{8}$ of the distance the first day, and $\frac{5}{16}$ of the distance the second day. How many miles did he go each day?

15. A farmer who had 51 sheep, sold $\frac{2}{3}$ of them. How many did he sell?

16. A man paid $75 for a watch, which was 5 times as many dollars as he paid for a chain. How many dollars did he pay for the chain?

17. Albert weighs 105 lbs., and George weighs $\frac{6}{7}$ as many pounds. How many pounds does George weigh?

18. The lessons in arithmetic for one week contained 90 problems. Frank solved $\frac{2}{3}$ of them, and his sister $\frac{5}{6}$ of them. Which solved the more? How many more?

19. How many are $\frac{5}{8}$ of 42 M bricks?

EXPLANATION: $\frac{1}{8}$ of 42 M bricks are $5\frac{1}{4}$ M bricks; and $\frac{5}{8}$ of the number are 5 times $5\frac{1}{4}$ M bricks, or $26\frac{1}{4}$ M bricks.

20. When hay is worth $16 a ton, what will $\frac{3}{5}$ of a ton cost?

21. If a vessel sails 18 mi. an hour, how many miles will it sail in $\frac{3}{4}$ of an hour?

22. In one month, Willie sawed 20 cd. of wood into stove-lengths. At that rate, how many cords did he saw in $\frac{5}{8}$ of a month?

23. A man who owned 26 A. of land sold $\frac{3}{7}$ of it. How many acres did he sell?

24. A man paid \$30 for a suit of clothes, and $\frac{2}{5}$ as many dollars for a hat and a pair of shoes. How many dollars did he pay for hat and shoes?

25. Agnes has read 38 pages of a certain book, and Hattie has read $\frac{9}{10}$ as many pages. How many pages has Hattie read?

26. A horse ate $\frac{8}{11}$ of 40 bu. of oats in one month. How many bushels did he eat?

27. A man's salary is \$96 a month. He pays $\frac{1}{6}$ of it for house rent, $\frac{5}{8}$ of it for provisions, and $\frac{1}{12}$ of it for labor. How many dollars does he pay for each?

28. What will $\frac{5}{8}$ of an acre of land cost, at \$91 an acre?

29. A and B bought a harvesting machine for \$125. A paid $\frac{5}{12}$ of the cost, and B the remainder. How many dollars did each pay?

30. A rectangular field is 66 rds. long; its width is $\frac{8}{15}$ of its length. What is its width?

31. The average rate of a fast express train is 52 mi. an hour. How many miles will it go in $\frac{5}{8}$ of an hour?

32. A farmer took 80 melons to market. He sold $\frac{1}{4}$ of them to one merchant, $\frac{5}{16}$ of them to another, $\frac{1}{5}$ of them to a third, and $\frac{3}{20}$ of them to a fourth. How many did he sell to each? How many had he left?

WRITTEN EXERCISES.

200. *33.* Multiply 64 by $\frac{25}{36}$.

EXPLANATION: This means, take $\frac{25}{36}$ of 64.

$\frac{1}{36}$ of $64=\frac{64}{36}$. $\frac{25}{36}$ of $64=\frac{64\times25}{36}=\frac{400}{9}=44\frac{4}{9}$.

Multiply (cancel when you can):

34. 105 by $\frac{7}{12}$.

35. 200 by $\frac{9}{14}$.

36. 19 by $\frac{13}{47}$.

37. 448 by $\frac{19}{56}$.

38. 239 by $\frac{9}{13}$.

39. 375 by $\frac{13}{15}$.

40. 42 by $\frac{51}{84}$.

41. 627 by $\frac{5}{7}$.

42. 572 by $\frac{23}{24}$.

43. 404 by $\frac{3}{11}$.

44. 391 by $\frac{11}{17}$.

45. 92 by $\frac{23}{35}$.

46. 384 by $\frac{31}{48}$.

47. 765 by $\frac{57}{85}$.

201. To Multiply a Fraction by a Fraction.

1. There is no essential difference between this process and the last one.

2. Define a compound fraction.

ORAL EXERCISES.

202. *1.* What part of a dollar is $\frac{1}{2}$ of $\frac{2}{5}$? (Can you show it?) $\frac{1}{3}$ of $\frac{6}{20}$? $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{1}{3}$ of $\frac{9}{10}$? $\frac{2}{3}$ of $\frac{9}{10}$? $\frac{1}{4}$ of $\frac{16}{20}$? $\frac{3}{4}$ of $\frac{16}{20}$? $\frac{1}{5}$ of $\frac{20}{25}$? $\frac{4}{5}$ of $\frac{18}{20}$? $\frac{3}{5}$ of $\frac{24}{25}$?

2. What part of a dollar is $\frac{1}{2}$ of $\$\frac{1}{2}$? (Can you show it?) What part of a dollar is $\frac{1}{2}$ of $\$\frac{1}{5}$? $\frac{1}{2}$ of $\$\frac{3}{5}$? $\frac{1}{5}$ of $\$\frac{1}{2}$? $\frac{4}{5}$ of $\$\frac{1}{2}$? $\frac{1}{5}$ of $\$\frac{1}{10}$? $\frac{1}{5}$ of $\$\frac{3}{10}$? $\frac{2}{5}$ of $\$\frac{6}{10}$? $\frac{1}{20}$ of $\$\frac{1}{5}$? $\frac{1}{20}$ of $\$\frac{3}{5}$? $\frac{7}{20}$ of $\$\frac{3}{5}$? $\frac{3}{10}$ of $\$\frac{8}{20}$? $\frac{1}{2}$ of $\$\frac{9}{25}$? $\frac{3}{4}$ of $\$\frac{11}{25}$?

3. What part of a dollar is $\frac{1}{2}$ of $\frac{1}{5}$ of $\$\frac{1}{10}$? $\frac{2}{5}$ of $\frac{1}{2}$ of $\$\frac{7}{10}$? $\frac{3}{10}$ of $\frac{2}{5}$ of $\$\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{5}$ of $\$\frac{3}{4}$? $\frac{1}{2}$ of $\frac{2}{3}$ of $\$\frac{3}{4}$? $\frac{2}{3}$ of $\frac{1}{2}$ of $\$\frac{3}{5}$? $\frac{1}{2}$ of $\frac{1}{5}$ of $\$\frac{3}{5}$?

4. What part of a dollar is $\frac{3}{4}$ of $\frac{2}{5}$ of $\frac{1}{5}$ of $\$\frac{9}{10}$? What is the continued product of the numerators? Of the denominators? What simple fraction in its lowest terms do they make? Make a rule for reducing a compound fraction to a simple fraction, or for multiplying one fraction by another.

5. At $\$\frac{4}{5}$ a gallon, what will $\frac{3}{4}$ gal. of syrup cost?

6. A lot containing $\frac{15}{16}$ of an acre was divided equally among 3 children. What part of an acre did each child receive?

7. If the fuel for a furnace costs $\$\frac{7}{8}$ a day, what will it cost for $\frac{6}{7}$ of a day?

8. Luke gathered $\frac{7}{8}$ of a peck of cherries, and sold $\frac{2}{3}$ of them. What part of a peck did he sell? How much did he receive for them at $\$\frac{9}{10}$ a peck?

9. A man who owned $\frac{5}{6}$ of a mill, sold $\frac{2}{5}$ of his share to his brother. What part of the mill did his brother buy?

10. If a field produces $\frac{6}{7}$ of a bale of cotton, what part of a bale does $\frac{5}{6}$ of the field produce?

11. If a pound of steak costs $\$\frac{3}{20}$, what must I pay for $\frac{15}{16}$ of a pound?

12. If a man can plow $\frac{7}{8}$ of a field in a week, what part of the field can he plow in $\frac{5}{6}$ of a week?

13. A man divided $\frac{6}{7}$ of a barrel of flour equally among 4 poor families. How much flour did each family receive?

14. A boy having $\$3\frac{3}{4}$ gave $\frac{4}{5}$ of it for a knife. How much did he pay for the knife?

15. What will $\frac{7}{9}$ of a yard of silk cost, at $\$1\frac{1}{2}\frac{5}{0}$ a yard?

16. If a horse eats $\frac{7}{8}$ of a bushel of oats in 6 feedings, how much does he eat in 4?

17. My strawberry bed is $\frac{10}{11}$ of a rod long, and $\frac{3}{5}$ as wide. How wide is it?

18. A grocer having $\frac{5}{6}$ of a barrel of vinegar, sold $\frac{3}{5}$ of it to one man. What part of a barrel did the man buy?

19. What is $\frac{3}{4}$ of $\frac{5}{8}$ of a mile?

20. A book cost $\$\frac{9}{10}$, and a slate $\frac{5}{6}$ as much. How much did the slate cost?

21. A man who sold his farm, loaned $\frac{4}{7}$ of his money, and invested $\frac{2}{3}$ as much in city property. What part of his money did he invest?

22. $\frac{7}{13}$ of the trees in an orchard are apple trees; there are $\frac{2}{3}$ as many peach trees as apple trees. What part of the orchard is in peach trees?

23. If $\frac{4}{9}$ of the population of a town is less than 21 years old, and $\frac{2}{3}$ of them are in school, what part of the population of the town attend school?

WRITTEN EXERCISES.

203. *24.* Multiply $\frac{32}{45}$ by $\frac{14}{15}$.

EXPLANATION: This means, find $\frac{14}{15}$ of $\frac{32}{45}$. $\frac{1}{15}$ of $\frac{32}{45}=\frac{32}{45\times15}$. $\frac{14}{15}$ of $\frac{32}{45}=\frac{32\times14}{45\times15}$. The result is a simple fraction with its terms partly factored. $\frac{32\times14}{45\times15}$ reduced to its lowest terms$=\frac{28}{225}$.

25. Multiply $\frac{5}{9}$ of $\frac{7}{16}$ by $\frac{9}{14}$ of $\frac{8}{15}$.

(1) $\frac{7}{16}$

(2) $\frac{7}{16\times9}$

(3) $\frac{7\times5}{16\times9}$

(4) $\frac{7\times5}{16\times9\times15}$

(5) $\frac{7\times5\times8}{16\times9\times15}$

(6) $\frac{7\times5\times8}{16\times9\times15\times14}$

(7) $\frac{7\times5\times8\times9}{16\times9\times15\times14}=\frac{1}{12}$

EXPLANATION: Express (1) $\frac{7}{16}$. (2) $\frac{5}{9}$ of $\frac{7}{16}$. (3) $\frac{5}{9}$ of $\frac{7}{16}$. (4) $\frac{1}{15}$ of this multiplicand. (5) $\frac{8}{15}$ of the multiplicand. (6) $\frac{1}{14}$ of the last result (7) $\frac{9}{14}$ of it. The result is a simple fraction with its terms partly factored. When reduced to its lowest terms, it equals $\frac{1}{12}$. All the common factors of numerator and denominator are canceled.

Multiply (cancel when you can):

26. $\frac{15}{17}$ by $\frac{51}{75}$.

27. $\frac{36}{65}$ by $\frac{52}{99}$.

28. $\frac{36}{49}$ by $\frac{14}{27}$.

29. $\frac{18}{19}$ by $\frac{11}{13}$.

30. $\frac{9}{56}$ by $1\frac{28}{153}$.

31. $\frac{45}{1296}$ by $\frac{7}{800}$.

32. $\frac{2}{3}\frac{6}{0}\frac{7}{0}$ by $\frac{5}{7}$.

33. $\frac{7}{8}\frac{1}{8}\frac{5}{2}$ by $\frac{9}{4}\frac{8}{2}\frac{8}{9}$.

34. $\frac{7}{8}$ of $\frac{4}{5}$ by $\frac{1}{2}\frac{3}{1}$.

35. $\frac{1}{1}\frac{}{8}$ of $\frac{3}{1}\frac{}{4}$ by $\frac{4}{5}$.

36. $\frac{1}{2}\frac{6}{5}$ by $\frac{8}{3}$ of $\frac{7}{9}\frac{5}{4}$.

37. $\frac{8}{9}$ by $\frac{3}{7}$ of $\frac{1}{3}\frac{1}{8}$.

38. $\frac{3}{4}\frac{3}{0}$ of $\frac{9}{1}\frac{}{1}$ by $\frac{8}{1}\frac{}{5}$ of $\frac{1}{1}\frac{3}{7}$.

39. $\frac{8}{3}\frac{5}{2}$ of $\frac{1}{2}\frac{8}{0}$ by $\frac{4}{5}$ of $\frac{1}{1}\frac{8}{9}$.

40. $\frac{4}{8}\frac{8}{1}$ of $\frac{4}{7}$ by $\frac{5}{6}$ of $\frac{9}{2}\frac{}{4}$.

41. $\frac{6}{7}$ of $\frac{8}{9}$ by $\frac{1}{1}\frac{1}{6}$ of $\frac{2}{4}\frac{1}{4}$.

42. $\frac{1}{2}\frac{9}{1}$ of $\frac{2}{4}\frac{5}{7}$ of $\frac{1}{3}\frac{5}{8}$ by $\frac{9}{2}\frac{4}{7}\frac{}{5}$.

43. $\frac{1}{1}\frac{7}{8}\frac{1}{0}$ by $\frac{3}{4}\frac{6}{7}$ of $\frac{2}{5}\frac{2}{7}$ of $\frac{1}{2}\frac{0}{1}\frac{5}{6}$.

204. Multiplication with a Mixed Number for One or Both Factors.

All such problems may be solved by changing the mixed numbers to improper fractions, and then proceeding as we have already learned to do. But in most cases it is better to use the mixed numbers without change, as in the following illustrations:

1. Find the cost of 7 cows, at $\$35\frac{3}{4}$ apiece.

$\$\ 35\frac{3}{4}$
$\quad\ 7$
$\quad 5\frac{1}{4}$
$\quad 245$
$\$250\frac{1}{4}$

EXPLANATION: 7 times $\$\frac{3}{4}=\$5\frac{1}{4}$. 7 times $\$35=\$245$. $\$245+\$5\frac{1}{4}=\$250\frac{1}{4}$.

2. At $\$2\frac{3}{4}$ a yard, what will $6\frac{5}{8}$ yds. of silk cost?

$\$\ 2\frac{3}{4}$
$\quad 6\frac{5}{8}$
$\quad\ \frac{1}{3}\frac{5}{2}$
$\quad 1\frac{1}{4}$
$\quad 16\frac{1}{2}$
$\$18\frac{7}{3}\frac{}{2}$

EXPLANATION: Multiplying by the parts of the multiplier, we have, first, $\frac{5}{8}$ of $\$\frac{3}{4}=\$\frac{1}{3}\frac{5}{2}$. Next, $\frac{5}{8}$ of $\$2=\$\frac{1}{8}^{0}$, or $\$1\frac{1}{4}$. Then, 6 times $\$2\frac{3}{4}=\$16\frac{1}{2}$, found as in the preceding example. $\$\frac{1}{3}\frac{5}{2}+\$1\frac{1}{4}+\$16\frac{1}{2}=\$18\frac{7}{3}\frac{}{2}$.

ORAL EXERCISES.

205. *1.* What must I pay for a railroad ticket for 15 mi., at $2\frac{2}{3}$ cts. a mile?

2. How many yards are there in 14 rds.?

3. What is the cost of a dozen mahogany chairs, at $\$3\frac{5}{8}$ apiece?

4. If 5 men can mow a field in $6\frac{7}{16}$ hrs., how many hours will it take 1 man?

5. A boy paid $\$12\frac{3}{8}$ for a coat, and $\frac{1}{3}$ as much for a hat. How much did he pay for the hat?

6. Harry has $\$3\frac{1}{4}$, and Laura has $\frac{3}{5}$ as much. How much money has Laura? (First find $\frac{1}{5}$.)

7. If a bird flies $10\frac{1}{3}$ mi. in $\frac{1}{3}$ of an hour, how far can it fly in 1 hr.?

8. If a horse eats $2\frac{5}{8}$ bu. of oats in a week, how many bushels would 10 horses eat?

9. What will $2\frac{3}{4}$ doz. of eggs cost, at 16 cts. a dozen?

10. What is the cost of 3 lbs. of steak, at $12\frac{1}{2}$ cts. a pound?

11. How far will James row in 4 hrs., if he rows $3\frac{5}{8}$ mi. an hour?

12. A boy paid $\$1\frac{5}{8}$ for a bridle, and 9 times as much for a saddle. What did the saddle cost?

13. What will 7 cd. of wood cost, at $\$6\frac{2}{10}$ a cord?

14. If a family uses $3\frac{7}{16}$ lbs. of sugar a week, how many pounds will they use in 6 wks.?

15. How much will it cost to build $5\frac{1}{2}$ rds. of fence, at $\$3\frac{3}{5}$ a rod?

16. What will 20 lbs. of nails cost, at $4\frac{4}{5}$ cts. a pound?

17. At $12\frac{1}{2}$ cts. a pound, what cost $5\frac{3}{4}$ lbs. of pepper?

18. How far can a man walk in $8\frac{3}{5}$ hrs., if he averages $3\frac{4}{5}$ mi. an hour?

19. What will 12 boxes of berries cost, at $8\frac{1}{3}$ cts. a box?

20. If a boy picks $4\frac{3}{5}$ qts. of berries in an hour, how many quarts will he pick in $8\frac{3}{4}$ hrs.?

21. How many hours are there in $2\frac{7}{12}$ days?

22. If cork weighs 15 lbs. per cubic foot, what is the weight of $6\frac{3}{5}$ cu. ft.?

23. What will $2\frac{4}{5}$ A. of land cost, at $\$40$ an acre?

WRITTEN EXERCISES.

206. *24.* What will 16 bu. of oats cost, at $37\frac{1}{2}$ cts. a bushel?

25. Find the cost of 36 yds. of carpet, at $\$4\frac{3}{5}$ a yard.

26. A water-pipe discharges $18\frac{3}{4}$ gals. an hour. How many gallons will it discharge in $26\frac{8}{15}$ hrs.?

27. What will $120\frac{1}{2}$ A. of land cost, at $\$85\frac{9}{10}$ an acre?

28. A steamer that sails $16\frac{7}{8}$ mi. an hour, will sail how many miles in $34\frac{4}{5}$ hrs.?

29. A certain strip of land is $23\frac{9}{11}$ rds. wide and $17\frac{4}{5}$ times as long. How long is it?

Multiply:

30.	$40\frac{1}{3}$	by 63.	*38.*	$156\frac{2}{3}$	by $54\frac{9}{10}$.
31.	$81\frac{7}{18}$	by 27.	*39.*	$28\frac{1}{3}$	by $31\frac{9}{16}$.
32.	$39\frac{3}{10\,5}$	by 75.	*40.*	$44\frac{3}{11}$	by $77\frac{1}{4}$.
33.	$125\frac{5}{16}$	by 80.	*41.*	$365\frac{7}{8}$	by $126\frac{1}{4}$.
34.	85	by $13\frac{1}{6}$.	*42.*	$746\frac{1}{6}$	by $98\frac{4}{6}$.
35.	96	by $19\frac{5}{12}$.	*43.*	$247\frac{1}{3}$	by $76\frac{2}{3}$.
36.	145	by $11\frac{2}{7}$.	*44.*	$327\frac{1}{2}$	by $18\frac{9}{2\,5}$.
37.	715	by $14\frac{4}{15}$.	*45.*	$64\frac{7}{15}$	by $135\frac{1}{15}$.

207. Denominate Numbers.

EXERCISES.

1. Multiply 38 rds. 5 yds. 2 ft. 10 in. by 9.

mi.	rds.	yds.	ft.	in.
	38	5	2	10
				9
1	31	4		

EXPLANATION: Write as in the margin. 9 times 10 in. are 90 in.=7 ft. 6 in. 9 times 2 ft. are 18 ft. 18 ft.+7 ft. 6 in.=25½ ft.=8½ yds. 9 times 5 yds. are 45 yds. 45 yds.+8½ yds.=53½ yds.=9 rds. 4 yds. 9 times 38 rds.=342 rds. 342 rds.+9 rds.=351 rds.=1 mi. 31 rds. Hence, 9 times 38 rds. 5 yds. 2 ft. 10 in.=1 mi. 31 rds. 4 yds.

NOTE: By a little skill in handling the numbers, the solution of such problems may often be accomplished more briefly and with smaller numbers. For instance, in this example it may be seen that 5 yds. 2 ft. 10 in.=1 rd. 1½ ft. 9 times 1½ ft.=12 ft.=4 yds.; and 9 times 1 rd.=9 rds., etc.

2. Find 9 times 3 bu. 1 pk. 5 qts.

bu.	pks.	qts.
3	1	5
		9
30	2	5

EXPLANATION: 9 times 1 qt.=1 pk. and 1 qt.; hence, 9 times 5 qts.=5 pks. and 5 qts. The 5 qts. we express; the 5 pks.=1 bu. and 1 pk. 9 times 1 pk. give 2 more bushels and another peck. We express the 2 pks. and add the 3 bu. we now have, to 9 times 3 bu.

In the following problems try to find short solutions:

3. What is the weight of a dozen gold cups, each weighing 9 oz. 15 pwt. 8 gr.?

4. A farmer sold 7 loads of oats, averaging 39 bu. 3 pks. 6 qts. per load. How many bushels, etc., did he sell?

5. What is the value of 15 A. of land, at £15 12s. 5d. 3 far. per acre?

6. If a water-pipe discharges 2 hhds. 28 gals. 3 qts. 1 pt. of water an hour, how much will it discharge in 10 hrs.?

7. How much hay is there in 14 stacks, averaging 4 T. 1,500 lbs. per stack?

8. If a man cut 3 cd. 6 cd. ft. 14 cu. ft. of wood in a day, how much wood will he cut in 6 days?

9. How much land in 8 fields, each containing 36 A. 120 sq. rds. 28 sq. yds.?

10. If the moon moves in her orbit, 10° 11′ 35″ in a day, how far will it move in 18 days?

11. If a man builds 8 rds. 4 yds. 1 ft. 8 in. of fence in a day, how much will he build in 6 days?

12. How much land is there in 5 lawns, each containing 11 sq. rds. 25 sq. yds. 8 sq. ft. 108 sq. in.?

13. Multiply 19 wks. 5 days 21 hrs. 45 min. by 9.

14. Multiply 12 T. 1,600 lbs. 15 oz. by 24.

15. Multiply 54 mi. 75 ch. 3 rds. 20 l. by 35.

16. Multiply 96 bu. 2 pks. 5 qts. 1 pt. by 23.

17. Multiply 112 cd. 7 cd. ft. 10 cu. ft. by 48.

18. Multiply 25 lbs. 8 oz. 18 pwt. 21 gr. by 11.

19. Multiply £12 14s. 9d. 3 far. by 14.

20. Multiply 126° 36′ 48″ by 27.

21. Multiply 28 gals. 3 qts. 1 pt. 2 gi. by 100.

22. 64 cu. yds. 24 cu. ft. 684 cu. in. × 42 = ?

23. 10 gt. gross 9 gross 7 doz. 11 ones × 81 = ?

24. 7 bales 4 bun. 1 rm. 14 qu. 20 sheets × 8 = ?

25. 8 mi. 160 rds. 3 yds. 2 ft. 9 in. × 16 = ?

26. 5 cen. 75 common years 275 days 19 hrs. 58 min. 44 sec. × 24 = ?

208. Multiplication Review.

EXERCISES.

1. What will be the cost of a bag of rice containing 97.75 lbs., at $.024 a pound?

2. Find the cost of 54$\frac{1}{8}$ T. of hay, at 16\frac{3}{4}$ a ton.

3. How far can a man walk in 8 days, walking 10 hrs. a day, if he averages 3 mi. 240 rds. an hour?

4. How many cubic yards of earth will be removed from a drain 189$\frac{3}{4}$ yds. long, $\frac{2}{3}$ of a yard wide, and 1$\frac{1}{2}$ yds. deep?

5. How many pounds of cement will be required to plaster a cistern, whose dimensions are $5\frac{2}{3}$ ft. square and $14\frac{1}{2}$ ft. deep, if the cement on a square foot weighs $6\frac{5}{8}$ lbs.?

6. The multiplier is 8.412 and the multiplicand is $887.03. What is the product?

7. A farmer put his crop of rye into 65 bags, each holding 2 bu. 1 pk. 5 qts. How many bushels of rye had he?

8. Find 9 times 3 Sec. 500 A. 130 sq. rds. 15 sq. yds. 6 sq. ft. 114 sq. in.

9. What will 8 T. of steel cost, at $.0125 a pound?

10. At $.0275 a mile, what is the cost of a railroad ticket for 56.84 mi.?

11. How many bushels of potatoes will $18\frac{5}{8}$ A. of land produce, at the rate of $124\frac{3}{4}$ bu. per acre?

12. If an acre of land produces .875 of a bale of cotton, what will .75 of an acre produce?

13. Multiply $\frac{3}{13}$ of $16\frac{4}{11}$ by $\frac{11}{15}$ of $90\frac{5}{8}$. (Arrange the work in the form of a simple fraction, with its terms partly factored.)

14. Multiply 5 T. 1,800 lbs. 14 oz. by 23.

15. Multiply 4.128 by .00025.

16. What will $88\frac{3}{10}$ A. of land cost, at $\$88\frac{3}{10}$ an acre?

17. $1413.6 \times .00045 = ?$

18. $50.05 \times 70.007 = ?$

19. 38 cu. yds. 26 cu. ft. 1,628 cu. in. $\times 27 = ?$

20. 5 wks. 5 days 21 hrs. 59 min. 45 sec. $\times 25 = ?$

21. $384 \times 45\frac{1}{3} = ?$

22. $57\frac{1}{4} \times 48\frac{2}{3} = ?$

23. $70\frac{8}{15} \times 50\frac{3}{4} = ?$

24. $.00186 \times .00047 = ?$

DIVISION.

209. *1.* Define division. Partition. Dividend. Divisor. Quotient.

2. If the dividend is a concrete number, what kind of a number is the quotient in division? In partition?

3. Define primary unit. Relative unit.

EXERCISES.

210. *1.* The divisor is 4 and the dividend 12. What is the quotient?

2. The divisor is 5 times 4 and the dividend 5 times 12. What is the quotient?

3. The divisor is 7 and the dividend 28. What is the quotient?

4. The divisor is 3 times 7, and the dividend 3 times 28. What is the quotient?

NOTE: The preceding problems illustrate the truth that: If both divisor and dividend are multiplied by the same number, the quotient is not changed.

5. When rice is selling at .015 of a dollar a pound, how many pounds will .75 of a dollar buy?

$.015)$.750
50 times.

EXPLANATION: $.75 will buy as many pounds as there are *times* $.015 in $.75. This is a problem in division, and divisor and dividend must be alike. Consider the dividend as $.750. $.015 is in $.750 50 times. Hence, $.75 will buy 50 lbs. of rice, at $.015 a pound.

6. If 15 bu. of apples cost $9.375, what is the price per bushel?

15)$9.375
$.625

EXPLANATION: One bushel of apples will cost $\frac{1}{15}$ of $9.375, which is $.625.

7. $.075 ÷ .15 = what? (This means, What is 100 fifteenths of $.075, or $\frac{1}{15}$ of 100 times $.075?)

15)$7.5
$.5

ANOTHER EXPLANATION: Make the divisor an integer, by moving the figures of divisor and dividend two places to the left; that is, by multiplying both by 100. The problem may then be stated in this way: What is $\frac{1}{15}$ of $7.5? $\frac{1}{15}$ of $7.5 is $.5.

8. $0.15 ÷ 7.5 = what?

75)$.150
$.002

EXPLANATION: Multiply both terms by 10; then the question is, "What is $\frac{1}{75}$ of $.15?" $\frac{1}{75}$ of $.150 is $.002.

In each of the problems 5 to 8, how many decimal places in the divisor? How many in the dividend? How many in the quotient? How does the sum of the number in divisor and quotient compare with the number in the dividend?

9. .7 × .5 = ? .6 × .09 = ? .04 × .012 = ?

How many places in the product of two decimal factors?

If the product and one of the factors are given, how may the number of decimal places in the other factor be found?

Since the divisor and quotient are the factors that make the dividend, how can the number of decimal places in the quotient be found, from the number in the divisor and dividend?

Make a rule for pointing off the quotient in division of decimals.

10. At $.18 a yard, how many yards of gingham will $12.06 buy?

11. A merchant paid $49.95 for 27 yds. of silk. What was the price per yard?

12. How many bushels of wheat can be bought for $87.75, at $.65 a bushel?

13. At $.125 a dozen, how many dozen eggs can be bought for $12.25?

14. If 93 baskets of peaches are sold for $125.55, what is the price per basket?

15. An errand boy earns $2.75 a week. In how many weeks will he earn $49.5?

16. At $3.25 a box, how many boxes of oranges can be bought for $78?

17. If a man walks 3.5 mi. an hour, in how many hours can he walk 588 mi.?

18. A merchant paid $4,009 for 844 bbls. of flour. What did he pay per barrel?

19. 31.5 gals. of water make a barrel. How many barrels are there in a cistern that contains 5,567.625 gals.?

20. There are 23.22 gr. of pure gold in a gold dollar. How many dollars can be made from 580.5 gr. of gold?

21. A perch of masonry contains 24.75 cu. ft. How many perches are there in 497.2275 cu. ft.?

22. A dealer paid $641.25 for 38 T. of hay. How much did he pay a ton?

23. A merchant paid $44.37 for a hogshead of sugar, weighing 986 lbs. What did he pay a pound for the sugar?

24. A pound sterling is worth $4.8665. How many pounds are worth $344.5482?

25. A dairyman sold in one summer 47,296 qts. of milk, for which he received $2,128.32. At what price per quart did he sell it?

26. The circumference of a circle is 3.1416 times its diameter. What is the diameter of a circle whose circumference is .23562 of a yard?

27. A bushel contains 2,150.42 cu. in. How many bushels will a bin hold that contains 827,911.7 cu. in.?

28. A gallon of water weighs 8.3388 lbs. How many gallons are there in 90.809532 lbs.?

29. A mark is worth $.2385. How many marks are worth $1.1626875?

30. There are 231 cu. in. in a gallon. How many gallons are there in 209.748 cu. in.?

31. The price of a railroad ticket for 568 mi. is $15.62. What is the rate per mile?

WRITTEN EXERCISES.

Divide:

1. 4096 by .032.	*11.* 1.8375 by 375.
2. 21.13212 by .916.	*12.* 2483.25 by 5.15625.
3. 36.72672 by .5025.	*13.* 142.0281 by 9.2376.
4. 1.288 by 23.	*14.* 9000 by .009.
5. 81.2096 by 1.28.	*15.* .009 by 9000.
6. 1 by 100.	*16.* 2401 by 21.4375.
7. .3 by 100.	*17.* 46 by 800.
8. 8.9 by 8900.	*18.* .07 by .0001.
9. .42 by .00875.	*19.* .003 by 30000.
10. .0066 by .22.	*20.* 16.275 by .41664.

211. To Divide a Fraction by an Integer.

ORAL EXERCISES.

1. At \$2 a yard, how many yards of cloth will \$4 buy? What kind of units is the dividend? Divisor? Quotient?

2. At \$2 a yard, how much cloth will \$$\frac{4}{5}$ buy? Which number is the divisor? What kind of units is the divisor? The dividend? Can the units of the divisor and the dividend be made the same? What will those units be? How many fifths of a dollar in \$2? What part of \$$\frac{10}{5}$ are \$$\frac{4}{5}$? Then what part of a yard will \$$\frac{4}{5}$ buy?

3. If 6 yds. of cloth cost $\$\frac{9}{10}$, what is the price per yard? Is this a problem in division or partition? How will the quotient in this problem differ from the quotient in the last?

EXPLANATION: 1 yd. will cost $\frac{1}{6}$ as much as 6 yds., or $\frac{1}{6}$ of $\$\frac{9}{10}$, which is $\$\frac{3}{10}$, found as already learned in multiplication.

4. At $3 a yard, how much silk will $\$\frac{4}{5}$ buy?

5. If 5 lbs. of butter cost $\$\frac{3}{5}$, what is the price per pound?

6. If a man walks $\frac{3}{5}$ of a mile in 7 min., how far does he walk each minute?

7. How long will it take a boy to earn $\$\frac{10}{11}$ at $5 a week?

8. If a boy picks $\frac{9}{10}$ of a crate of berries in 6 hrs., what part of a crate does he pick per hour?

9. If 4 bu. of seed are required to plant an acre of land, how much land will $\frac{3}{5}$ of a bushel plant?

10. If $\frac{3}{4}$ of a bushel of walnuts fill 12 bags of the same size, what part of a bushel does each bag hold?

11. A man distributed $\frac{3}{25}$ of a ton of coal equally among 16 poor families. What part of a ton did each receive?

12. If 3 T. of hay feed a horse a year, how long will $\frac{11}{12}$ of a ton feed it?

13. If a boy hoes an acre of potatoes in 4 hrs., how much can he hoe in $\frac{10}{13}$ of an hour?

14. If $\frac{3}{4}$ of a section of land be divided into 4 farms of equal size, what part of a section in each farm?

15. When 3 doz. oranges cost $\$\frac{9}{10}$, what is the cost of 1 orange?

16. If an acre of land yields 9 bu. of flaxseed, how much land yields $\frac{7}{8}$ of a bushel?

17. At \$7 a cord, how many cords of wood can be bought for $\$39\frac{1}{5}$? Is this a problem in division or partition?

$\$7)\$39\frac{1}{5}$
$5\frac{3}{5}$ times.

EXPLANATION: $\$39\frac{1}{5}$ will buy as many cords at \$7 a cord as there are *times* \$7 in $\$39\frac{1}{5}$. \$7 is in \$39 5 *times*, with a remainder of \$4. Change the remainder to the next lower denomination. $\$4=\$\frac{20}{5}$. $\$\frac{20}{5}+\$\frac{1}{5}=\$\frac{21}{5}$. \$7 is in $\$\frac{21}{5}$ $\frac{3}{5}$ of 1 time. Hence, $\$39\frac{1}{5}$ is $5\frac{3}{5}$ times \$7, and, therefore, $\$39\frac{1}{5}$ will buy $5\frac{3}{5}$ cd. of wood at \$7 a cord.

18. If 5 plows cost $\$68\frac{3}{4}$, what is the cost of 1 plow? Is this a problem in division or partition? How will the explanation differ from the explanation of the preceding problem?

19. If I pay $81\frac{1}{3}$ cts. for 7 lbs. of coffee, what is the price per pound?

20. At the rate of 4 mi. an hour, how long will it take a man to walk $25\frac{3}{5}$ mi.?

21. Lewis earned $\$7\frac{1}{3}$ in 6 days. How much did he earn a day?

22. I paid $\$48\frac{4}{7}$ for board at \$6 a week. How many weeks' board did I pay?

23. If 8 boxes of raisins weigh 57½ lbs., what is the average weight per box?

24. At $5 a barrel, how many barrels of apples will $43¾ buy?

25. 10 lbs. of nails cost 46 cts. What was the price per pound?

26. At 9 cts. a quart, how many quarts of vinegar can be bought for 60¾ cts.?

27. I paid 50 cts. for cabbage at 6 cts. a pound. How many pounds did I buy?

28. At $3 a rod, how much fence can be built for $41⅝?

29. A mason paid $61½ for 9 M bricks. What was the price per thousand?

30. How long will it take a boy to row 45⅜ mi., at the rate of 6 mi. an hour?

31. A man paid $73¾ for 10 T. of coal. What did he pay per ton?

32. If 10 doz. pencils cost $4⅘, what is the cost per dozen? (Change the mixed number to an improper fraction.)

33. If 7 men consume 3$\frac{1}{16}$ lbs. of meat in a day, how much does 1 man consume?

34. If 12 boxes of berries cost 100 cts., what is the price per box?

35. How many times will 49⅜ gals. fill a vessel that holds 4 gals.?

36. How long will it take a family that uses 7 lbs. of sugar a week, to use $72\frac{3}{4}$ lbs.?

37. At \$12 a ton, how many tons of hay can be bought for $\$85\frac{1}{2}$?

38. A man sold 11 turkeys for $\$28\frac{3}{5}$. What was the average price apiece?

39. When 12 cd. of wood cost $\$76\frac{5}{16}$, what is the price per cord?

40. A field that is 12 rds. long contains $111\frac{1}{3}$ sq. rds. What is its width?

41. If 20 lbs. of rice cost $\$1\frac{3}{5}$, what is the price per pound?

WRITTEN EXERCISES.

212. Divide:

42. $\frac{8831}{316}$ by \$21.

43. $\frac{64}{97}$ gal. by 16.

44. $\frac{78}{95}$ lb. by 13 lbs.

45. $\frac{1886}{651}$ bu. by 27.

46. $\frac{1588}{3005}$ yd. by 19 yds.

47. $\frac{48}{45}$ yd. by 12.

48. $\frac{6884}{10084}$ cu. ft. by 36. cu. ft.

49. $\frac{1}{131}$ A. by 5 A.

50. $\frac{3681}{3684}$ mi. by 9 mi.

51. $\frac{8964982}{5858817}$ by 11.

52. $\frac{67}{91}$ by 60.

53. $\frac{4500}{5371}$ by 75.

54. $65\frac{4}{13}$ by 9.

55. $57\frac{5}{9}$ by 16.

56. $\frac{43820}{53652}$ by 864.

57. $107\frac{7}{10}$ by 18.

58. $2\frac{1}{7}$ by 28.

59. $4\frac{4128}{5241}$ by 368.

60. $\frac{5}{871}$ by 31.

61. $326\frac{5}{17}$ by 19.

213. To Divide an Integer by a Fraction.

ORAL EXERCISES.

1. At \$2 a yard, how many yards of cloth will \$6 buy?

2. At \$$\frac{2}{3}$ a yard, how many yards of cloth will \$6 buy?

EXPLANATION: Make the units of dividend and divisor alike \$6=\$$\frac{18}{3}$. \$$\frac{18}{3}$ will buy as many yards at \$$\frac{2}{3}$ a yard as there are *times* \$$\frac{2}{3}$ in \$$\frac{18}{3}$, or as there are times 2 in 18, which is 9 *times*. Hence, \$6 will buy 9 yds. of cloth, at \$$\frac{2}{3}$ a yard.

3. If a boy earns \$$\frac{3}{5}$ a day, how many days will it take him to earn \$9?

4. How many penknives, at \$$\frac{3}{4}$ apiece, can be bought for \$6?

5. If a man builds $\frac{5}{8}$ of a rod of fence a day, how long will it take him to build 7 rds.?

6. If a cord 10 in. long be cut into pieces $\frac{5}{6}$ of an inch long, how many pieces will there be?

7. How long will it take a man to plow 14 A., at the rate of $\frac{6}{7}$ of an acre a day?

8. At \$2$\frac{1}{3}$ a box, how many boxes of raisins can be bought for \$10? (Change both numbers to thirds of a dollar.)

9. If 2$\frac{3}{4}$ bu. of oats will keep a horse a week, how long will 15 bu. keep it?

10. A pleasure yacht was hired for $12. Each person paid $\$1\frac{1}{2}$. How many persons were there in the company?

11. If a man moves forward $\frac{1}{4}$ of a rod at each step, how many steps must he take to go 16 rds.?

12. A man bought $2\frac{1}{2}$ lbs. of butter for 60 cts. What was the price per pound?

13. There is 1 cu. ft. in $\frac{4}{5}$ of a bushel. How many cubic feet are there in 20 bu.?

14. How long will it take a train to go 17 mi., at the rate of $\frac{2}{3}$ of a mile a minute?

15. If a barrel holds $2\frac{3}{4}$ bu. of apples, how many barrels will be required to hold $16\frac{1}{2}$ bu.?

16. If a boy saws $\frac{7}{8}$ of a cord of wood a day, how long will it take him to saw 9 cd.?

17. A man paid $6 for molasses, at $\$\frac{4}{15}$ a gallon. How many gallons did he buy?

18. If a boy goes $5\frac{1}{3}$ ft. at a leap, how many leaps must he make to go $37\frac{1}{3}$ ft.?

19. A piece of land containing 21 A. was laid off into lots, each containing $\frac{3}{4}$ of an acre. How many lots were there?

20. In how many weeks will a boy earn $14, at $\$3\frac{1}{2}$ a week?

21. How long will 5 lbs. of butter last a family that uses $\frac{3}{16}$ of a pound a day?

22. Mrs. Martin received \$6 for chickens, which she sold at $\$\frac{3}{10}$ apiece. How many did she sell?

23. A farmer bought a stove for \$11, paying for it with apples at $\$2\frac{1}{2}$ a barrel. How many barrels did it take?

24. A girl learned a page of music in $\frac{1}{10}$ of an hour. How many such pages could she learn in 9 hrs.?

25. How many cords of wood, at $\$6\frac{2}{3}$ a cord, will \$40 buy?

26. At $\$1\frac{1}{12}$ a yard, how many yards of silk can be bought for \$9?

27. If $\frac{9}{16}$ of a ton of coal supplies a furnace one day, how long will 7 T. last?

28. How many rosettes, each requiring $\frac{2}{3}$ of a yard, can be made from 10 yds. of ribbon?

29. If a miller takes for toll $\frac{6}{25}$ of a bushel for every bushel of wheat he grinds, how many bushels must he grind to get 4 bu. of toll?

30. A strawberry bed that is $\frac{10}{11}$ of a rod wide must be how long to contain 12 sq. rds.?

WRITTEN EXERCISES.

214. *31.* Divide 42 by $\frac{28}{31}$.

EXPLANATION: If the divisor were 28, the quotient would be $\frac{42}{28}$. But as the true divisor $\frac{28}{31}$ is only $\frac{1}{31}$ of 28, the true quotient is 31 times $\frac{42}{28}$, which is indicated by writing 31 as a factor of the numerator, thus, $\frac{42 \times 31}{28}$. The result is a simple fraction with its numerator partly factored. Hence, reduced to its lowest terms, $\frac{42 \times 31}{28} = \frac{3 \times 31}{2} = 46\frac{1}{2}$.

32. A locomotive ran 185 mi. in $6\frac{4}{5}$ hrs. What was the average rate per hour? (Make the mixed number an improper fraction.)

33. If a family consumes $18\frac{7}{16}$ lbs. of flour a week, how long will a barrel of flour last it?

34. How many tons of hay, at $\$14\frac{3}{8}$ a ton, can be bought for \$75?

35. A field containing $18\frac{3}{4}$ A., produced 650 bu. of corn. What was the average yield per acre?

Divide:

36. 49 by $\frac{14}{25}$.

37. 63 by $\frac{12}{21}$.

38. 118 by $\frac{7}{8}$.

39. 60 by $9\frac{6}{17}$.

40. 100 by $12\frac{17}{25}$.

41. 420 by $3\frac{9}{11}$.

42. 396 by $27\frac{18}{11}$.

43. 84 by $\frac{24}{25}$.

44. 212 by $5\frac{15}{16}$.

45. 108 by $\frac{36}{37}$.

46. 70 by $\frac{44}{65}$.

47. 98 by $19\frac{7}{10}$.

48. 65 by $28\frac{5}{7}$.

49. 625 by $\frac{125}{218}$.

50. 476 by $\frac{11}{143}$.

51. 889 by $31\frac{87}{370}$.

215. To Divide a Fraction by a Fraction.

ORAL EXERCISES.

1. How many yards of ribbon, at $\$\frac{3}{7}$ a yard, will $\$\frac{8}{9}$ buy? Is this a problem in division or partition?

EXPLANATION: Make the units alike. $\$\frac{3}{7}=\$\frac{27}{63}$. $\$\frac{8}{9}=\$\frac{56}{63}$. $\$\frac{56}{63}$ will buy as many yards at $\$\frac{18}{63}$ a yard as there are *times* $\$\frac{18}{63}$ in $\$\frac{56}{63}$, or 18 in 56, which is $3\frac{1}{9}$ times. Hence, $\$\frac{8}{9}$ will buy $3\frac{1}{9}$ yds. at $\$\frac{3}{7}$ a yard.

2. A lady paid $\$\frac{9}{10}$ for brooms, at $\$\frac{3}{10}$ apiece. How many brooms did she buy?

3. How many pounds of beefsteak will $\$\frac{4}{6}$ buy, at $\$\frac{1}{7}$ a pound?

4. Among how many children can $\frac{3}{4}$ of a melon be divided so that each child may receive $\frac{1}{8}$ of the melon?

5. How long will a boy have to work at $\$\frac{2}{5}$ an hour, to earn $\$\frac{9}{10}$?

6. A lady paid $\$1\frac{1}{2}$ for goods, at $\$\frac{1}{8}$ a yard. How many yards did she buy?

7. A gentleman distributed $\frac{6}{7}$ of a barrel of flour to a number of poor families, giving each $\frac{3}{14}$ of a barrel. How many families received flour?

8. How much cheese can be bought for $\$1\frac{1}{10}$, at $\$\frac{1}{5}$ a pound?

9. How many pounds of butter, at $\$\frac{1}{4}$ a pound, can be bought for $\$2\frac{7}{8}$? (Change both numbers to the same denomination.)

10. How many pounds of tea, at $\$\frac{4}{5}$ a pound, can be bought for $\$5\frac{2}{3}$?

11. How many badges, each $\frac{1}{18}$ of a yard long, can be made from $\frac{5}{6}$ of a yard of ribbon?

12. At $\$3\frac{1}{3}$ a ton, how many tons of coal will $\$12\frac{1}{2}$ buy?

13. A lady having $1\frac{1}{2}$ of a gallon of jelly, put it into glasses, each holding $\frac{1}{3}$ of a gallon. How many glasses did she use?

14. When cream is worth $\$\frac{1}{10}$ a pint, how many pints will $\$\frac{4}{5}$ buy?

15. At $\$2\frac{2}{3}$ a bushel, how many bushels of beans can be bought for $\$9\frac{3}{4}$?

16. $\frac{8}{11}$ of the trees in an orchard are apple trees, and $\frac{2}{3}$ of them cherry trees. How many times as many apple trees as cherry trees are there?

17. How long will it take a student to read $\frac{9}{11}$ of a certain book, if he reads $\frac{1}{8}$ of it a day?

18. If a turkey costs $\$1\frac{3}{5}$, how many will $\$13\frac{3}{4}$ buy?

19. If it takes a member of this class $\frac{1}{18}$ of an hour to explain one of these problems, how many such problems can he explain in $\frac{7}{8}$ of an hour?

20. If a man consumes $1\frac{1}{3}$ lbs. of meat in a day, how many men would consume $16\frac{4}{5}$ lbs.?

WRITTEN EXERCISES.

216. *21.* Divide $\frac{8}{7}$ by $\frac{6}{11}$.

(1) $\frac{8}{7}$

(2) $\frac{8}{7\times 6}$

(3) $\frac{8\times 11}{7\times \cancel{6}_{3}}=\frac{44}{21}=2\frac{2}{21}$

EXPLANATION: (1) Write the dividend. (2) Indicate 6 as a divisor. But as 6 is 11 times as large as the true divisor $\frac{6}{11}$, (3) make the quotient right by entering 11 as a multiplier. The result is a simple fraction, with its terms partly factored. Reduced to its simplest form, it becomes $\frac{44}{21}=2\frac{2}{21}$. (Note that in the solution the divisor appears *inverted.*)

Divide (change mixed numbers to improper fractions):

22. $1\frac{8}{13}$ by $\frac{8}{11}$.

23. $1\frac{8}{31}$ by $\frac{3}{5}$.

24. $\frac{25}{36}$ by $\frac{5}{6}$.

25. $\frac{32}{40}$ by $\frac{4}{5}$.

26. $\frac{28}{56}$ by $\frac{7}{8}$.

27. $4\frac{20}{27}$ by $\frac{6}{7}$.

28. $1\frac{3}{16}$ by $\frac{9}{77}$.

29. $1\frac{8}{49}$ by $4\frac{5}{7}$.

30. $5\frac{4}{5}$ by $3\frac{1}{2}$.

31. $7\frac{37}{39}$ by $2\frac{4}{13}$.

32. $\frac{3}{5}$ by $\frac{1}{40}$.

33. $\frac{45}{99}$ by $\frac{15}{27}$.

34. $\frac{114}{315}$ by $7\frac{6}{19}$.

35. $\frac{106}{351}$ by $\frac{74}{351}$.

36. $\frac{74}{351}$ by $\frac{106}{351}$.

37. $24\frac{5}{9}$ by $38\frac{25}{27}$.

38. $\frac{18}{731}$ by $\frac{195}{731}$.

39. $\frac{38}{275}$ by $\frac{133}{385}$.

40. $8\frac{5}{16}$ by $3\frac{7}{8}$.

41. $15\frac{5}{7}$ by $45\frac{10}{21}$.

42. $47\frac{5}{6}$ by $7\frac{9}{14}$.

43. $53\frac{16}{17}$ by $3\frac{2}{43}$.

217. Denominate Numbers.

EXERCISES.

1. How many cans, each holding $3\frac{1}{2}$ pts., can be filled from a cask of syrup containing 36 gals.? Is this a problem in division or partition?

EXPLANATION: Make the units of divisor and dividend alike. $3\frac{1}{2}$ pts. $=\frac{7}{2}$ pts. 36 gals. $=\frac{576}{2}$ pts. As many cans, each holding $3\frac{1}{2}$ pts., can be filled from 36 gals. as there are *times* $\frac{7}{2}$ pts. in $\frac{576}{2}$ pts.; that is, as many as there are *times* 7 in 576, which is $82\frac{2}{7}$ times.

2 A farm containing 58 A. 50 sq. rds. 8 sq. yds. is in 4 fields of equal size. What is the size of each field? Is this a problem in division or partition?

EXPLANATION: ¼ of 58 A.=14 A., with 2 A. remaining. 2 A.=320 sq. rds. 320 sq. rds.+50 sq. rds.=370 sq. rds. ¼ of 370 sq. rds.=92 sq. rds., with 2 sq. rds. remaining. 2 sq. rds.=60½ sq. yds. 60½ sq. yds.+8 sq. yds.=68½ sq. yds. ¼ of 68½ sq. yds.=17⅛ sq. yds. Hence, each field contains 14 A. 92 sq. rds. 17⅛ sq. yds.

	A.	sq. rds.	sq. yds.
4)	58	50	8
	14	92	17⅛

3. How many cans, each holding 2 lbs. 3 oz., will 98 lbs. 7 oz. of tomatoes fill? (Make dividend and divisor alike.)

4. A planter put 1,534 gals. 1 qt. 1 pt. of molasses into 25 hhds. of the same size. How much did each hold?

5. If 52 cu. yds. 26 cu. ft. 1,386 cu. in. of earth were removed in 18 equal loads, how much did each load contain?

6. How many thimbles, each weighing 1 pwt. 4 gr., can be made from 5 oz. 16 pwt. 16 gr. of silver?

7. If a ship sails 31° 45′ 52″ in 14 days, how far does it average a day?

8. If £20 12s. 1d. 2 far. be divided equally among 9 poor families, how much will each receive?

9. In how many loads can a man deliver 35 cd. 1 cd. ft. 4 cu. ft. of wood, if the loads average 1 cd. 3 cd. ft. 4 cu. ft.?

10. A farmer put 127 bu. 2 pks. 5 qts. of rye into 43 equal bags. How much did each bag hold?

11. The entire weight of 37 hhds. of sugar is 17 T. 368 lbs. 6 oz. What is the average weight of each?

12. How many revolutions will a carriage wheel, 11 ft. 7 in. in circumference, make in going 2 mi. 24 rds. 1 ft. 10 in.?

13. How many barrels of water, each holding 36 gals., will a rectangular cistern contain, that is 11 ft. deep and 7 ft. square at the bottom?

14. How many building lots, each 3 rds. front and 8 rds. deep, are there in 14 A. 64 sq. rds.?

15. What is one thirteenth of 853 mi. 9 ch. 9 l.?

16. Divide 23 yrs. 235 days 16 hrs. 12 min. by 9.

17. Divide 47 lbs. 9 oz. 17 pwt. 12 gr. by 15.

18. Find one eighth of 18 bales 1 bun. 1 rm. 19 qu. 16 sheets of paper.

19. 81 A. 127 sq. rds. 28 sq. yds. 3 sq. ft.$\div 7 =$?

20. What is one twelfth of 28 gt. gross 10 gross 10 doz. of buttons?

218. Division Review.

EXERCISES.

1. If in a watch-chain there are 1 oz. 5 pwt. 7 gr. of gold, how many such chains can be made from 4 lbs. 1 pwt. 2 gr.?

2. At \$1.68 a rod, how many rods of fence can be built for \$149.1?

3. If it takes $\frac{5}{8}$ of a yard of cloth to make a vest, how many vests can be made from $51\frac{1}{4}$ yds.?

4. A drover paid $297\frac{3}{5}$ for 38 sheep. What was the average price per head?

5. If the circumference of a wheel is 13.85 ft., how many revolutions must it make to go 6,597.309 ft.?

6. A gentleman sold 50 cd. 7 cd. ft. 13 cu. ft. of wood, in equal lots, to 9 persons. How much did each buy?

7. If $32\frac{3}{4}$ A. produce $900\frac{5}{8}$ bu. of wheat, what is the average yield per acre? (First, find the yield for $\frac{1}{4}$ of an acre.)

8. A silver dollar weighs 412.5 gr. How many dollars weigh 31,350 gr.?

9. How many days will it take a vessel, that averages 13.125 mi. an hour, to sail 1,260 mi.?

10. If a man builds 196 rds. 5 yds. of fence in 12 days, how much does he average a day?

11. A bushel contains 2,150.42 cu. in. How many bushels does a bin hold that is $10\frac{3}{8}$ ft. long, 6 ft. wide, and $7\frac{3}{4}$ ft. deep?

12. A gentleman divided 551 A. 12 sq. rds. of land equally among his 3 sons and 4 daughters. How much land did each receive?

13. If 54 horses cost $\$4,622\frac{1}{2}$, what is the cost of each?

14. What number multiplied by $33\frac{1}{3}$ will produce $297\frac{1}{2}$?

15. What is one twelfth of 47 wks. 5 days 23 hrs. 43 min. 36 sec.?

16. The product of two numbers is .0016016. One of the numbers is .143. What is the other?

17. A man divided 520 A. of land equally among his children, giving each $74\frac{2}{7}$ A. How many children had he?

18. The dividend is $145.764, and the quotient is 100. What is the divisor?

19. $214\frac{3}{4} \div 25\frac{11}{12} = ?$

20. $.031812 \div 4.82 = ?$

21. 165 cu. yds. 15 cu. ft. 1,252 cu. in. $\div 17 = ?$

22. 803 mi. 67 ch. 2 rds. 4 l. $\div 23 = ?$

23. $.69825 \div 735 = ?$

24. $238 \div 68\frac{2}{7} = ?$

25. $87\frac{5}{8} \div 113\frac{5}{8} = ?$

COMPLEX FRACTIONS.

219. *1.* Define a complex fraction.

In the complex fraction $\frac{2\frac{1}{2}}{4\frac{1}{2}}$, what is the numerator? How do you tell? Such a complex fraction is read, "The complex fraction whose numerator is $\frac{5}{2}$ and whose denominator is $\frac{9}{2}$."

Read the following:

$$\frac{\frac{3}{4}}{\frac{5}{6}} \quad \frac{5}{\frac{3}{7}} \quad \frac{\frac{3}{10}}{8} \quad \frac{2\frac{1}{2}}{\frac{1}{3}} \quad \frac{\frac{3}{8}}{3\frac{1}{4}} \quad \frac{2\frac{3}{5}}{5\frac{2}{3}} \quad \frac{8}{4\frac{4}{9}} \quad \frac{3\frac{5}{11}}{11}$$

Since a fraction expresses an unexecuted division whose dividend is the numerator and whose divisor is the denominator, it may be changed to a simple fraction by performing the division indicated.

EXERCISES

220. *1.* Reduce $\frac{2\frac{4}{7}}{\frac{2}{3}}$ to a simple fraction.

SOLUTION: $2\frac{4}{7}=\frac{18}{7}$. $\frac{18}{7}\div\frac{2}{3}=\frac{27}{7}=3\frac{6}{7}$.

Change the following to simple fractions:

2. $\frac{\frac{5}{8}}{\frac{3}{4}}$	*5.* $\frac{5\frac{5}{6}}{2\frac{1}{3}}$	*8.* $\frac{4\frac{2}{7}}{6}$	*11.* $\frac{\frac{7}{16}\times\frac{20}{49}}{\frac{3}{4}\times 8}$
3. $\frac{\frac{9}{13}}{\frac{2}{5}}$	*6.* $\frac{7}{\frac{5}{8}}$	*9.* $\frac{19}{2\frac{4}{11}}$	*12.* $\frac{\frac{4}{5}\div\frac{2}{3}}{\frac{5}{8}\div\frac{3}{10}}$
4. $\frac{4\frac{2}{3}}{\frac{7}{8}}$	*7.* $\frac{\frac{8}{7}}{3\frac{1}{5}}$	*10.* $\frac{12\frac{3}{5}}{10\frac{5}{7}}$	*13.* $\frac{\frac{5}{6}\times 3\frac{1}{3}}{2\frac{7}{9}\div\frac{25}{27}}$

Each of these complex fractions may be arranged as a simple fraction, with its terms partly factored. Number 11 thus arranged becomes $\frac{7\times 20\times 4\times 8}{16\times 49\times 3}$. (See whether you can arrange each of the others in a similar way, and explain every step.)

A complex fraction may be made simple, by multiplying its terms by the least common multiple of the denominators in the terms. For example, multiplying both terms of $\frac{3}{8\frac{1}{4}}$ by 8, the numerator becomes 3, and the denominator 26; hence, $\frac{3}{8\frac{1}{4}}=\frac{8}{26}$.

221. To Find the Part which One Number is of Another.

EXERCISES.

1. \$1 is what part of \$2? Of \$3? Of \$4? Of \$5? Of \$9? Of \$14? Of \$38? Of \$85? Of \$150?

2. \$2 are what part of \$3?

EXPLANATION: \$1 is $\frac{1}{3}$ of \$3. \$2 is 2 times $\frac{1}{3}$ of \$3, or $\frac{2}{3}$ of \$3.

3. 3 mi. are what part of 5 mi.? Of 7 mi.? Of 9 mi.? Of 10 mi.? Of 12 mi.? Of 18 mi.?

4. 6 trees are what part of 7 trees? Of 10 trees? Of 8 trees? Of 12 trees? Of 13 trees? Of 18 trees? Of 20 trees?

5. 9 bu. are what part of 10 bu.? 4 in. of 12 in.? 5 qts. of 8 qts.? 8 oz. of a pound? 10 books of 15 books? 9 oranges of a dozen oranges? 20 cts. of a dollar?

6. What part of 7 is $\frac{2}{3}$?

EXPLANATION: $7=\frac{21}{3}$. $\frac{2}{3}$ is the same part of $\frac{21}{3}$ that 2 is of 21. 2 is $\frac{2}{11}$ of 21; hence, $\frac{2}{3}$ is $\frac{2}{11}$ of $\frac{21}{3}$, or 7. (We can compare such numbers only as are alike.)

7. What part of 6 is $\frac{3}{4}$? Of 3 is $\frac{4}{5}$? Of 4 is $\frac{1}{2}$? Of 8 is $\frac{2}{3}$? Of 5 is $\frac{5}{6}$? Of 10 is $\frac{6}{7}$? Of 3 is $\frac{2}{5}$?

8. What part of 4 is $1\frac{1}{3}$? Of 5 is $2\frac{1}{2}$? Of 6 is $2\frac{1}{5}$? Of 8 is $3\frac{3}{4}$? Of 10 is $3\frac{1}{3}$? Of 11 is $2\frac{3}{4}$? Of 13 is $7\frac{1}{3}$?

9. What part of $\frac{3}{4}$ is $\frac{2}{3}$?

EXPLANATION: $\frac{3}{4}=\frac{9}{12}$. $\frac{2}{3}=\frac{8}{12}$. $\frac{8}{12}$ is $\frac{8}{9}$ of $\frac{9}{12}$.

10. What part of $\frac{2}{3}$ is $\frac{2}{5}$? Of $\frac{1}{4}$ is $\frac{1}{6}$? Of $\frac{4}{5}$ is $\frac{3}{7}$? Of $\frac{3}{7}$ is $\frac{4}{5}$? Of $\frac{5}{6}$ is $\frac{2}{3}$? Of $\frac{2}{5}$ is $\frac{5}{6}$?

11. What part of $2\frac{1}{3}$ is $\frac{2}{3}$? Of $\frac{2}{3}$ is $2\frac{1}{2}$? Of $3\frac{1}{3}$ is $2\frac{1}{4}$? Of $2\frac{1}{4}$ is $3\frac{1}{3}$? Of $5\frac{1}{3}$ is $3\frac{1}{4}$? Of $6\frac{2}{3}$ is $2\frac{2}{5}$? Of $1\frac{2}{3}$ is $1\frac{5}{6}$?

12. If a man can dig a ditch in 11 days, what part of it can he dig in 1 day? In 3 days? In 6 days? In 9 days?

13. A boy having $45, paid $15 for an overcoat, $20 for a suit, and $6 for a railroad ticket. What part of his money did he give for each? What part of his money had he left?

14. If $56 buys an acre of land, what part of an acre will $28 buy? $14? $8? $24?

15. A and B hired a pasture. A pastured 14 cows and B 16 cows. What part of the price should each pay?

16. Walter gathered $3\frac{1}{4}$ pks. of pecans, and Hugh gathered $5\frac{1}{8}$ pks. If Hugh's pecans are worth $1, what part of a dollar are Walter's worth?

17. If a man can do a piece of work in $5\frac{1}{2}$ days, what part of the work can he do in $3\frac{1}{3}$ days? (Find what he can do in a day.)

18. A tree whose height was 85 ft., was broken off in a gale, 51 ft. from the top. What part of the tree was left standing?

19. Three men were paid $45 to mow a certain field. A mowed 12 A., B 10 A., and C 8 A. How many dollars should each man receive?

222. To Find a Number when a Fractional Part of it is Given.

ORAL EXERCISES.

1. $4 is $\frac{1}{2}$ of how many dollars? $6 is $\frac{1}{2}$ of how many dollars? $8 is $\frac{1}{3}$ of how many dollars? $9 is $\frac{1}{3}$ of how many dollars?

2. If 18 yrs. is $\frac{3}{4}$ of my age, how old am I?

EXPLANATION: If 18 yrs. is $\frac{3}{4}$ of my age, $\frac{1}{4}$ of my age is $\frac{1}{3}$ of 18 yrs., or 6 yrs. My age is 4 times 6 yrs., or 24 yrs.

3. If \$15 is $\frac{5}{8}$ of John's money, how many dollars has he?

4. How far is it to Chicago, if $\frac{7}{9}$ of the distance is 28 mi.?

5. Henry lost $\frac{3}{5}$ of his marbles and has 12 marbles left. How many had he at first?

6. A boy gave $\frac{5}{8}$ of his money for a watch which cost \$48. How many dollars had he left?

7. Mr. Jones sold a horse for $\frac{4}{5}$ of its cost, and received \$60. What was the cost of the horse?

8. A farmer sold $\frac{5}{7}$ of his sheep and kept 20. How many sheep had he at first?

9. If $\frac{1}{10}$ of a ton of coal cost \$3.50, what will 8 T. cost? What will 1 T. cost?

10. Mr. R. sold a cow for \$36, which was $\frac{1}{3}$ more than the cost. What did she cost him? (How many thirds of the cost did he get?)

11. Lewis had $\frac{3}{4}$ as many marbles as George, and together they have 35 marbles. How many has each? What part of the whole number had Lewis?

12. If $\frac{3}{8}$ of a peck of cranberries cost 27 cts., what will $\frac{1}{8}$ of a peck cost?

13. Mr. B. sold a carriage for \$70, losing $\frac{1}{8}$ of the cost. What did the carriage cost him?

14. One half of Jane's money is 15 cts. more than $\frac{1}{3}$ of Ellen's. How many cents has each?

15. A grocer sold $\frac{5}{8}$ of a gallon of molasses for 40 cts. What was the price per gallon?

16. An army besieged, is put on short allowance. Each man receives per day, 12 oz. of bread, which is $\frac{4}{7}$ of a full ration, and 10 oz. of meat, which is $\frac{5}{7}$ of a full ration. What are the daily rations?

17. A man had $\frac{5}{7}$ of his money stolen from him. The thief was caught, but not till he had spent $\frac{3}{5}$ of what he had stolen. The remainder, $50, was given back. How much money had the man at first?

18. Fred's weight is $\frac{5}{12}$ of the sum of his and Harry's weights. Harry's weight is 77 lbs. What is Fred's weight?

WRITTEN EXERCISES.

223. *19.* If $\frac{5}{8}$ of a farm is worth $6,240, what is $\frac{3}{4}$ of it worth? (What is the farm worth?)

20. A man paid me $184.75, which is equal to $\frac{5}{8}$ of what he still owes me. How much did he owe me at first?

21. A man bequeathed to his wife $35,000, which was $\frac{7}{16}$ of his estate. The remainder was divided equally among his 3 children. How many dollars did each receive?

22. A man owning $\frac{5}{8}$ of a mill, sold $\frac{3}{5}$ of his share for $4,560$\frac{5}{8}$. What was the value of the mill?

23. Frank and Ida together have $387. Ida's part is $\frac{7}{11}$ as great as Frank's. How much has each?

24. A boy who lost $\frac{1}{3}$ of his kite string, added $55\frac{1}{3}$ ft. The string was then $\frac{5}{7}$ of its original length. What was the original length?

25. A farmer gave $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ of his property to his 3 children, and had $24,000 left. How much did he give to each?

26. A merchant sold flour for $2,035.35, and gained a sum equal to $\frac{3}{7}$ of the cost. What was the cost?

MISCELLANEOUS PROBLEMS.

224. *1.* If 15 bbls. of apples were given for $28\frac{3}{4}$ yds. of cloth valued at $2\frac{1}{2}$ a yard, what was the value of 1 bbl. of apples?

2. Two men, 86.394 mi. apart, are traveling toward each other. When they meet, one has traveled 8.916 mi. farther than the other. How far has each man traveled?

3. A merchant bought $\frac{4}{5}$ of a vessel, and sold $\frac{1}{3}$ of his share for $1,756. What was the value of the whole ship at that rate?

4. A owns 280 A. of land; $\frac{3}{5}$ of A's farm equals $\frac{4}{7}$ of B's. How many acres does B own?

5. A farmer sold 85 sheep at $6.75 a head, and a number of others at $7.5 a head. He received $1,001.25 for all. How many did he sell?

6. A flour dealer bought 167 bbls. of flour at $3\frac{1}{8}$ a barrel. He sold 123 bbls. at $4\frac{1}{8}$ a barrel, and the remainder, being damaged, brought only $2\frac{1}{8}$ a barrel. How much did he gain?

7. How many barrels of potatoes, at \$3.45 a barrel, must be given for 4 T. 500 lbs. of coal, at \$6.75 a ton?

8. A school enrolls 798 pupils. $\frac{11}{21}$ of them are boys. How many boys in the school?

9. Another school enrolls 312 boys. $\frac{13}{25}$ of the pupils are girls. How many pupils enrolled?

10. A grocer bought flour at \$4.35 a barrel, paying \$243.6 for it. He sold it at \$5.09 a barrel. How much did he gain?

11. A man bequeathed \$3,752 to his wife, which was $\frac{14}{31}$ of his estate. What was his estate?

12. A farmer brought to town 3 jars of butter, weighing 24 lbs., $32\frac{1}{2}$ lbs., and $27\frac{3}{4}$ lbs., respectively. The jars weighed $2\frac{3}{4}$ lbs., $4\frac{5}{8}$ lbs., and $3\frac{1}{2}$ lbs. He received $\$23\frac{13}{20}$ for the butter. What was the price per pound?

13. A merchant sold $\frac{3}{5}$ of a piece of cloth at \$1.85 a yard, receiving \$33.3 for it. How many yards were there in the piece at first? What was the value of the remainder at the same price?

14. A merchant tailor has $57\frac{3}{4}$ yds. of cloth, from which he wishes to cut an equal number of coats, trousers, and vests. What is the number of each, if they contain $3\frac{5}{6}$ yds., $2\frac{3}{4}$ yds., and $1\frac{3}{5}$ yds., respectively?

15. A farmer divided $\frac{1}{3}$ of his estate of 358 A. 80 sq. rds. of land equally between his 2 daughters; and the remainder, after setting off 75 A. 120 sq. rds., equally between his 2 sons. What was the share of each son and each daughter?

16. A gentleman paid $\$106\frac{5}{16}$ for 18 loads of wood, each measuring 1 cd. 2 cd. ft. 8 cu. ft. What was the price per cord?

17. A dealer sold 235 T. of coal, at \$2.65 a ton; and a number of tons of another kind, at \$6.85 a ton. He received for all, \$1,273.5. How many tons did he sell?

18. If a drover buys 357 sheep, at \$4.15 a head, for how much must he sell them per head to gain \$267.75?

19. A owns $\frac{3}{7}$ of a section of land, B owns $\frac{4}{9}$ of a section, and C owns $\frac{3}{16}$ as much as both A and B. What is C's farm worth, at $\$64\frac{3}{8}$ an acre?

20. A man sold $5\frac{1}{2}$ T. of chalk for \$31.71. What did he receive per pound?

21. From two fields, 712 bu. of corn were gathered. The first field yielded $\frac{4}{5}$ as much as the second. How many bushels did each field yield?

22. Two trains, an express and a freight, leave the same station at the same time, and move in opposite directions. The express makes 32 mi. 216 rds. 5 yds. per hour. At the end of 5 hrs. the trains are 256 mi. 223 rds. 1 yd. apart. What was the rate of the freight per hour?

23. A man gave $\frac{11}{27}$ of a farm to his daughter, and the remainder to his son. The difference between their shares was 195 A. How many acres did the son receive?

24. A grocer having $\frac{4}{5}$ of a barrel of sugar, sold $\frac{2}{3}$ of it for $\$5\frac{3}{8}$. What was the value of all of the sugar, at the same rate?

25. $\frac{4}{5}$ of a stock of goods was destroyed by fire, $\frac{3}{5}$ of the remainder was damaged by water, and the uninjured goods were sold at cost, for $4,192. What was the cost of the entire stock?

26. A man's income is $16.5 a week, and his weekly expenses are $20.45. If he has $79 in reserve, how many weeks can he live without incurring debt?

27. A farmer sold $\frac{1}{3}$ of his wheat to one man, $\frac{2}{5}$ of it to another, and kept 356 bu. What was the whole crop worth, at $1.03 a bushel?

28. How long will a barrel of pork (200 lbs.) last 7 persons, at the rate of 2 lbs. 7 oz. a day for each person?

29. A dealer buys 275 bu. of corn, at $.465 a bushel. For how much a bushel must he sell it to gain $29.5625?

30. A, B, and C have 27,794 head of cattle on the same range. How many has each if A has 7 times as many as C, and B has 5 times as many as C?

31. A seedsman bought $26\frac{1}{4}$ bu. of grass seed for 152\frac{1}{4}$. He sold $18\frac{1}{2}$ bu. at a profit of 1\frac{3}{8}$ per bushel. For what price must he sell the remainder so as to gain $40 on the whole purchase?

32. What will it cost to grade 2 mi. 245 rds. of road, at $4.85 a rod?

33. A box that will hold a cord of wood will hold how many bushels of wheat? How many barrels of water?

34. At $196 a pound, what would be the cost of 8 oz. 8 pwt. 8 gr. of gold?

PERCENTAGE.

225. *1.* What is $\frac{1}{100}$ of \$200? $\frac{3}{100}$ of \$200? $\frac{11}{100}$ of \$200?

2. What is $\frac{8}{100}$ of 800 yds.? $\frac{7}{100}$ of 800 yds.? $\frac{12}{100}$ of 800 yds.?

3. What is $\frac{1}{100}$ of 30 T.? $\frac{6}{100}$ of 30 T.? $\frac{10}{100}$ of 30 T.?

4. What is $\frac{1}{100}$ of 8 A.? $\frac{5}{100}$ of 8 A.? $\frac{9}{100}$ of 8 A.?

5. What part of \$100 is \$1? Are \$2? Are \$7? Are \$13?

6. What part of 200 pts. are 2 pts.? How many hundredths?

7. What part of 300 men are 12 men? How many hundredths?

8. 18 mi. are how many hundredths of 300 mi.?

9. 20 sheep are how many hundredths of 400 sheep?

10. 90 lbs. are how many hundredths of 1,000 lbs.?

11. What part of 50 cts. are 2 cts.? How many hundredths?

12. What part of 10 pens is 1 pen? How many hundredths?

13. .7 posts are how many hundredths of 25 posts? 7 posts are what part of 25 posts? (How can you change $\frac{7}{25}$ to hundredths?)

14. 4 ships are how many hundredths of 5 ships?

15. $8 are how many hundredths of $40?

16. 9 days are how many hundredths of 30 days?

(The system of making computations by hundredths is called *Percentage.*)

Per Cent is a contraction of the Latin *per centum,* and means hundredth or hundredths. Thus, 1 per cent means $\frac{1}{100}$; 9 per cent means $\frac{9}{100}$; $\frac{1}{8}$ per cent means $\frac{1}{8}$ of $\frac{1}{100}$; $2\frac{1}{3}$ per cent means $\frac{2\frac{1}{3}}{100}$.

Business men use the sign % instead of the words *per cent.* Per cent may also be written decimally. Hence:

6 per cent = 6 % = $\frac{6}{100}$ = .06 = $\frac{3}{50}$.
25 per cent = 25 % = $\frac{25}{100}$ = .25 = $\frac{1}{4}$.
$\frac{1}{2}$ per cent = $\frac{1}{2}$ % = $\frac{\frac{1}{2}}{100}$ = .005 = $\frac{1}{200}$.
$16\frac{2}{3}$ per cent = $16\frac{2}{3}$ % = $\frac{16\frac{2}{3}}{100}$ = .16$\frac{2}{3}$ = $\frac{1}{6}$.
240 per cent = 240 % = $\frac{240}{100}$ = 2.40 = $2\frac{2}{5}$.

Express similarly the following:

2 %	28 %	91 %	$\frac{3}{8}$ %	$5\frac{5}{6}$ %
5 %	38 %	99 %	$\frac{3}{5}$ %	$6\frac{1}{4}$ %
8 %	45 %	103 %	$\frac{1}{6}$ %	$8\frac{1}{3}$ %
10 %	50 %	110 %	$\frac{5}{8}$ %	$33\frac{1}{3}$ %
12 %	$56\frac{1}{4}$ %	125 %	$\frac{4}{5}$ %	270 %
15 %	63 %	144 %	$2\frac{1}{2}$ %	300 %
17 %	75 %	150 %	$3\frac{1}{3}$ %	$312\frac{1}{2}$ %
20 %	80 %	180 %	$1\frac{1}{4}$ %	$218\frac{3}{4}$ %
23 %	$87\frac{1}{2}$ %	200 %	$4\frac{1}{6}$ %	$483\frac{1}{3}$ %

226. Commit thoroughly to memory the following

TABLE OF ALIQUOT PARTS.

$2\% = \frac{1}{50}$	$31\frac{1}{4}\% = \frac{5}{16}$	$62\frac{1}{2}\% = \frac{5}{8}$	$75\% = \frac{3}{4}$
$4\% = \frac{1}{25}$	$8\frac{1}{3}\% = \frac{1}{12}$	$87\frac{1}{2}\% = \frac{7}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$
$5\% = \frac{1}{20}$	$41\frac{2}{3}\% = \frac{5}{12}$	$16\frac{2}{3}\% = \frac{1}{6}$	$66\frac{2}{3}\% = \frac{2}{3}$
$10\% = \frac{1}{10}$	$58\frac{1}{3}\% = \frac{7}{12}$	$83\frac{1}{3}\% = \frac{5}{6}$	$50\% = \frac{1}{2}$
$6\frac{1}{4}\% = \frac{1}{16}$	$12\frac{1}{2}\% = \frac{1}{8}$	$20\% = \frac{1}{5}$	$3\frac{1}{3}\% = \frac{1}{30}$
$18\frac{3}{4}\% = \frac{3}{16}$	$37\frac{1}{2}\% = \frac{3}{8}$	$25\% = \frac{1}{4}$	$2\frac{1}{2}\% = \frac{1}{40}$

ORAL EXERCISES.

227. *1.* A man having \$80, gave 5% of it for a hat. What did the hat cost?

EXPLANATION: $5\% = \frac{1}{20}$. $\frac{1}{20}$ of \$80 = \$4. Or, 1% of \$80 = \$.80, and 5% ($\frac{5}{100}$) = 5 times \$.80 = \$4. Hence, the hat cost \$4. (Compare this problem with Problem 1, Section 198, p. 188.) (Is there anything new in this problem?)

2. A man having \$80, gave \$4 for a hat. What per cent (or how many hundredths) of his money did he give for the hat?

EXPLANATION: \$4 is $\frac{4}{80}$ of \$80. $\frac{4}{80} = \frac{1}{20} = \frac{5}{100} = 5\%$. Hence, he gave 5% of his money for the hat.

3. There were 40 sheep in a pasture. 7 were killed. What per cent of the whole number were killed?

EXPLANATION: 7 is $\frac{7}{40}$ of 40. We want to change $\frac{7}{40}$ to a fraction whose denominator is 100. By multiplying 40, the present denominator, by $2\frac{1}{2}$, we shall have 100 for a new denominator; but to preserve the value of the fraction, the numerator 7 must also be multiplied by $2\frac{1}{2}$. (See Prin. I, p. 107.) $\frac{7}{40} = \frac{17\frac{1}{2}}{100} = 17\frac{1}{2}\%$. Hence, $17\frac{1}{2}\%$ of the flock were killed.

4. A farmer raised 600 bu. of wheat, and sold all but 108 bu. What per cent of the crop did he keep?

EXPLANATION: 108 bu. $=\frac{108}{600}$ of 600 bu. To change $\frac{108}{600}$ to hundredths, we can divide both terms by 6. (See Prin. II, p. 107.) $\frac{108}{600}=\frac{18}{100}=18\%$. Hence, he kept 18% of the crop.

5. A man paid $40 for a cow, which was $62\frac{1}{2}\%$ of his money. How much money had he at first?

EXPLANATION: $62\frac{1}{2}\%=\frac{5}{8}$. If $40 is $\frac{5}{8}$ of his money, $\frac{1}{8}$ of his money is $\frac{1}{5}$ of $40, which is $8; and his money at first was 8 times $8, or $64. Hence, he had $64 at first. (Compare this problem with Problem 2, Section 222, p. 230.) (Is there anything new in this problem?)

6. A man gave 30% of his money for a cart, and had $56 left. How many dollars had he at first?

EXPLANATION: $30\%=\frac{3}{10}$. If he gave $\frac{3}{10}$ of his money for a cart, he had $\frac{7}{10}$ of his money left. Then, $56 is $\frac{7}{10}$ of the money he had at first. $\frac{1}{10}$ of the money he had at first is $\frac{1}{7}$ of $56, which is $8, and the money he had at first is 10 times $8, or $80.

7. John has $80, which is 25% more money than Elmer has. How many dollars has Elmer?

EXPLANATION: $25\%=\frac{1}{4}$, and $80 is $\frac{5}{4}$ of Elmer's money. $\frac{1}{4}$ of his money is $\frac{1}{5}$ of $80, which is $16; and his money is 4 times $16, which is $64.

8. A man having 400 A. of land, gave his son 25% of it. How many acres did his son get?

9. A laborer agreed to dig potatoes for $16\frac{2}{3}\%$ of the number of bushels he should dig. If he dug 180 bu., how many bushels did he receive?

10. Out of a cask containing 30 gals. of vinegar, 15 gals. were drawn. What per cent of the vinegar was drawn?

11. In a class of 24 pupils, 15 are girls. What per cent of the class are girls? (Change $\frac{15}{24}$ to hundredths.)

12. A clerk's expenses are \$400, which is 80 % of his salary. What is his salary?

13. If to John's age, 50 % of it be added, the sum will be 24 yrs. How old is he?

14. \$75 is 25 % less than Ellen's money. How much money has she?

15. I bought 24 lbs. of sugar, but lost 6 lbs. by breaking the bag which contained it. What per cent of the purchase did I lose?

16. A gardener sold $66\frac{2}{3}$ % of 150 choice fruit trees. How many did he sell?

17. A man earns \$18 a week, which is 20 % more than his son earns. How much does his son earn?

18. A fire destroyed 70 % of a stock of goods, and the value of the goods destroyed was \$1,400. What was the value of the entire stock?

19. A school enrolled 280 pupils, and the average number in daily attendance was $87\frac{1}{2}$ % of the enrollment. What was the daily attendance? (Subtract $\frac{1}{8}$ of 280 from 280. Why?)

20. In a class of 48 pupils, 4 errors were made in spelling 1 word each. What per cent of the words were missed?

21. The population of a certain town is 2,500, which is 125 % of what it was one year ago. What was the population a year ago? (2,500—$\frac{1}{4}$ of 2,500.)

22. A father's age diminished by 40 % is equal to the age of his son, which is 36 yrs. What is the father's age?

23. If wood which should be cut 4 ft. long falls short 3 in., what per cent should be deducted from the price?

24. A farm contains 160 A. 15 % is woodland, $6\frac{1}{4}$ % is pasture, 30 % is meadow, and 40 % is sowed in wheat. How many acres are devoted to each of these purposes?

25. A tree 81 ft. in height, was broken by a storm. The length of the part left standing is 50 % of the part broken off. What is the length of each part?

26. In a school of 120 pupils, 90 study reading, 80 study arithmetic, 60 study geography, 50 study grammar, 48 study history, and 20 study algebra. What per cent of the school study each of these branches?

27. A farmer, after selling 65 % of his sheep, had 140 left. What number had he at first?

28. My wages were increased $12\frac{1}{2}$ %, and I then received $81. What were my wages at first?

29. A has 640 A. of land, which is 200 % as much as B has. How many acres has B?

30. A boy travels 24 mi. on his bicycle, which is 4 % of the entire journey he will make. What is the length of the journey?

31. A boy deposited $50 in the bank, and each year increased his deposit 100 %. How much had he deposited at the end of 4 yrs.?

32. A and B each bought 600 cattle. A lost 15 % of his stock, and B gained 9 % of his. How many more cattle did B then have than A?

33. C and D began business each with $200. C lost 100 % of his capital, and D gained 100 % of his. How much more had D than C when they closed business?

34. If the annual rent of a house at 7 % of its value is $560, what is its value?

WRITTEN EXERCISES.

228. *35.* If the population of a certain county is 47,900, and 37 % of the population is engaged in agriculture, how many persons are engaged in agriculture?

EXPLANATION: $37\% = \frac{37}{100}$. $\frac{1}{100}$ of 47,900 persons = 479 persons, and $\frac{37}{100}$ of the population = 37 times 479 persons, or 17,723 persons. Hence, there are 17,723 persons engaged in agriculture.

36. 49 mi. is what per cent of 72 mi.?

EXPLANATION: 49 mi. is $\frac{49}{72}$ of 72 mi. $\frac{49}{72}$ may be reduced to hundredths, as we have already learned to do, or both terms may be multiplied by 100, and then both divided by the present denominator 72. This will give a fraction equivalent to $\frac{49}{72}$, but having 100 for a denominator. (See Prin. I and II, p. 107.)
$\frac{49}{72} = \frac{4900}{7200} = \frac{4900/72}{100} = \frac{68\frac{1}{18}}{100} = 68\frac{1}{18}\%$.

(Compare this problem with the first, second, and third numbers in Problem 3, and the third number Problem 4, Section 116, p. 120. How did you change those numbers to hundredths?)

The general process for finding what per cent one number is of another may be stated in this way: Find what fraction expresses the *part* that one number is of the other; then make the denominator of the fraction 100, in the best way it can be done and preserve the value of the fraction. The numerator will be the required per cent.

37. If 27% of a cargo of flour is 1,053 bbls., how many barrels are there in the cargo?

EXPLANATION: $27\% = \frac{27}{100}$. If 1,053 bbls. are $\frac{27}{100}$ of the cargo, $\frac{1}{100}$ of the cargo is $\frac{1}{27}$ of 1,053 bbls., or 39 bbls., and the cargo is 100 times 39 bbls., or 3,900 bbls.

38. 32 T. of hay are 79% of how many tons of hay?

EXPLANATION: $79\% = \frac{79}{100}$. If 32 T. are $\frac{79}{100}$ of some number of tons, $\frac{1}{100}$ of the number of tons is $\frac{1}{79}$ of 32 T., or $\frac{32}{79}$ T., and the required number of tons is 100 times $\frac{32}{79}$ T., or $40\frac{40}{79}$ T. Therefore, 32 T. of hay are 79% of $40\frac{40}{79}$ T.

39. The population of a certain city in 1890 was 95,744, which is 36% more than it was in 1880. What was the population in 1880?

EXPLANATION: $36\% = \frac{36}{100}$. The population in 1890 was $\frac{136}{100}$ of the population in 1880. Hence, 95,744 is $\frac{136}{100}$ of the population in 1880. $\frac{1}{100}$ of the population in 1880 is $\frac{1}{136}$ of 95,744, or 704; and the population in 1880 was 100 times 704, or 70,400.

40. If in a bed of iron ore 35% of the ore is pure iron, how much iron will 9,756 lbs. of ore produce?

41. A man has $3,450 deposited in a bank, and wishes to draw out 18% of it. How much must he draw?

42. A dealer shipped 2,800 bu. of wheat from Duluth. 525 bu. were thrown overboard during a gale. What per cent of his shipment did he lose?

43. What per cent of a pound Avoirdupois is a pound Troy? What per cent of a pound Troy is a pound Avoirdupois?

44. A farmer sold 1,755 bu. of corn, and had 35% of his crop left. What was his entire crop?

45. A speculator sold 50 bales of cotton for $2,366, losing 9% of its cost. What was the cost?

46. A farmer raised 2,600 bu. of wheat. He contracted 23% of it at $.89 a bushel, 50% of it at $.95 a bushel, and the remainder at $1.08½ a bushel. How much did he receive in all?

47. The population of a certain town was 6,192. Ten years later it was 8,256. What was the per cent of increase?

48. Last year a gentleman traveled 1,596 mi. by water, which was 24% less than the distance he traveled by land. How far did he travel by land?

49. A father divided $4,500 between a son and daughter so that the son received 25% more than the daughter. How much did each receive?

50. A merchant owes $18,360, and his assets are $15,300. What per cent of his debts can he pay?

51. A man owning a cotton mill, sold 38% of it for $18,240. What was the value of the mill?

52. A man receives a salary of $3,200 a year. He pays 8⅓% of it for board, 6¼% for clothing, 15% for traveling expenses, and 12½% for incidentals. What are his yearly expenses?

53. In a class examination, 180 questions were submitted to each of the 6 members. A answered 120, B 150, C 160, D 135, E 175, and F 125. What per cent of the questions did each member answer?

54. B sold his house for $4,371, which was 93 % of its cost. What did it cost?

55. A young man having received a fortune, deposited 85 % of it in a bank. He afterward drew out 30 % of his deposit, and then had $7,140 in the bank. How much was his entire fortune?

56. A man owning 75 % of a factory, sold 33⅓ % of his share for $13,570. At this rate what is the value of the factory?

57. One trader made in one year $5,200, which was 62½ % of what another trader lost in the same time. If the second trader closed with $3,500, how much had he at the beginning?

58. An orchard contains 1,400 trees, of which 37 % are apple, 26 % plum, 15½ % pear, and the remainder cherry. How many trees of each kind are there in the orchard?

59. A flock of sheep has been increased by 175 % of its number, and now numbers 902. What was the original number?

60. A man having an estate worth $90,000, bequeathed 8⅓ % of it to a public library, 12½ % of it to a college, 2⅔ % to a church, 9⅙ % to a hospital, 30 % to his wife, and the remainder he divided equally between his two daughters. How many dollars did he bequeath to each object? How much did each daughter receive? What per cent of his estate did he bequeath?

61. A mass of gunpowder is made of 2,691 lbs. of nitre, 547 lbs. of charcoal, and 348 lbs. of sulphur. What per cent of the mass does each ingredient constitute?

62. A man owning 60 % of a mine gave his son 50 % of his share, which was worth $15,000. What was the value of the mine?

63. A man drew 50 % of his bank deposits, and expended 66⅔ % of the money thus drawn in the purchase of a building lot, paying $480 for it. How much money had he in the bank at first?

64. F deposited 75 % of his money in a bank, and afterward drew out 20 % of the sum deposited. He then had $2,460 in the bank. What was the total amount of his money?

65. Two contractors agree to pave a certain street. A paves 901 rds., which is 53 % of the length of the street. What is the length of the street?

66. A gentleman owned a farm worth $9,313, which was 39 % more than the value of the house which he built upon it. What was the value of the house?

APPLICATIONS OF PERCENTAGE.

229. The chief applications of Percentage are Profit and Loss, Commission, Commercial Discount, Insurance, Taxes, Duties or Customs, Stocks and Brokerage, Interest, Discount, and Exchange. The last three involve the element of *time*. This new element will demand special treatment.

1. The difference between the cost of an article, bought or produced, and the selling price is called *profit* or *loss*, according as the selling price is more or less than the cost.

2. *Commission* is compensation paid to a person who buys or sells goods, or transacts certain other kinds of business, for another. A person doing such a business is called a commission merchant, factor, broker, or agent.

(*a*) His commission is some per cent of the money which he collects or invests.

(*b*) The sum left after the commission and all expenses are deducted is the proceeds.

3. *Commercial Discount* is a reduction made by manufacturers and wholesale dealers from the list price of goods, or from the amount of a debt.

(*a*) Price lists of articles manufactured and sold are issued. The market price of these articles changes, while the list price remains the same. Discounts are made and changed to meet the rise and fall in the market prices, to avoid the necessity of changing the list or catalogue prices.

(*b*) Cash discounts are discounts made from bills for payment before they are due.

(*c*) Sometimes several discounts are allowed the purchaser. In such cases, the first discount is deducted; the second is computed upon the remainder and deducted; and so on for each successive discount.

4. *Insurance* is security guaranteed by one party to another against financial loss on account of the destruction of property, or by the injury or death of a person.

(*a*) Property insurance includes fire insurance, tornado insurance, marine insurance, and live-stock insurance.

(*b*) Personal insurance includes life insurance, accident insurance, and health insurance.

(*c*) The party who agrees to make good the losses is called the Insurer, or Underwriter.

(*d*) The written contract between the insurer and the insured is called a Policy.

(*e*) The sum paid the insurer for taking the risk is called a Premium.

In property insurance the premium is a certain per cent of the amount insured. In life insurance it is a certain amount per year for each thousand dollars of insurance. In accident insurance it is a certain amount for the protection afforded for a given time. Personal insurance is not computed by percentage.

5. A *Tax* is a sum of money assessed upon a person or upon property, to meet public expenses.

(*a*) A poll tax is a certain sum required of each male citizen liable to taxation without regard to his property. Each person so taxed is called a poll.

(*b*) A property tax is a tax on property. It is a certain rate per cent of the estimated, or assessed, value of the property.

(*c*) Property is of two kinds, personal property and real estate. Personal property consists of movable property, such as money, household goods, cattle, ships, etc. Real estate consists of immovable property, such as lands and improvements put upon them.

(*d*) Assessors are officers whose duty is to ascertain the value of taxable property, and the number of polls, and to apportion the tax to each person.

6. *Duties* are taxes levied by the general government on goods imported from other countries.

Duties are of two kinds, *ad valorem* and *specific*.

(*a*) *Ad valorem* duty is a certain per cent of the cost of the goods in the country from which they are imported.

(*b*) *Specific* duty is a sum imposed upon goods according to the quantity, but without regard to their value. Such duties are not computed by percentage.

7. Capital is money employed in business. When the business is carried on by a corporation its capital is represented by *Stock*.

(*a*) Stock is divided into equal portions, usually $100, called shares.

(*b*) Stock may be bought and sold as any other property. The *par value* of stock is its original or face value.

(*c*) The *market value* is its selling price. When shares sell for more than their face or par value, the stock is said to be *above par*, or at a *premium*. When they sell for less than their par value, stock is *below par*, or at a *discount*.

(*d*) Persons engaged in the business of buying and selling stocks for others are called brokers; and their compensation, called *brokerage*, is some per cent, generally, of the par value of the stocks they buy or sell.

ORAL EXERCISES.

(Be ready to tell what the per cent is hundredths of, in every case.)

230. *1.* What part of the cost is gained when 10 % is gained? When 12½ % is gained? When 25 % is gained?

2. What part of the cost is lost when 5 % is lost? When 33⅓ % is lost? When 62½ % is lost?

3. If a merchant buys hats at $4 apiece and sells them at 25 % profit, what does he gain? What is the selling price?

4. If a grocer buys sugar at 5 cts. a pound and sells it at 20 % loss, how much does he lose per pound? What is the selling price?

5. When shoes that cost $4 a pair are sold at $6 a pair, how much is gained on each pair? The gain is what part of the cost? What per cent of the cost?

6. When flour that costs $6 a barrel is sold at $5 a barrel, how much is lost per barrel? The loss is what part of the cost? What per cent of the cost?

7. A merchant gained 50 cts. on the sale of an umbrella, which was 40 % of the cost. What was the cost? What was the selling price?

8. A grocer sold rice at 6 cts. a pound, and gained 20 % of the cost. What was the cost per pound? What was the gain per pound? (The selling price was $\frac{6}{5}$ of the cost.)

9. At what price per pound must a grocer sell tea that costs 60 cts. a pound, so as to gain 30 %?

10. If butter that costs 32 cts. a pound is sold at a loss of $6\frac{1}{4}$ %, what is the selling price?

11. If a grocer sells coffee that cost 18 cts. a pound, at 24 cts. a pound, what per cent does he gain?

12. If cheese that cost 12 cts. a pound is sold at 10 cts. a pound, what per cent is lost?

13. A tailor sold cloth at a profit of $2 a yard, and gained $33\frac{1}{3}$ %. What was the cost?

14. A jeweler lost $15 by selling a watch at 20 % below cost. What was the cost?

15. A newsboy, by selling his papers at 4 cts. each, gained $33\frac{1}{3}$ %. What did they cost him?

16. If a grocer sells nutmegs at 33 cts. a pound and loses $8\frac{1}{3}$ %, what do they cost him?

17. If, by selling books at $2.50 a volume, 25 % is gained, at what price per volume must they be sold to lose 25 %?

18. Sold melons at 60 cts. each, that cost 40 cts. each. What per cent was gained?

19. A commission merchant sold a consignment of cotton for $800. How much was his commission at 2%?

20. I paid an attorney $15 for collecting a debt of $300. What was his rate of commission?

21. An agent sold a sewing machine and sent the company $56, after deducting his commission of 12½%. What did he receive for the machine?

22. I paid an auctioneer $27 for selling my personal property, which was a commission of 3%. What was the amount of the sales?

23. What is the premium for insuring a house for $2,000, at 2½%?

24. How much must be paid for insuring a house and its furniture for $5,000, at 1½%?

25. A merchant bought goods amounting to $500, and was allowed 6% off for cash. How much was the discount? How much did he pay?

26. Mr. B. bought books amounting to $360, from which a discount of 16⅔% from the list price was made, and a further discount of 5% of the balance for cash. How much did the books cost him? What was the amount of the discount?

27. What is the amount of tax on property valued at $1,000, at 1½%?

28. If a tax of $12 is paid on a house and lot valued at $1,200, what is the rate per cent of tax?

29. What is the duty at 15%, on a piano invoiced at $400?

30. What will 7 shares of railroad stock cost, at 8% premium? At 10% premium?

31. What will 10 shares of telephone stock cost, at $6\frac{1}{2}$% discount? At 12% discount?

32. What per cent is lost by selling a cow for $40, that cost $60?

33. A horse that cost $150, was sold at a gain of 10%. What was received for him?

34. A grocer sells tea at 15 cts. a pound more than cost, and gains $16\frac{2}{3}$%. What does it cost? What is the selling price?

35. What per cent is gained on an article that is bought for $2 and sold for $4? If sold for $5? If sold for $6?

36. What per cent is lost on an article that cost $6 and is sold for $5? Sold for $4? Sold for $3?

37. An agent sold a sewing machine for $70, and gained 40%. What was the cost of the machine?

38. An attorney received $56 for collecting a note, which was a commission of 7%. What amount did he collect?

39. What will be the annual premium of insurance, at $\frac{3}{4}$%, on a mill valued at $40,000?

40. A grocer bought a bill of $500, from which the seller made a discount of 20% and 8%. What was the amount paid?

41. An organ was sold for $180, at a profit of $12\frac{1}{2}$%. What was the cost?

42. What is the duty at 25 %, on oil paintings valued at $36,000?

43. What must be the amount of my sales for one month that I may gain $900, at a profit of 15 %?

44. What per cent will be lost by selling gloves for 75 cts. that cost 80 cts.?

45. A commission merchant received $210 on a sale of wool, at 6 % commission. What was the amount of the sale?

46. $90 was paid for insuring a load of horses at 5 %. For what amount were the horses insured?

47. What per cent is gained by selling goods for 2 times their cost? For $3\frac{1}{2}$ times their cost?

48. What per cent is lost by selling goods for $\frac{3}{4}$ of their cost? For $\frac{1}{2}$ of their cost? For $\frac{1}{3}$ of their cost?

49. A broker negotiated the sale of $4,800 United States securities, at a brokerage of $\frac{1}{4}$ %. What was the brokerage?

50. If 20 % is lost by selling wheat at 60 cts. a bushel, how many cents would have been gained if sold at 20 % profit?

51. A commission merchant receives $618 with which to buy flour, after deducting his commission of 3 %. How much did he invest in flour? How much was his commission?

52. If 25 % is gained by selling tea at 90 cts. a pound, what is the cost?

53. If $75 is paid for an insurance of $2,500, what is the rate of insurance?

54. At what price must an article that costs $5 be sold so as to gain 100 %? To gain 150 %? To gain 200 %? To gain 340 %? (The selling price is how many times the cost, in each case?)

55. What was my property worth 6 yrs. ago, if it has since increased 100 %, and is now worth $5,000?

56. What per cent is gained on the sale of a reaper that is bought for $60 and sold for $70?

57. 10 % is lost on hay sold at $18 a ton. At what price per ton must it be sold to gain 15 %?

58. Two pictures were sold for $63 each. 12½ % was gained on one, and on the other there was a loss of 12½ %. Which was greater, the gain or the loss? How much greater?

59. If 10 % is gained on the sale of a watch that is sold for $55, what per cent would be gained if sold for $65?

60. If a stove was sold for $18 less than cost, and 37½ % was lost, what per cent would have been gained if it had been sold for $56?

WRITTEN EXERCISES.

231. *61.* What is the net amount of a bill of $680.50 subject to a discount of 20 %, 12½ %, and 8 %? (See 3 (*c*), p. 247.)

62. ⅝ of an estate, valued at $45,000, is insured in one company at 1⅝ %, and the remainder in another company at ⅝ %. What is the annual premium for the whole?

63. French silk costs $1.60 a yard. What is the duty, at 2½ %, on 30 cases of silk, of 60 yds. each?

64. A commission merchant sold 250 bbls. of potatoes at $2.90 a barrel, and 328 bbls. of apples at $4.15 a barrel. What was his commission at 4½ %?

65. If a man invests $1,750 in wheat, and sells so as to gain 14⅓ %, what is his profit?

66. What was the cost of corn that was sold at a profit of $2,000, or a gain of 16 %?

67. The taxable property of a certain town is $368,425, on which a tax of $5,894.80 is to be raised. What is the rate per cent? How much will be the collector's commission at 3 %? A's property is valued at $4,850, and B's at $5,765. How much tax will each have to pay?

68. A man bought a house and lot for $2,500, and he sold them for $2,150. What per cent did he lose?

69. A commission merchant sold a consignment of iron for $15,504, and gained 14 %. What was the cost of the iron? How much was his commission at 3½ %? How much did he remit to the consignor?

70. A dealer imported from Geneva, 50 watches invoiced at $80 each, and 20 clocks at $12 each. What was the duty, the rate being 30 % on the watches and 18 % on the clocks?

71. An insurance company took a risk of $14,640 on a vessel to Liverpool and return, at a premium of 1¼ %, and on the cargo each way $11,340, at 1⅛ % out and 1⅛ % return. What was the whole premium?

72. What must a grocer ask for tea that cost 75 cts. a pound so that he may fall 10 % and still make 20 %?

EXPLANATION: If he makes 20 %, or $\frac{1}{5}$, on the cost, he must get 75 cts. plus $\frac{1}{5}$ of 75 cts., or 90 cts. a pound. And if he falls 10 % from his asking price, the 90 cts. must be $\frac{9}{10}$ of the asking price, which is $1 a pound.

73. What must a dealer ask for pianos that cost $350, so that he can fall 12½ % and still make a profit of 14 %?

74. A merchant sold goods for $3,113.20, and gained 7½ %. What was the cost?

75. If a merchant buys goods amounting to $2,820, and is given a discount of 33⅓ %, 10 %, and 4 %, how much does he pay for the goods?

76. What is the difference on a bill of $450, between a direct discount of 30 % and successive discounts of 25 % and 5 %?

77. A vessel worth $28,000 was insured at 1¾ %, and the cargo worth $15,000, at 2½ %. Both were totally lost. What was the loss to the insurer?

78. The net amount of a tax collection was $2,790.06. If the collector's commission was 2 %, what was the whole amount collected? If the value of the taxable property was $113,880, what was the rate per cent of tax? How much tax will Mr. Hunt have to pay, if his property is assessed at $3,480?

79. What was the original cost of a lot of land which, selling at 28 % below cost, brought $1,283.76? How much was the loss?

80. Bought goods for $11,600. Half of them I am obliged to sell at a loss of 16%. If I sell the other half at a gain of 20%, how much do I gain on the whole? The gain is what per cent of the whole investment?

81. A lawyer collected 85% of a note for $980, and charged 6½% commission on what he collected. How much did the man who held the note get for it?

82. A grocer lost 9% by selling 50 lbs. of butter for $1.08 less than cost. What did it cost him per pound?

83. A herd of cattle was bought and shipped to Chicago, at an expense of $3,825. They were sold for $4,322.25. What was the gain per cent? How much did the commission merchant receive, at a commission of 2%?

84. Find the duty at 12½% on 100 cases of indigo, each weighing 185 lbs., and invoiced at 70 cts. a pound.

85. A mill valued at $64,000 was insured for ⅝ of its value, at 1½%. What would be the actual loss to the insurer if the property were destroyed within 6 months? What would be the owner's loss?

86. How must carpet, costing $2 a yard, be marked so that 10% may be deducted from the marked price and still a profit of 17% be made?

87. An agent sold real estate for Mr. Dale, and remitted $3,000, after deducting a commission of 4%. How much was received for the real estate, and what was the agent's commission?

88. Find the total amount of duty on the following importations:

$750 worth of toys, at 30 %.

$1,250 worth of lace, at 50 %.

$860 worth of penknives, at 40 %.

80 bags of coffee, 156 lbs. each, invoiced at 8 cts. a pound, at $4\frac{2}{3}$ %.

89. An organ was sold for $1,512 less than its value, which was a loss of 27 %. What would have been the gain per cent if it had been sold for $6,944?

90. A farm was sold for $5,814, which was 40 % less than its value. What was the value of the farm?

91. A merchant can buy goods from A, B, or C at the same list price; but A will allow him successive discounts of 25 %, 12 %, and 8 %; B will allow him successive discounts of 12 %, 25 %, and 8 %; and C, 8 %, 12 %, and 25 %. Which makes him the best offer, and how much, on a bill of goods amounting to $5,000?

92. A broker received $7,052.50 to be invested in stocks, after deducting $\frac{3}{4}$ % brokerage. What was the amount invested in stocks?

93. A furniture dealer sold two parlor sets for $385 each. On one he gained $37\frac{1}{2}$ %, and on the other lost $37\frac{1}{2}$ %. What did each cost him?

94. Mr. Green's broker sold 125 shares of railroad stock, at 16 % premium. How much did Mr. Green receive after a deduction of $\frac{1}{4}$ % brokerage?

95. A and B make $17,520 by trading in company. A's money is 200 % of B's money. What part of the profits should each receive?

96. E and F are in partnership. E receives of the profits $490, which is 40% of the whole profit. How much was the total profit?

97. A merchant started in business with a certain capital. The first year he lost 6¼% of his capital; the second year he lost 16⅔% of the remainder; and the third, 4% of the rest, when he had $4,800 left. How much was his capital at first?

98. Suppose the merchant in the preceding problem had gained 25% the first year. The other conditions remaining the same, what would have been his capital at the first?

99. L and M are partners in business. $4,410 is 58⅓% of their profits for a year. A's capital is 118¾% of B's capital. How much is each man's share of the profits?

100. An agent bought for a merchant in Chicago, 2,600 bbls. of flour, at $4 a barrel, and charged 2% commission. What was the amount of the bill, including the commission?

101. A man sold his farm for $10,000, for which he had paid $7,500. What was the gain per cent?

102. A grain buyer in Nebraska received $19,125 with which to buy wheat. After deducting 2% commission, how many bushels of wheat did he buy at $.75 a bushel?

103. A real estate agent received $80 for selling a piece of property. His commission being 2%, what was the amount of the sale?

INTEREST.

232. *1.* *Interest* is money paid for the use of money.

2. The *Principal* is the money for the use of which interest is paid.

3. The *Amount* is the sum of principal and interest.

4. The *Rate Per Cent,* or *Rate of Interest,* is the per cent of the principal paid for its use for *one year,* or other specified time.

Money paid for labor is usually called wages. Money paid for the use of houses and lands is called rent. Wages and rent are usually reckoned by the day, the month, or the year. Hence, time is an essential element in reckoning such compensation. So in interest, the longer the time one uses money, the larger the compensation. The customary unit of time for reckoning interest is one year.

The rate of interest fixed by law is the *legal rate.* Any higher rate is usury. The legal rate in many of the States and on debts due the United States is 6%.

ORAL EXERCISES.

233. *1.* What is the interest on \$400 for a year, at 6%? At 9%? At 4½%? At 7½%? At 12%? At 5½%?

2. What is the interest on \$600 for 1 yr., at 5%? At 8½%? At 6⅓%? At 12%? At 10⅔%?

3. What is the interest on \$120 for 1 yr., at 3%? At 5½%? At 12%? At 6¼%? At 7¾%?

4. What is the interest on $200, at 6%, for 1 yr.? For 2 yrs.? For 5 yrs.? For 3 yrs. 3 mo. ($3\frac{1}{4}$ yrs.)? For 6 yrs. 6 mo.?

5. What is the interest on $500, at 4%, for 2 yrs. 6 mo.? For 3 yrs. 9 mo.? For $4\frac{2}{3}$ yrs.? For 6 yrs. 3 mo.? For $8\frac{1}{3}$ yrs.?

6. What is the interest on $300 for 2 yrs. 2 mo., at 4%? For 3 yrs. 4 mo., at 5%? For 1 yr. 8 mo., at 8%? For 5 yrs. 6 mo., at $3\frac{1}{3}$%? For 2 yrs. 10 mo., at $9\frac{2}{3}$%?

234. Six Per Cent Method.

At 6% the interest on any principal for 1 yr. is $\frac{6}{100}$ of the principal; for 2 mo. ($\frac{1}{6}$ of a year) it is $\frac{1}{100}$ of the principal; for 200 mo. it equals the principal.

1. What part of the principal is the interest for 100 mo.? For 50 mo.? For 40 mo.? For 25 mo.? For 20 mo.? For 10 mo.? For 2 mo.? For $16\frac{2}{3}$ mo.? For $33\frac{1}{3}$ mo.? For $66\frac{2}{3}$ mo.?

2. In computing interest, 1 mo.=30 days, and 2 mo.=60 days. Since the interest for 60 days is $\frac{1}{100}$ of the principal, the interest for 6 days ($\frac{1}{10}$ of 60 days) is $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$ of the principal. Hence, to compute interest on any principal, at 6%, by this method, *the following facts must be remembered:*

(1) The interest equals the principal in 200 mo.

(2) The interest equals $\frac{1}{100}$ of the principal in 2 mo.

(3) The interest equals $\frac{1}{1000}$ of the principal in 6 days.

3. Find the interest of $2,440, for 3 yrs. 11 mo. 21 days, at 6%.

EXPLANATION:

3 yrs. 11 mo. = 47 mo.

$2440 = principal.
$305 = interest for 25 mo. = $\frac{1}{8}$ of the principal.
244 = interest for 20 mo. = $\frac{1}{10}$ of the principal.
24.40 = interest for 2 mo. = $\frac{1}{100}$ of the principal.
6.10 = interest for 15 days = $\frac{1}{4}$ of $\frac{1}{100}$ of the principal.
2.44 = interest for 6 days = $\frac{1}{1000}$ of the principal.
$581.94

If the amount is to be found, do not draw a line under the principal, because it is to be added with the interest.

WRITTEN EXERCISES.

235. Solve the following problems by the above method. Do not carry results farther than tenths of mills:

Find the interest, at 6%, on:

1. $380 for 3 yrs. 8 mo. 24 days.

2. $540 for 5 yrs. 3 mo. 20 days.

3. $1,060 for 4 yrs. 10 mo. 8 days.

4. $460 for 6 yrs. 5 mo. 26 days.

5. $872 for 2 yrs. 9 mo. 25 days.

6. $687 for 7 yrs. 2 mo. 19 days.

7. $900 for 3 yrs. 6 mo. 12 days.

8. $215 for 6 yrs. 11 mo. 28 days.

9. $80.60 for 8 yrs. 1 mo. 10 days.

10. $2,850 for 4 yrs. 7 mo. 14 days.

11. $64.82 for 7 yrs. 3 mo. 6 days.

12. $1,400 for 10 mo. 21 days.

13. $740 for 5 yrs. 8 mo. 13 days.

14. $100 for 9 yrs. 9 mo. 9 days.

15. $76.50 for 3 yrs. 7 mo. 27 days.

16. $40.40 for 6 yrs. 2 mo. 16 days.

Find the amount, at 6%, of:

17. $320 for 4 yrs. 3 mo. 28 days.

18. $892 for 10 yrs. 4 mo. 5 days.

19. $500 for 9 mo. 11 days.

20. $463.50 for 8 yrs. 6 mo. 24 days.

21. $325.75 for 3 yrs. 8 mo. 15 days.

22. $1,000 for 7 yrs. 10 mo. 26 days.

23. $2,890 for 2 yrs. 11 mo. 16 days.

24. $96.70 for 5 yrs. 4 mo. 3 days.

25. $56.48 for 6 yrs. 5 mo. 20 days.

236. Rule for General Method.

Since, at 6%, the interest for 200 mo. equals the principal, for 2 mo. it is .01 of the principal, and for 6 days it is .001 of the principal. Therefore, express the given time in months and days. Find what part of any principal the interest for the given time is, and take such a part of the given principal.

EXERCISES.

237. *1.* Find the interest on $678 for 2 yrs. 4 mo. 16 days, at 6%.

$678.
.142⅔
$ 96.728

EXPLANATION: 2 yrs. 4 mo.=28 mo. The interest for 28 mo. is .14 of the principal. 16 days=2⅔ times 6 days. The interest for 16 days=2⅔ times .001 of the principal, or .002⅔ of the principal. The interest for 28 mo. 16 days=.14 of the principal+.002⅔ of the principal, or .142⅔ of the principal. .142⅔ of $678=$96.728, the interest on $678 for 2 yrs. 4 mo. 16 days, at 6%.

Find the interest, at 6%, on:

2. $650 for 3 yrs. 1 mo. 18 days.

3. $280 for 5 yrs. 7 mo. 24 days.

4. $500 for 2 yrs. 10 mo. 12 days.

5. $390 for 4 yrs. 3 mo. 21 days.

6. $800 for 1 yr. 5 mo. 9 days.

7. $735 for 11 mo. 15 days.

8. $126 for 9 yrs. 6 mo. 27 days.

9. $84.60 for 7 yrs. 2 mo. 3 days.

10. $1,500 for 1 yr. 8 mo. 20 days.

11. $475.50 for 3 yrs. 8 days.

12. $18.90 for 8 yrs. 5 mo. 26 days.

13. $578.10 for 2 yrs. 7 mo. 14 days.

Find the amount, at 6 %, of:

14. $2,700 for 3 mo. 10 days.

15. $75 for 4 yrs. 2 mo. 22 days.

16. $900 for 5 yrs. 1 mo. 7 days.

17. $870 for 1 yr. 9 mo. 19 days.

18. $54.30 for 6 yrs. 8 mo. 11 days.

19. $4,176 for 4 mo. 23 days.

20. $312.60 for 3 yrs. 28 days.

21. $85 for 9 yrs. 6 mo. 15 days.

22. $29.45 for 7 mo. 29 days.

238. To Find the Interest when the Rate is Other than 6 per cent.

Find the interest at 6 %, and then add to it or subtract from it the proper amount to give the interest at the required rate.

Having the interest at 6 %—

Add $\frac{1}{6}$ of it for 7 %.	Subtract $\frac{1}{6}$ of it for 5 %.
Add $\frac{1}{4}$ of it for 7$\frac{1}{2}$ %.	Subtract $\frac{1}{3}$ of it for 4 %.
Add $\frac{1}{3}$ of it for 8 %.	Subtract $\frac{1}{4}$ of it for 4$\frac{1}{2}$ %, etc.

Or divide the interest at 6 % by 6, and multiply the quotient by the given rate.

Review the second method of finding the time between two dates (p. 182).

EXERCISES.

239. *1.* Find the interest on $284.36 from April 8, 1891, to November 5, 1895, at $5\frac{1}{2}\%$.

EXPLANATION: From April 8, 1891, to April 8, 1895, is 4 yrs.; from April 8, 1895, to October 8, 1895, is 6 mo. There are 23 days more in October and 5 in November. Hence, the time is 4 yrs. 6 mo. 28 days, or 54 mo. 28 days. The interest on $284.36, at 6%, is $78.104. $5\frac{1}{2}\%$ is $\frac{11}{12}$ of 6%, or $\frac{1}{12}$ of 6% less than 6%. Subtracting $\frac{1}{12}$ of the interest at 6% from the interest at 6%, leaves $71.595, the interest at $5\frac{1}{2}\%$.

Find the interest on:

2. $572.84 from November 15, 1893, to August 15, 1894, at 5%.

3. $87.50 from February 13, 1887, to October 10, 1893, at 7%.

4. $3,000 from December 2, 1895, to June 21, 1896, at 4%.

5. $845 from July 5, 1894, to May 25, 1898, at $7\frac{1}{2}\%$.

6. $672.20 from March 20, 1891, to January 18, 1895, at 3%.

7. $486.15 from October 27, 1890, to November 13, 1897, at 7%.

8. $165 from January 15, 1896, to December 3, 1896, at $4\frac{1}{2}\%$.

9. $750 from September 7, 1892, to July 20, 1897, at 8%.

10. $1,200 from June 17, 1891, to June 27, 1894, at $5\frac{1}{2}\%$.

11. $530.88 from February 28, 1892, to May 15, 1897, at 4%.

12. $810.35 from August 17, 1890, to June 13, 1895, at 7½%.

13. $37.50 from November 1, 1893, to March 27, 1894, at 8%.

14. $700 from May 10, 1893, to January 3, 1897, at 6½%.

15. $8,005.61 from April 14, 1896, to September 14, 1896, at 7%.

16. $313.18 from September 5, 1891, to July 1, 1897, at 8%.

17. $28.28 from December 11, 1895, to October 27, 1898, at 9%.

18. $910 from March 19, 1892, to March 29, 1896, at 5%.

19. $17.25 from January 20, 1896, to November 30, 1896, at 12%.

20. $222.20 from October 9, 1896, to December 1, 1900, at 5½%.

Find the amount of:

21. $1,800 from November 20, 1896, to March 12, 1897, at 6½%.

22. $172.16 from May 27, 1890, to February 21, 1894, at 10%.

23. $695 from October 18, 1895, to December 22, 1896, at 4½%.

24. $3,460 from February 13, 1894, to July 13, 1894, at 8½%.

25. $1,643 from December 30, 1893, to September 10, 1896, at 4%

26. $948.15 from August 4, 1895, to August 11, 1896, at 7%.

27. $630 from June 12, 1892, to January 10, 1895, at 7½%.

28. $400 from March 31, 1890, to April 30, 1894, at 5%.

29. $68 from January 6, 1896, to August 16, 1896, at 8%.

30. $48 from July 13, 1888, to May 1, 1896, at 5½%.

31. $5,000 from November 1, 1894, to July 1, 1895, at 9%.

32. $320 from September 5, 1892, to February 29, 1896, at 4½%.

33. $670 from April 10, 1891, to April 13, 1895, at 6½%.

34. $200 from August 19, 1893, to October 8, 1896, at 8½%.

35. $1,600 from February 7, 1895, to September 30, 1895, at 7%.

NOTE: Bankers and others who have occasion to compute interest frequently, rarely work by any of the methods given in the books. They use interest tables.

PARTIAL PAYMENTS.

240. FORM OF NOTE.

$315.50. BLOOMINGTON, ILL., July 27, 1896.

Eight months after date, for value received, I promise to pay J. T. Hicks or order, three hundred fifteen $\frac{50}{100}$ dollars, with interest at six per cent.

S. N. MOORE.

1. The written promise above is a *promissory note.* J. T. Hicks is the *payee,* and S. N. Moore the *maker.* The $315.50 is the *face,* or *principal.* Make a definition for each of these terms.

2. There are several forms of notes. The one given above is a *time note.* If the words "on demand" occurred in the note instead of the words "eight months after date," it would be called a *demand note.* If instead of the words "I promise," it contained the words "we or either of us promise," and was signed by two or more persons, it would be called a *joint and several note.*

(*a*) The amount of the note should be written in words, to avoid error or fraud.

(*b*) If the words "for value received" are omitted, the owner of the note may have to prove that the maker received the value specified.

(*c*) If the words "with interest" are omitted, the note will not draw interest till it is due. After it is due, it will draw interest at the legal rate prevailing at the place where it was made.

3. Some notes may be bought and sold. Such notes are called *negotiable notes.* If J. T. Hicks should sell the above note, he would write his name across the back. This will make it payable to the

holder, and will guarantee its payment. This is called indorsing the note "in blank." It is then an order to S. N. Moore to pay the note to the person who holds it when it becomes due. If he fails to pay it, the indorser is liable for its payment. If the indorsement names some person to whom, or upon whose order, the note is to be paid, it is called "indorsement in full." If the indorser declines to guarantee the payment, he adds to his indorsement the words, "without recourse." If a note contains the words "or bearer," instead of the words "or order," it needs no indorsement, but is payable to the person who presents it for payment at maturity.

A note that is payable to the payee only, contains neither the words "or bearer" nor "or order." It is non-negotiable.

4. A note is payable at the time specified, but three days are usually allowed before the note is *legally* due. The three days are called "days of grace."

5. Partial payments are sometimes made on notes and other obligations bearing interest; records of these payments made on the back of the note are called "indorsements." The following rule has been adopted by the Supreme Court of the United States for finding the value of interest-bearing notes:

241. The United States Rule.

Find the amount of the principal to the time when the payment, or the sum of the payments, equals or exceeds the interest due. From this amount subtract the payment or the sum of the payments. With the remainder as a new principal, proceed as before.

EXERCISES.

242. *1.* A note for $760, dated June 15, 1881, bearing interest at 6%, had payments indorsed upon it as follows: Jan. 3, 1882, $150; Nov. 8, 1883, $20; May 10, 1885, $45; Sept. 24, 1886, $230. How much was due March 12, 1889?

EXPLANATION: The interest on $760 from June 15, 1881, to Jan. 3, 1882, is $25.08, which is less than the payment. The amount is $785.08. Deducting the payment, $150, from this amount, there is left $635.08 for a new principal. The interest on $635.08 from Jan. 3, 1882, to Nov. 8, 1883, is more than the payment. The interest from Jan. 3, 1882, to May 10, 1885, is greater than the sum of the payments, $20 and $45. The interest from Jan. 3, 1882, to Sept. 24, 1886, is $180.044. The sum of the payments for the same time is $295, which, deducted from the amount of the note for the time, leaves $520.124 for a new principal. The amount of this principal from Sept. 24, 1886, to March 12, 1889, is $591.901. Hence, there was due March 12, 1889, $596.928.

2. A note of $650, dated Feb. 16, 1875, was indorsed as follows: Oct. 10, 1875, $90; Sept. 20, 1876, $28; May 6, 1877, $118; Aug. 17, 1878, $210. How much was due July 1, 1879, at 6% interest?

3. What is due at the end of a year, on a note for $1,200, dated June 15, 1895, with interest at 7%, indorsements as follows: Aug. 20, 1895, $275; Dec. 1, 1895, $190; Feb. 3, 1896, $500; April 25, 1896, $210.

4. What sum will discharge a note for $900, dated Sept. 1, 1889, settled Aug. 1, 1894, with interest at 8%, indorsements as follows: June 1, 1891, $120; Dec. 16, 1892, $350; Oct. 1, 1893, $265.

5. Find the balance due Sept. 25, 1894, on a note of $1,240.60, dated June 1, 1892, with interest at 6%, containing the following indorsements: Sept. 25, 1892, $340; May 1, 1893, $360; Aug. 25, 1893, $310; Feb. 1, 1894, $80.60.

6. $1,000. DETROIT, MICH., July 10, 1890.

On or before July 10, 1895, we, or either of us, promise to pay to B. F. Rogers & Co., or order, one thousand dollars, with interest at 7%. Value received. A. T. CAMPBELL,
J. R. MASON.

If the following indorsements were written across the back of this note, how much was still due July 10, 1895?

Received on the within note, Feb. 20, 1891, $250.
Received on the within note, Dec. 1, 1892, $215.50.
Received on the within note, Aug. 25, 1894, $390.

243. The Mercantile Rule.

The following method of computation is often used by business men, in the settlement of notes and interest accounts running a year or less, upon which partial payments have been made. It is called the *Mercantile Rule:*

Find the amount of the principal at the time of settlement. Find the amount of each payment from the time it was made to the time of settlement. From the amount of the principal subtract the amount of the payments.

EXERCISES.

244. *1.* On a note for $820, dated May 12, 1895, were the following indorsements: Aug. 20, 1895, $210; Oct. 1, 1895, $175; Dec. 28, 1895, $128; March 10, 1896, $350. How much remained due April 25, 1896, at 5% interest?

2. How much is due at the end of a year, on a note for $1,200 dated July 15, 1895, with interest at 7 %, on which the following payments have been made: Sept. 25, 1895, $300; Nov. 8, 1895, $185; Jan. 30, 1896, $316; May 2, 1896, $435.

3. Write two forms of negotiable notes from the following data: Face, $800; maker, yourself; payee, P. S. Eldred; time, 10 mo. 24 days; rate of interest, 6 %.

Make three indorsements on each of the above notes and find the amount still due at the end of the time, by the mercantile rule.

4. Write a joint and several note for $1,450, non-negotiable, to N. R. Davis, due in 11 mo. 21 days from date, with interest at 7 %, and signed by D. P. Lynch and C. O. Barlow.

Make four indorsements on the note, and find how much is still due at maturity.

ANNUAL INTEREST.

245. Sometimes when money is loaned, the note is made to read, "with interest payable annually," or "with annual interest." This means that the interest on the principal is due at the end of each year of the contract. If the interest is not paid when due, it draws simple interest until it is paid.

Simple interest on the principal, and on the interest from the time it is due until it is paid, is called *Annual Interest.* (In some States annual interest is illegal.)

EXERCISES.

246. *1.* Find the amount of $2,400 for 4 yrs. 8 mo., with interest payable annually, at 6%.

$2400
672
74.88
$3146.88

EXPLANATION: The interest on $2,400 for 4 yrs. 8 mo. is $672. The annual interest is $144. The first year's interest remained unpaid for 3 yrs. 8 mo.; the second, 2 yrs. 8 mo., etc. Therefore, the unpaid interest, $144, drew interest for 8 yrs. 8 mo. The interest on $144 for 8 yrs. 8 mo. is $74.88. The amount due is the sum of $2,400, $672, and $74.88, or $3,146.88.

Find the amount of the following, with annual interest:

2. $1,600 for 2 yrs. 9 mo., at 6%.

3. $896 for 5 yrs. 3 mo., at 7%.

4. $1,250 for 3 yrs. 10 mo., at 7½%.

5. $1,000 for 4 yrs. 6 mo., at 5½%.

6. $1,480 for 6 yrs. 4 mo., at 5%.

7. $3,600 for 2 yrs. 11 mo., at 4%.

8. $998 for 4 yrs. 4 mo., at 6½%.

9. $1,570 for 5 yrs. 7 mo., at 4½%.

10. $2,500 for 3 yrs. 1 mo., at 8%.

COMPOUND INTEREST.

247. *Compound Interest* differs from annual interest in one particular: the interest is added to the principal at the end of each period, thus forming a new principal each time. Hence, compound interest is more than annual interest.

The interest may be added to the principal annually, semi-annually, or quarterly, as the parties agree.

EXERCISES.

248. *1.* What is the amount of $800 for 4 yrs., at 6%, compound interest?

$ 800	1st principal.
48	1st interest.
$ 848	2d principal.
50.88	2d interest.
$ 898.88	3d principal.
53.93	3d interest.
$ 952.81	4th principal.
57.16	4th interest.
$1009.97	amount.
800.	
$ 209.97	compound interest.

Find the compound interest on the following:

2. $600 for 3 yrs. 10 mo., at 6%.

3. $560 for 4 yrs. 6 mo., at 7%.

4. $480 for 2 yrs. 9 mo., at 5½%.

5. $900 for 3 yrs. 3 mo., at 4½%.

6. $320 for 2 yrs. 6 mo., at 6%, payable semi-annually.

7. $740 for 2 yrs. 8 mo., at 5%, payable quarterly.

Compound interest is rarely paid. When it is, it is almost invariably computed from a table.

TRUE DISCOUNT.

249. *1. Discount* is the amount allowed for the payment of money before it is due.

2. The *Present Worth* of a debt, due at some future time without interest, is such a sum of money as would produce the debt in the given time at the given rate.

3. True Discount is the difference between the debt and its present worth.

EXERCISES.

250. *1.* A man sold a horse for $222 and gained 11% of the cost. What did the horse cost him?

2. A real estate dealer sold a building lot for $654, gaining 9% of the cost. What was the cost?

3. What sum of money put at interest at 6% for 1 yr. will amount to $106?

4. How many dollars must I lend at 6% so as to get $112 at the end of 2 yrs.?

5. What principal will amount to $208 in 1 yr. at 4%?

6. What principal put at interest at 7% for 1 yr. will amount to $321?

7. What is a note for $218, due 1 yr. and 6 mo. hence, worth to-day, if interest is at 6%?

8. What is the present worth of $960, due 3 yrs. 6 mo. hence, at 8%?

EXERCISES.

251. *1.* A sold B a piano for $460, taking B's note for it, due in 16 mo. without interest. C offered to buy the note if A would make an equitable discount, money being worth 6%. A proposed a discount of $30, C refused it. Why should C refuse to buy the note at a discount of $30? How much discount should A justly make?

EXPLANATION: The note will be worth $460, 16 mo. after date. The question is, How much was the note worth the day it was made? Or, What principal would amount to $460, in 16 mo. at 6%? The interest on any principal for 16 mo. at 6% is .08 of the principal. The amount, or $460, is the principal + .08 of itself, or 1.08 of the present worth of the note. Hence, the note was worth $\frac{108}{108}$ of $460, or $425.92, the day it was made. A should make a discount equal to the difference between $460 and $425.92, which is $34.08.

PROOF: Find the amount of $425.92 for 16 mo., at 6%.

2. In the problem explained above, how much would A have received for the note if he had sold it 4 mo. after date? (The note would then have been discounted for 12 mo. instead of 16 mo. Why?)

3. What is the present worth and the true discount of $672 due in 1 yr. 6 mo., discounted at 6%?

4. How much should be discounted for the present payment of a note for $386, without interest, due in 1 yr. 7 mo. 12 days, money being worth 6%?

5. Find the present worth and discount of $720 payable in 10 mo. 18 days, at 7½%; $290 payable in 1 yr. 2 mo., at 6%; $1,356.50 payable in 4 mo. 21 days, at 4½%.

6. A owes $326.80, due in 2 yrs. 9 mo., which he wishes to pay at once. How much should he pay if money is worth 8 %? How much should he pay when money is worth 5 %?

7. Mr. Moore owes me $647.30, due in 21 mo. 27 days. If he pays me now, how much ought I to receive, if money is worth 7 %? How much if money is worth 5½ %?

8. What is the present worth of a debt bearing interest, if the rate of discount is the same as the rate of interest?

9. A note for $900, bearing interest at 5½ %, dated Aug. 15, 1890, and due April 10, 1892, was discounted Jan. 23, 1891, at 6 %. What was it worth?

(The amount due at the maturity of the note is the debt whose present worth is to be found.)

BANK DISCOUNT.

252. *1. Bank Discount* is the simple interest on the sum due at maturity, computed from the day it is discounted. In most States the time of maturity is counted three days beyond the time specified in the note. These three days are called "days of grace."

This method of discounting is practiced at banks, and in business transactions generally, where paper is discounted for a *short time*. Bank discount is greater than true discount, since it is computed on a larger sum. On account of the "days of grace," the real time is three days more than the time specified, except in California, Connecticut, District of Columbia, Idaho, Illinois, Maryland, Montana, New Jersey, New York, North Dakota, Ohio, Oregon, Pennsylvania, Utah, Vermont, and Wisconsin.

2. The sum left after the discount is deducted is called the *Proceeds* or *Avails* of the note.

EXERCISES.

253. *1.* What are the bank discount and the proceeds on a note for $540, for 60 days, discounted at 6%? (Take 63 days for the time.)

EXPLANATION: The interest on $540 for 63 days, at 6%, is $5.67. Hence, the bank discount is $5.67, and the proceeds $540−$5.67=$534.33.

Find the bank discount and the proceeds on the following:

2. $782 for 90 days, at 6%.

3. $900 for 60 days, at 7%.

4. $375.60 for 90 days, at 5%.

5. $1,200 for 45 days, at 6½%.

6. $496 for 60 days, at 4½%.

7. $94.80 for 4 mo., at 8%.

8. $2,650 for 30 days, at 5%.

9. $876 for 40 days, at 5½%.

NOTE: Compute the above without the three days of grace.

254. To Find the Face of a Note.

EXERCISES.

1. If I go to a bank to borrow $380 for 90 days, for how much must I make a note when the current rate of interest is 6%?

EXPLANATION: Since $380 is the proceeds of the note, I must make a note for more than $380.

The interest for 93 days at 6%=.0155 of the face of the note. Since .0155 of the face of the note is discount, the remainder,

which is .9845 of the face of the note, must be the proceeds, or $380. Hence, the note must be given for $\frac{10000}{9845}$ of $380, or $385.98.

PROOF: Find the proceeds of $385.98 discounted for 93 days, at 6%.

For how much must a note be drawn to yield:

2. $460 for 60 days, at 6%?

3. $900 for 30 days, at 7%?

4. $94 for 4 mo., at 8%?

5. $546 for 90 days, at 7½%?

6. $1,478 for 45 days, at 5½%?

7. $850 for 50 days, at 5%?

8. $756.68 for 80 days, at 6%?

9. $2,850 for 30 days, at 4½%?

10. $638 for 90 days, at 9%?

NOTE: Compute the above without the three days of grace.

EXCHANGE.

255. 1. *Exchange* is a method of making payments at distant places by transmitting drafts or bills of exchange instead of money.

2. A *Draft* is a written order made by one party upon another, directing him to pay a specified sum of money to the party named in the draft. A draft made by a party on a personal creditor is an *order;* made by a depositor on his bank, it is a *check.*

(*a*) The party who issues the draft is the *Maker.* The party upon whom the order is drawn is the *Drawee.* The party to whom the money is to be paid is the *Payee.*

(*b*) When a draft can be bought for its face, it is said to be *at par*. When the cost is less than the face, it is said to be *at a discount*. When the cost is more than the face, it is *at a premium*.

3. A draft made payable when presented to the drawee is called a *Sight Draft*. A draft made payable at a specified time after it is presented to the drawee is called a *Time Draft*.

EXERCISES.

256. *1.* Find the cost of a draft on New York for $1,500, at $\frac{1}{4}\%$ premium.

EXPLANATION: $\frac{1}{4}\%=\frac{1}{400}=\frac{1}{4}$ of .01. $\frac{1}{4}$ of .01 of $1,500=$3.75, the premium. $1,500+$3.75=$1,503.75, the cost of the draft.

Find the cost of a draft for:

2. $1,200 on New York, at $\frac{3}{8}\%$ premium.

3. $690 on Chicago, at $\frac{1}{10}\%$ premium.

4. $2,400 on Boston, at $\frac{1}{8}\%$ premium.

5. $1,000 on New Orleans, at $\frac{1}{4}\%$ discount.

6. $1,600 on San Francisco, at $\frac{1}{8}\%$ premium.

7. $950 on St. Louis, at $\frac{1}{10}\%$ discount.

8. $1,895 on Cincinnati, at $\frac{1}{8}\%$ discount.

9. $12,698 on Philadelphia, at $\frac{3}{10}\%$ premium.

10. If exchange is $\frac{1}{8}\%$ premium, how large a draft will $1,201.50 buy?

EXPLANATION: $\frac{1}{8}\%=\frac{1}{800}$. Then $1,201.50 is $\frac{801}{800}$ of the face of the draft. The face of the draft must be $\frac{800}{801}$ of $1,201.50, or $1,200.

11. How large a draft will $2,997 buy if exchange is $\frac{1}{10}\%$ discount?

12. How large a draft will $6,850 buy if exchange is $\frac{3}{8}\%$ premium? $\frac{3}{8}\%$ discount? $\frac{7}{8}\%$ premium? $\frac{7}{8}\%$ discount?

13. Find the cost of a draft for $500, payable 45 days after sight, when exchange is $\frac{1}{8}\%$ premium, and interest 6%.

EXPLANATION: The interest on $500 for 48 days is $4. If exchange were at par, the draft would cost $500−$4, or $496. But the exchange is $\frac{1}{8}\%$ premium, or $1. Hence, the cost of the draft is $496+$1=$497.

ANOTHER EXPLANATION: The interest not yet earned is .008 of $500; the premium is .002 of $500. Hence, the draft should cost .006 of $500, or $3 less than the face.

14. Find the cost of a draft for $1,450, payable 90 days after sight, when exchange is $\frac{1}{4}\%$ premium, and interest 6%.

15. Find the cost of a 60-day draft for $2,600, exchange $\frac{1}{10}\%$ discount, and the rate of interest 7%.

16. Find the value of a 75-day draft for $1,756, exchange $\frac{1}{4}\%$ premium, and the rate of interest 5%.

17. Find the cost of the following draft, interest 6%, exchange $\frac{1}{8}\%$ discount:

$768. KANSAS CITY, MO., Oct. 26, 1896.

Fifteen days after sight, pay to the order of Baker & Williams seven hundred sixty-eight dollars, value received, and charge to the account of ALONZO KING.

To GEORGE BENNETT, Chicago.

LONGITUDE AND TIME.

257. *1.* Define circle. Circumference. Degree.

2. Turn, in your geography, to the map of the hemispheres. Find the equator. What is it? How many degrees in its circumference?

3. In the Western and Eastern Hemispheres, what are the lines running up and down the page and crossing the equator called? What is a meridian? What is the meridian from which the numbering begins called? What meridian is used as prime meridian? What number is given to it? In which direction from the prime meridian are the others numbered? Why are they numbered in both directions? How far are they numbered in each direction? Why not farther? What is the distance east of the prime meridian called? West of the prime meridian? How many degrees of east longitude? Of west longitude?

4. What is the longitude of Boston, Mass.? Of Denver, Colo.? What is their difference in longitude? How do you find it? What is the difference in longitude between New York City and Omaha, Neb.? Between Washington, D. C., and Helena, Mont.? Between Charleston, S. C., and Salem, Ore.? Between Salt Lake City and Sitka, Alaska? Between City of Mexico and Sandwich Islands? Between Dublin, Ireland, and Rio de Janeiro, Brazil?

5. What is the longitude of Athens, Greece? Of Detroit, Mich.? What is their difference? How do you find it? What is the difference in longitude between Mobile, Ala., and Constantinople, Turkey?

EXERCISES.

258. *1.* In what direction does the earth rotate? When is it noon at any place? (When the meridian of that place is directly under the sun's rays.) How long does noon last? How many degrees of longitude pass under the sun's rays in 1 day, or 24 hrs.? In 1 hr.? In 2 hrs.? In 9 hrs.? From midnight to noon? From 3 o'clock A. M. to 11 o'clock A. M.? From 10 o'clock A. M. to 5 o'clock P. M.?

2. If 15° of longitude pass under the sun's rays in 1 hr., how long will it take 45° to pass under? 75°? 120°? 1 sextant? 1 quadrant? A semi-circumference?

3. If 15° of longitude pass under the sun's rays in 1 hr., or 60 min., how many minutes will it take 1° to pass under? 2°? 9°? 14°?

4. How many degrees of longitude will pass under the sun's rays in 8 min.? In 36 min.? In 52 min.?

5. If 15′ of longitude pass under the sun's rays in 1 min. or 60 sec. of time, how many seconds will it take 1′ to pass under? 3′? 7′? 11′? 14′?

6. How many minutes of longitude will pass under the sun's rays in 8 sec.? In 20 sec.? In 44 sec.? In 56 sec.?

7. What places have noon at the same time? Midnight? 9 o'clock A. M.? 5 o'clock P. M.?

TABLE OF RELATIONS BETWEEN LONGITUDE AND TIME.

259. *8.* A difference between two places of—

15° in longitude makes a difference of 1 hr. in time.
15′ in longitude makes a difference of 1 min. in time.
15″ in longitude makes a difference of 1 sec. in time.
1° in longitude makes a difference of 4 min. in time.
1′ in longitude makes a difference of 4 sec. in time.
1″ in longitude makes a difference of $\frac{1}{15}$ sec. in time.
(Can you show that this table is correct?)

9. When it is noon at a place, is the time earlier or later east of that place? Why? West of that place? Why?

10. When it is noon at Chicago, what is the time 15° east of Chicago? 30° east? 120° east? 1° east? 8° east? 13° east? 15′ east? 45′ east? 30′ east? 1′ east? 7′ east? 12′ east?

11. When it is noon at Washington, D. C., what is the time 15° west of Washington? 30° west? 135° west? 1° west? 2° west? 11° west? 15′ west? 45′ west?

12. When the days and nights are of equal length, and it is noon at the first meridian, on what meridian is it sunrise? Sunset? Midnight?

13. When it is 10.00 A. M. at Philadelphia, longitude 75° west, it is 6.00 A. M. at Sitka. What is the longitude of Sitka?

14. When it is 10.00 P. M. at Austin, Tex., longitude $97\frac{3}{4}$° west, it is 1.00 A. M. next day at St. Johns, N. F. What is the longitude of St. Johns?

15. When the time at Richmond, longitude $77\frac{1}{2}$° west, is 8.00 A. M., what is the time at Rome, Italy, longitude $12\frac{1}{2}$° east?

16. When the time at Rome is 8.00 P. M., what is the time at Richmond?

17. When the time at Quebec, longitude 71½° west, is 6.00 A. M., the time at the Cape of Good Hope is 12.00 M. What is the longitude of the Cape of Good Hope?

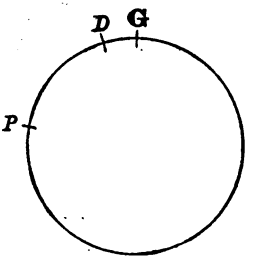
TABLE OF LONGITUDES.

260. These longitudes are given in the records of the U. S. Coast Survey. They are to be used for reference in the solution of the problems which follow them. They may be used by the teacher for the making of additional problems, if needed. They need not be memorized:

Boston, Mass.,	71°	3′	50″	W.
Charleston, S. C.,	79°	55′	49″	W.
Chicago, Ill.,	87°	35′	0″	W.
Cincinnati, O.,	84°	29′	45″	W.
Denver, Colo.,	104°	59′	33″	W.
Detroit, Mich.,	83°	3′	0″	W.
Honolulu, Sandwich Islands,	157°	52′	0″	W.
New York City,	74°	0′	24″	W.
New Orleans, La.,	90°	3′	28″	W.
Rio Janeiro, Brazil,	43°	20′	0″	W.
San Francisco, Cal.,	122°	27′	49″	W.
Washington, D. C.,	77°	0′	36″	W.
Greenwich, England,	0°	0′	0″	
Berlin, Germany,	13°	23′	0″	E.
Bombay, India,	72°	48′	0″	E.
Constantinople, Turkey,	28°	59′	0″	E.
Pekin, China,	116°	26′	0″	E.
Sydney, Australia,	151°	11′	0″	E.

WRITTEN EXERCISES.

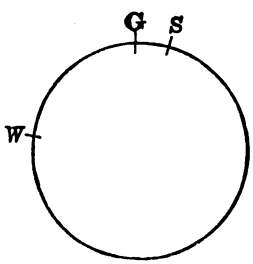
261. *1.* What is the time at Pittsburg, longitude 79° 58′ W., when it is 3.00 A. M. at Dublin, longitude 6° 20′ 30″ W.?



EXPLANATION: Draw a diagram (see margin) to represent a section of the earth made by the plane of the equator, and viewed from the south side so that east shall be toward the right hand and west toward the left hand. Mark on it the locations of the meridians of Greenwich, Dublin, and Pittsburg. The arc from Greenwich to Dublin is 6° 20′ 30″. The arc from G. to P. is 79° 58′. The arc from D. to P. is the difference in longitude, which is found by subtraction. Its length is 73° 37′ 30″. The difference in time is $\frac{1}{15}$ as much as the difference in longitude, or $\frac{1}{15}$ of 73 hrs. 37 min. 30 sec. As Pittsburg is west of Dublin, the time is 4 hrs. 54 min. 30 sec. earlier there, or 1 hr. 54 min. 30 sec. before midnight—that is, 10 hrs. 5 min. 30 sec. P. M. of the previous day. (For every problem in longitude and time, make a diagram similar to this one.)

	hrs.	min.	sec.
15)	73	37	30
	4	54	30

2. When it is 10.00 A. M. at Stockholm, it is 3 hrs. 24 min. 58 sec. A. M. at Wheeling (longitude 80° 42′ W.) What is the longitude of Stockholm?



EXPLANATION: The time at Stockholm is 6 hrs. 35 min. 2 sec. later than at Wheeling. We see at once that Stockholm must be more than 90° east of Wheeling; hence, it must be in east longitude, as the diagram shows. As the difference of longitude is 15 times as much as the difference of time, the arc W. S. must be 15 times 6° 35′ 2″, or 98° 45′ 30″, as found in the margin. To find the length of the arc G. S., or the longitude of Stockholm, subtract the length of the arc G. W., the longitude of Wheeling, from 98° 45′ 20″. The result is 18° 3′ 30″ E.

6°	35′	2″
		15
98°	45′	30″

3. When it is 12 o'clock M. at Rochester, N. Y., it is 9 hrs. 1 min. 37 sec. A. M. at San Francisco. What is the longitude of Rochester? (See table.)

4. When it is half-past 3 o'clock P. M. at Boston, what is the time at Bombay?

5. When it is 9 hrs. 27 min. 36 sec. P. M. at Chicago, it is 10 hrs. 36 min. 55 sec. P. M. at Portland, Me. What is the longitude of Portland?

6. A captain at sea finds by his chronometer that it is 2 hrs. 15 min. 30 sec. P. M. at Greenwich when it is 12.00 M. on board his vessel. In what longitude is his vessel?

7. The time at Rio Janeiro is 3 hrs. 7 min. 29 sec. earlier than at St. Louis. What is the longitude of St. Louis?

8. If a message is sent by telegraph, without any loss of time, at 12.00 M., from Berlin to Cincinnati, what is the time of its receipt at Cincinnati?

9. A gentleman left Detroit, Mich., and traveled till his watch was 11 hrs. 13 min. 56 sec. too slow. In what direction did he travel? In what longitude was he?

10. When it is 7 hrs. 45 min. P. M. at Constantinople, what is the time at Washington?

11. Time is 7 hrs. 55 min. 26 sec. later at St. Petersburg (longitude 30° 19′ 46″ E.) than at Madison, Wis. What is the longitude of Madison?

12. When it is 1.00 P. M. at Ithaca (longitude 75° 13′ W.) it is 11 hrs. 52 min. 4 sec. A. M. at Little Rock. What is the longitude of Little Rock?

RATIO AND PROPORTION.

262. *1.* What is ratio? What is the ratio of 3 to 4? Of 5 to 8? Of 7 to 6?

2. What is the ratio of 8 men to 12 men? Of 6 cts. to 8 cts.? Of 15 horses to 10 horses?

3. What is the ratio of 5 yds. to 4 boys?

The last question is evidently absurd. A ratio can exist between like quantities only, because any quantity can be a part of another quantity only when it is like it. Nor can the ratio between two like quantities be found until they are measured by the same kind of unit. Hence, one can not at once tell the ratio of $4\frac{1}{2}$ to $3\frac{1}{6}$. (Review problems 6–12, p. 228.)

Two or more equal ratios make a proportion; for proportion is an *equality of ratios.* A proportion may be written in three ways, as in the margin. The first may be read, "3 is to 5 as 6 is to 10"; the second may be read, "3 to 5 equals 6 to 10"; the third, "3 fifths equals 6 tenths." All these expressions mean the same thing. The first is the common way of writing a proportion; the colon, or two dots, signifies a ratio; the four dots mean the same as the sign =. But every proportion can be thrown into the third form—that of equal fractions.

(1) $3 : 5 :: 6 : 10$

(2) $3 : 5 = 6 : 10$

(3) $\frac{3}{5} = \frac{6}{10}$

If we multiply two equal fractions by their denominators, the products will be equal; thus, taking the preceding equal fractions, $3\times10=6\times5$. Now, if we compare these products with the first form of the proportion, we shall find that one pair of factors constitutes the extreme terms, and the other pair the mean terms, of that proportion. Hence, we see the grand principle that:

In any proportion the product of the means equals the product of the extremes.

On this principle we can always find any term of a proportion when the other three terms are given.

4. Find the fourth term in these proportions:

(1) $2:7::8:?$

(2) $4:3\frac{1}{2}::12:?$

(3) $5:9::2\frac{1}{2}:?$

How is the fourth term found in every case?

263. Solving Problems in Proportion.

Very many problems may be solved by proportion, as, for instance:

1. If a man earn $6 in 3 days, how many dollars will he earn in 7 days?

The first work is to put the question in the form of a proportion; this is called "stating the question." In order to do this, (1) write the form of a proportion; (2) mark the place of the lacking term; (3) put the number which is of the same kind as the lacking term into the same ratio; (4) determine by the conditions of the problem which of these two terms is the greater;

(5) put the two terms of the other ratio in such order that the ratios shall be equal.

(1) : : :

(2) dol. dol.
: : 6 : ?

(3) days. days. dol. dol.
3 : 7 : : 6 : ?

EXPLANATION: In the margin we have written, first, the framework of the proportion; next we have indicated the fourth term as the lacking term. This is dollars; hence, we write $6 as the other term of the same ratio. The lacking term is the number of dollars earned in 7 days, while the third term is the number of dollars earned in 3 days. Hence, the fourth term must be greater than the third. Therefore, in the other equal ratio, the second term must be greater than the first, so we write "3 days : 7 days," for the first ratio.

2. If 5 men do a piece of work in 8 days, in how many days will 4 men do it? (This problem is stated as in the margin.)

men. men. days. days.
4 : 5 : : 8 : ?

EXPLANATION: The unknown number of days is the time in which 4 men do the work, which must be more than 8 days, the number of days in which 5 men will do it. Hence, the second term must be greater than the first.

Any other term might be made the lacking term, just as well as the fourth; and, reasoning in exactly the same way, the statement can be completed.

When the statement is made, the method of solution is very evident from the principle that the product of the extremes is equal to the product of the means.

EXERCISES.

264. Solve the following problems by proportion:

1. If 9 men build 20 ft. of wall in a certain time, how many men can build 100 ft. of the same wall in the same time?

2. If 75 A. of land produce 5,250 bu. of corn, how many bushels will 56 A. produce at the same rate?

3. If a staff 5 ft. long casts a shadow 4 ft. long, what is the height of a pole casting a shadow 72 ft. long, at the same time of the day?

4. If the interest on a note for 28 mo. is $112, what is the interest on the same note, at the same rate, for 17 mo.?

5. A man sold $\frac{3}{8}$ of his farm for $3,900. What was $\frac{1}{8}$ of the farm worth at the same rate?

265. Compound Proportion.

In simple proportion the value of the unknown quantity depends on one condition only, and both ratios are simple. When the value of the unknown term depends on two or more conditions, one of the ratios must be compound. Such a proportion is called a *compound proportion.* There are more steps in a compound proportion, but the work is no more difficult than in simple proportion. Each step must be taken without regard to the others.

1. If 6 men, in 10 days of 9 hrs. each, build 25 rds. of fence, how many hours a day must 8 men work to build 48 rds. in 12 days?

men.	men.				
8	: 6				
days.	days.	hrs. a day.	hrs. a day.		
12	: 10	:: 9	: ?		
rds.	rds.				
25	: 48				

EXPLANATION: The value of the unknown quantity depends on three conditions; hence, the compound ratio must consist of three simple ratios, as in the margin. The unknown term is hours a day, so we put it in the same ratio with 9, the given number of hours a day. On condition of the number

of men who work, the fourth term is smaller than the third; hence, in the ratio expressing that condition, the second term must be less than the first, so we write "8 men : 6 men." On condition of the number of days in which they work, the fourth term is also smaller than the third, so we express the condition by writing, "12 days : 10 days." On condition of the amount of work to be done, the fourth term must be greater than the third; hence, we express this condition by writing, "25 rds. : 48 rds." Now that the statement is made, the fourth term is found, as in simple proportion, by dividing the product of all the means by the product of the given extremes.

EXERCISES.

266. Solve these problems by compound proportion:

1. If 9 men, in 4 days of 8 hrs. each, build a wall 20 ft. long, 8 ft. high, 2 ft. thick, how many feet of wall 10 ft. high, 3 ft. thick, could 16 men build in 12 days of 9 hrs. each?

2. If 810 bricks, 8 in. long and 4 in. wide, are required for a walk 36 ft. long and 5 ft. wide, how many bricks will be required for a walk 66 ft. long and 4 ft. wide?

3. If a ship's crew of 300 men have provisions to serve for 45 days, at the rate of 28 oz. a day for each man, how many men will the same provisions serve for 60 days, allowing each man 25 oz. a day?

4. If the interest on $640 for 4 yrs. 6 mo. is $172.8, what is the interest on $820 for 2 yrs. 8 mo., at the same rate?

5. If 7 men, working 12 days of 8 hrs. each, build 70 ft. of wall 10 ft. high, 2 ft. thick, how many men, working 8 days, 9 hrs. a day, would build 50 ft. of wall 1 ft. thick and 9 ft. high?

6. If 500 copies of a book containing 210 pages require 12 rm. of paper, how much paper will be required to print 1,200 copies of a book of 280 pages of the same size?

7. If 12 men can put $\frac{2}{5}$ of a cargo on board a ship in $\frac{3}{4}$ of a day, how many days will it take 5 men to load 3 such ships?

8. If A can do $\frac{2}{3}$ of a piece of work in 5 days, working 8 hrs. a day, how long will it take him to do the whole piece, working 10 hrs. a day?

9. If a workman earns $40 in 1 mo., working 8 hrs. a day, what ought he to earn in 6 mo., working 12 hrs. a day?

10. If 60 A. of land are bought for $3,000 and sold for $3,600, what is gained on 160 A. if bought and sold at the same rate?

267. General Analysis.

The method of solving problems by proportion is mechanical and cumbersome. It was much used formerly, and the process was called the "Rule of Three." All such problems can be better solved by *analysis;* and the analysis may always be put into such a form that the answer will appear as a simple fraction with one or both of its terms partly factored. In reducing this fraction to its simplest form, all common factors in numerator and denominator can be canceled.

The following solutions of problems already given in proportion will illustrate this method of analysis.

Any problem whose solution requires only the processes of multiplication and division may be solved in a similar manner.

Problem 5 in simple proportion, page 292:

SOLUTION:

(1) $3900

(2) $\frac{\$3900}{3}$

(3) $\frac{\$3900 \times 8}{3}$

(4) $\frac{\$3900 \times 8}{3 \times 5}$

EXPLANATION: In the margin, the first statement shows the worth of $\frac{3}{5}$ of the farm; the second shows the worth of $\frac{1}{5}$ of it; the third shows the worth of the whole farm; the fourth shows the worth of $\frac{1}{5}$ of it; the fifth shows the worth of $\frac{4}{5}$ of it. This is what was required. The fraction put in the simplest form equals $8,320, the answer.

(5) $\frac{\$3900 \times 8 \times 4}{3 \times 5} = \$8320$

Problem 4 in compound proportion, page 293:

SOLUTION:

(1) 17280 cts.

(2) $\frac{17280 \text{ cts.}}{640}$

(3) $\frac{17280 \text{ cts.} \times 820}{640}$

(4) $\frac{17280 \text{ cts.} \times 820}{640 \times 54}$

EXPLANATION: The second statement in the margin is the number of cents which is the interest on $1 for the given time, 54 months; the third is the interest on $820 for that time; the fourth is the interest on $820 for 1 month; the fifth is the interest on $820 for the required time, or 32 months. The value of this fraction is the answer.

(5) $\frac{17280 \text{ cts.} \times 820 \times 32}{640 \times 54} = 13120 \text{ cts.} = \$131.20$

Problem 7 in compound proportion, page 294:

SOLUTION:

(1) $\frac{3}{4}$ day.

(2) $\frac{3 \text{ days} \times 12}{4}$

(3) $\frac{3 \text{ days} \times 12}{4 \times 2}$

(4) $\frac{3 \text{ days} \times 12 \times 5}{4 \times 2}$

(5) $\frac{3 \text{ days} \times 12 \times 5 \times 3}{4 \times 2}$

(6) $\frac{3 \text{ days} \times 12 \times 5 \times 3}{4 \times 2 \times 5} = 13\frac{1}{2}$ days.

EXPLANATION: The first statement shows the number of days it takes 12 men to do a piece of work; the second shows the number of days it takes 1 man to do the same work — that is, to load $\frac{3}{4}$ of the cargo; the third shows the time it would take 1 man to load $\frac{1}{4}$ of the cargo; the fourth shows the number of days it would take him to load the whole cargo, or 1 ship; the fifth shows the number of days it would take him to load 3 ships; and the sixth shows the number of days it would take 5 men to load 3 ships. This is what was required, and the value of the fraction is $13\frac{1}{2}$ days.

268. Rules for Analysis.

In order to make the analysis perfect and the writing simple, observe the following:

1. Always write, first, *the number of the same kind with the answer sought.* All the other numbers enter as multipliers or divisors.

2. Take but *one* step at a time.

3. Enter no *mixed numbers* or *decimal fractions.*

4. As soon as the statement is complete, reduce the fraction to its *simplest form.*

The observation of these rules is illustrated in the solutions given.

It would be well for the student to solve the other problems in proportion by this method of analysis.

INVOLUTION.

269. Any number multiplied by itself is said to be *squared,* or raised to the *second power.*

What is the second power of 5? Of 9? Of 11? When a number is used as a factor three times, the product is the *third power* of that number, or its *cube.*

What is the cube of 3? Of 5? Of 7? Of 10?

So when a number is used as a factor four times, the product is the *fourth power* of that number, and so on.

How many times must a number be used as a factor to obtain its fifth power? Its eighth power? Its tenth power?

Find the third power of 5. Of 7. Find the sixth power of 2. Of 5. Of 6.

Raising numbers to their powers is called *Involution.* A little figure showing how many times the number is to be used as a factor is called an *Exponent.* It is written at the right of, and a little above, the figure it affects.

$3^4=81$. In this example, what is the exponent? How many times must 3 be used as a factor? How many multiplications are necessary to find the fourth power of a number? The fifth power? The eighth power?

Find the powers of the numbers indicated below:

$7^3=?$ $2^{10}=?$ $3^5=?$ $5^6=?$ $4^4=?$ $9^3=?$ $10^6=?$

EVOLUTION.

270. When the power is given and we are asked to find one of its equal factors, the process is *Evolution,* and the number which is found is a *Root.*

One of the two equal factors that make a number is called the *square root* of that number. One of its three equal factors is called its *cube root;* one of four is its *fourth root,* and so on.

The *radical sign,* made thus, $\surd$, before a number, signifies that a root of that number is to be taken. A little figure written within the sign, called an *index,* shows what root is to be taken; thus, $\sqrt[3]{}$ signifies the cube root. Usually the index for the square root is not written, but is understood; thus, $\sqrt{25}$ signifies the square root of 25, or 5. $\sqrt[4]{81}=?$ $\sqrt[5]{32}=?$ $\sqrt[4]{256}=?$ $\sqrt[3]{125}=?$

As we find the areas of squares by multiplying two equal factors, and the contents of cubes by multiplying three equal factors, text-books on arithmetic properly give rules for extracting square roots and cube roots. Higher roots are extracted by using what are called *logarithms;* such a process belongs to higher mathematics. One who had much of such work to do would also use logarithms in extracting square and cube roots. We attempt no explanation of the rules for extracting square and cube roots; for such an explanation, we must resort to algebra. Explanations by diagrams and blocks explain only *applications* of the process, not the *general principles.*

SQUARE ROOT.

271. Rule for Square Root.

1. Begin at the decimal point and strike off two figures each time until you can recognize the greatest square in what remains.

2. Subtract that square from what was not pointed off, and write its root like a quotient in long division.

3. Bring down the next period to the remainder, for a new dividend.

4. Double the root already found and use it as a trial divisor, omitting the right-hand figure of the new dividend.

5. Write the quotient in the root, and write the same figure at the right-hand of the divisor.

6. Multiply the complete divisor by the number just expressed in the root, and subtract the product thus obtained from the dividend.

7. To the remainder bring down the next period, and proceed as before.

It must be noticed that the squares of numbers as far as 9 must be thoroughly committed to memory, or we shall not be able to recognize the greatest square in the left-hand period. If the squares are known beyond the square of 9, we may be able to stop pointing off at an earlier stage, as we do in Example 2, next page.

Decimals must be pointed off toward the right, beginning at the decimal point; and all the periods *must be full;* if necessary, annex a zero to fill the last period. The number of decimal places in the root will equal the number of periods. Mixed decimals must be pointed both ways from the decimal point.

The square root of common fractions will be found by extracting the square root of numerator and of denominator.

The use of the trial divisor will sometimes give too large a figure for the root. When this is found to be the case, a smaller figure must be taken.

If the root can not be exactly found, decimal periods of zeros may be added to any desired extent, and so an approximate root may be found.

EXAMPLE 1.

```
51'40'89 (717
  49
 14 | 240
  1 |
141 | 141
142 | 9989
  7 |
1427 | 9989
     0000
```

ILLUSTRATION: We recognize 49 as the greatest square in 51. Our new dividend is 240, and our trial divisor is 14. The quotient of 24 divided by 14 is 1. The complete divisor is 141. The new dividend is 9,989, and the new trial divisor is 142. The quotient of 998 divided by 142 is 7, etc. The square root is found to be 717.

EXAMPLE 2.

```
156'25 (125
  144
24 | 1225
 5 |
245 | 1225
    0000
```

In Example 2 we recognize 144 as the greatest square in 156; hence, it is not necessary to point off any farther.

272. Extracting Square Root by Prime Factors.

If a number be resolved into its prime factors, the product of *half of the factors* will give the *square root of the number.* If the exponents of all of the prime factors are not even numbers, then the number is not a perfect square.

EXAMPLE 1.

$390625 = 5^2 \times 5^2 \times 5^4$
15625
625

ILLUSTRATION: Here we see that 390,625, is made up of eight factors 5; its square root will be four of these factors, or 625.

EXAMPLE 2.

$512 = 2^3 \times 2^6$
64

In Example 2 we find that 512 is made up of nine factors 2. Hence, we know 512 can not be a perfect square.

EXERCISES.

273. Find the square root of:

1.	4096	*5.*	.702244	*9.*	68425
2.	21025	*6.*	.001225	*10.*	$\frac{289}{324}$
3.	173056	*7.*	82.4464	*11.*	$\frac{576}{1849}$
4.	96100	*8.*	75.364	*12.*	$1040\frac{1}{16}$

(In Problem 12 change to an improper fraction.)

13. A square field contains 27,225 sq. rds. What is the length of each side?

14. A general has an army of 994,009 men. How many must he place rank and file to form them into a square?

15. A rectangular field is 624 rds. long and 156 rds. wide. How many more rods of fence will be required to inclose it, than to inclose a square field containing the same area?

CUBE ROOT.

274. In cube root, the pointing off is just like that in square root, only the periods must contain *three figures* instead of two. Cube root can be found by factoring; but in that case we take *one third* of the factors, instead of *one half* as in square root. If a number is a perfect cube, the exponent of each of its prime factors must be a multiple of 3.

To be able to use the rule for the extraction of cube root, the pupil must know the cubes of all numbers as far as the cube of 9.

275. Rule for Cube Root.

1. Point off as in square root, but into periods of three figures each.

2. Subtract the greatest cube in the left-hand period from that period, and write its root as a quotient.

3. To the remainder, bring down the next period to make a new dividend.

4. Take three times the square of the root already found as a trial divisor, and with it divide the hundreds of the dividend, writing the quotient in the root.

5. Complete the divisor by adding to it three times the product of the root already found multiplied by the last quotient, and also the square of the last quotient.

6. Multiply the completed divisor by the last quotient, and subtract the product from the dividend.

7. To the remainder, bring down the next period and proceed as before.

```
      13'312'053 (237
          8
     ------------
  12 | 5312
  18 |
   9 |
     ------------
1389 | 4167
     ------------
1587 | 1145053
 483 |
  49 |
     ------------
163579 | 1145053
     ------------
       0000000
```

ILLUSTRATION: Three times the square of 2 is 12, and 53 will contain 12 four times, but on completing the divisor and multiplying, it is found that 4 is too large a figure. The second trial divisor is 1,587, which is three times 529, the square of 23. This trial divisor is found 7 times in 11,450. The product of 3 times 7 times 23 is 483, etc.

EXERCISES.

276. What is the cube root of:

1.	54872?	*7.*	.000529475129?
2.	912673?	*8.*	25?
3.	12812904?	*9.*	$\frac{2744}{6859}$?
4.	8365.427?	*10.*	$\frac{68921}{59319}$?
5.	24.137569?	*11.*	$49\frac{8}{27}$?
6.	.091125?	*12.*	$31\frac{15}{343}$?

13. A block of granite in the form of a cube contains 46,656 cu. in. What is the length of its edge?

14. What is the length of one side of a cubical cistern that will hold 100 bbls. of water?

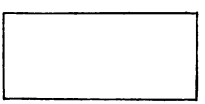
MENSURATION.

277. *1.* Define surface. Angle. Right angle. Acute angle. Obtuse angle.

2. Define rectangle. Square. Area. Unit of surface. Measure of area.

3. Review Section 141, p. 134.

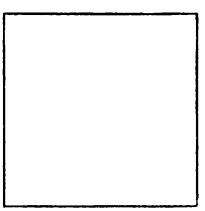
4. A polygon is any *plane surface having straight sides.*



RECTANGLE.

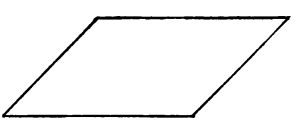
5. A quadrilateral is a *polygon having four sides.*

6. The sum of the lines which bound a polygon is called its *perimeter.* Thus, the perimeter of a quadrilateral is the sum of its four sides.



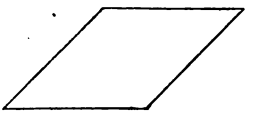
SQUARE.

7. A parallelogram is a *quadrilateral whose opposite sides are parallel.* The parallelograms are the rectangle, including the square, the rhomboid, and the rhombus.



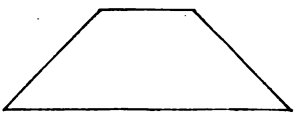
RHOMBOID.

8. A rhomboid is a *parallelogram having no right angles.*



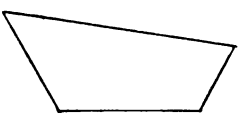
RHOMBUS.

9. A rhombus is a *rhomboid whose four sides are equal,* but whose angles are not right angles.



TRAPEZOID.

10. A trapezoid is a *quadrilateral having only two sides parallel.*

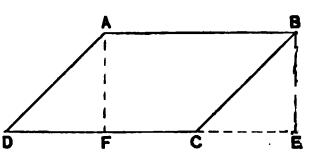


TRAPEZIUM.

11. A trapezium is a *quadrilateral having no parallel sides.*

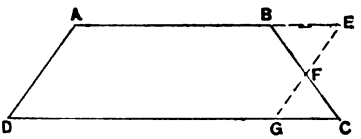
12. The area of any parallelogram is equal to the area of a rectangle having the same base and height.

This statement is illustrated by the drawing in the margin:



The lines A F and B E are drawn parallel to each other, and perpendicular to the lines A B and D C. The triangles A F D and B E C are equal. If the triangle A F D should be taken from the parallelogram A B C D, and the triangle B E C added to it, the resulting figure would be the rectangle A B E F, which is equivalent to A B C D.

13. The area of a trapezoid is equal to the area of a rectangle whose width is the same as the perpendicular distance between the parallel sides of the trapezoid, and whose length is half the sum of the parallel sides.



E G is drawn parallel to A D, making A E G D a parallelogram. The triangle B E F is equivalent to the triangle G F C. Hence, A B C D is equivalent to A E G D, and the length of G D is half the sum of the lengths of A B and D C.

EXERCISES.

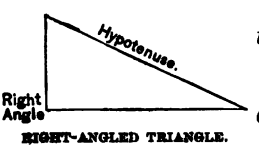
278. *1.* What is the area of a rhomboid whose base is 37 in. and whose height is 18 in.?

2. Find the area of a rhombus 46 in. on each side and 25 in. high. Compare it with a square 46 in. on each side.

3. Find the area of one side of a board in the form of a trapezoid whose parallel sides are 87 in. and 65 in., respectively, and whose width is 14 in.

TRIANGLES.

279. *1.* A triangle is a *polygon having three sides.*

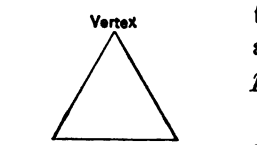


RIGHT-ANGLED TRIANGLE.

2. A right-angled triangle is a *triangle having a right angle.*

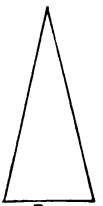
3. The hypotenuse is the *side opposite the right angle.*

4. The sides of a right-angled triangle adjacent to the right angle are called the *base* and the *perpendicular.*



EQUILATERAL TRIANGLE.

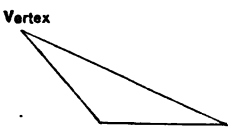
5. An equilateral triangle is a *triangle whose sides are all equal.*



ISOSCELES TRIANGLE.

6. An isosceles triangle is a *triangle two of whose sides are equal.*

7. A scalene triangle is a *triangle having no two sides equal.*



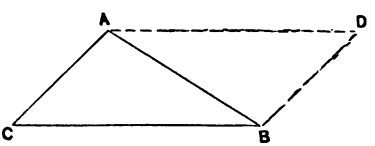
SCALENE TRIANGLE.

8. The base of a triangle is the side upon which the triangle is supposed to stand. Any side may be taken for the base.

9. The angle opposite the base of a triangle is the *vertex.*

10. The height or altitude of a triangle is the *length of the perpendicular line drawn from the vertex to the base.*

11. The area of a triangle is equal to one half the area of a parallelogram whose length equals the base of the triangle, and whose width equals the height of the triangle.

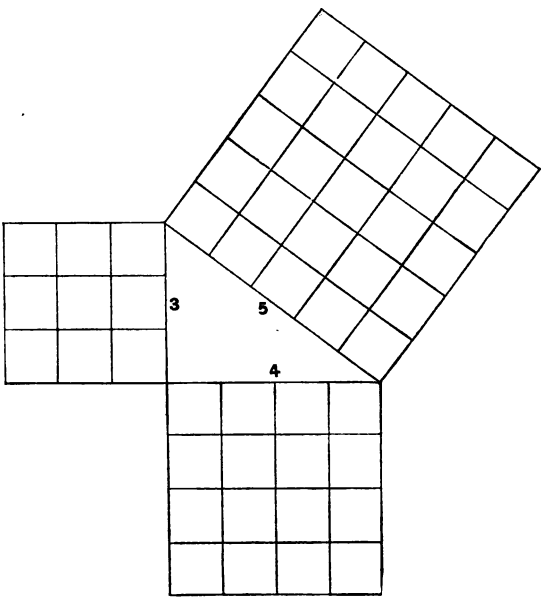


In the drawing in the margin, A D is drawn parallel to C B, and D B parallel to A C. A D B C is a parallelogram having the same base and height as the triangle. The triangle A B C is equal to the triangle A D B; hence, the triangle is one half of the parallelogram.

12. The altitude of an equilateral triangle bisects its base.

13. In every right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the base and perpendicular.

14. In every right-angled triangle the square of the base or the perpendicular is equal to the difference between the square of the hypotenuse and the square of the other side.



The last two statements, which are proved in geometry, are illustrated by a triangle whose sides are respectively 3, 4, and 5. The sum of the squares of the base and perpendicular is 25, which equals the square of the hypotenuse; hence, if two sides of a right-angled triangle are given, the third side can be found.

EXERCISES.

280. *1.* The base of a right-angled triangle is 16 in. and the perpendicular 12 in. Find the hypotenuse.

2. Base 36 ft., perpendicular 15 ft. Find the hypotenuse.

3. Hypotenuse 35 in., perpendicular 21 in. Find the base.

4. Hypotenuse 159 yds., base 135 yds. Find the perpendicular.

5. What is the diagonal of a slate 15 in. long and 11 in. wide?

6. Find the number of square feet in one side of a plank in the form of an isosceles triangle 14 ft. long and 16 in. wide at the base.

7. What is the area of a triangular piece of land, the base of which is 138 rds. and the perpendicular distance to the vertex of the opposite angle 96 rds.? Make a drawing to illustrate this problem.

8. How many acres in a rectangular farm whose length is 192 rds. and whose diagonal is 240 rds.?

9. If the gable end of a house 32 ft. wide is 12 ft. high, what is the length of the rafters? How many square feet in the gable?

10. What is the distance from one of the lower corners to the opposite upper corner of a room 22 ft. long, 18 ft. wide, and 12 ft. high? (Take the square root of the sum of the squares of 22, 18, and 12. Why?)

11. What is the area of an equilateral triangle, one of whose sides is 16 in.? 28 ft.? 36 yds.? 50 rds.? What is the height of each triangle?

CIRCLES.

281. *1.* Define circle. Circumference. Diameter. Radius.

2. The circumference of a circle is about 3.1416 times the diameter. Hence, *multiply the diameter by 3.1416 to find the circumference.*

3. The area of a circle equals *3.1416 times the square of the radius;* or, *it equals the product of its circumference by one half of its radius.*

EXERCISES.

282. *1.* What is the circumference of a tree whose diameter is 3 ft. 4 in.?

2. What is the diameter of a wheel whose circumference is 50 ft.?

3. Find the radius of a circle whose circumference is 31.416 in.

4. Find the area of a circular lake whose diameter is 280 rds.

5. If the distance around a circular park is 2 mi., how many acres does it contain?

6. A circular building covers 38.4846 sq. rds. What is the diameter?

283. Similar Surfaces.

1. Similar surfaces are such as have the same shape, such as circles, squares, right-angled triangles, etc.

2. Two similar surfaces are to each other as the squares of like dimensions.

EXERCISES.

284. *1.* A triangular lot whose base is 8 rds. has an area of 15 sq. rds. What is the area of a similar lot whose base is 12 rds.?

STATEMENT: rds. rds. sq.rds. sq.rds.
$8^2 : 12^2 :: 15 : ?$

2. The diameters of two circles are to each other as 2 to 3. If the smaller one contains 20 sq. in., how many square inches in the larger one?

3. A circle whose diameter is 3 ft. is what part of a circle whose diameter is 4 ft.? 6 ft.? 7 ft.? 9 ft.?

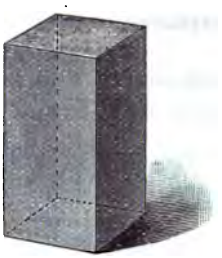
4. How many circles 10 in. in diameter contain the same area as a circle 20 in. in diameter?

5. A circle whose diameter is 3 ft. is how many times a circle whose diameter is 3 in.? 6 in.? 9 in.?

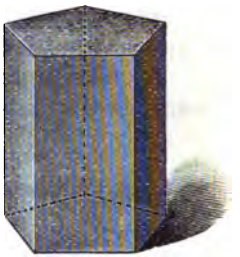
6. The areas of two similar triangles are respectively 32 in. and 50 in. The base of one is 8 in. What is the base of the other?

7. If the area of an equilateral triangle is 18 sq. ft., what is the area of a similar triangle, each of whose sides is twice as long? 3 times as long? 7 times as long?

TRIANGULAR PRISM.



QUADRANGULAR PRISM.



PENTAGONAL PRISM.

CYLINDER.

PYRAMID.

CONE.

SOLIDS.

285. *1.* Define solid. Rectangular solid. Cube. Unit of solid measure. Volume.

2. A prism is a *solid, two of whose faces, called ends or bases, are equal, parallel, and similar polygons, and the rest of whose faces are rectangles.*

3. A cylinder is a *solid whose bases are equal and parallel circles, and whose sides are perpendicular to the bases.*

4. The altitude of a prism or a cylinder is the *perpendicular distance between its bases.*

5. A pyramid is a *solid whose base is a polygon and whose other faces are triangles meeting in a point called the vertex.*

6. A cone is a *pyramid whose base is a circle.*

SPHERE.

7. The altitude of a pyramid or of a cone is the *perpendicular distance from its vertex to its base.*

8. A sphere is a *solid, all points of whose surface are equally distant from a point within called the center.*

9. The volume of any prism or cylinder equals the product of its base and its altitude. (See Section 149.)

10. The volume of a pyramid or a cone is one third of the volume of a prism or a cylinder having the same base and altitude.

11. The surface of a sphere is equal to 3.1416 times the square of its diameter.

12. The volume of a sphere is equal to 3.1416 times one sixth of the cube of the diameter.

EXERCISES.

286. *1.* How many cubic feet in a prism whose base is a rectangle 5 ft. long and 4 ft. wide, and whose altitude is 13 ft.? How many in a pyramid of the same dimensions?

2. What is the volume of a triangular prism whose altitude is 15 ft., and each side of whose base is 4 ft.? (See Problem 11, Section 280.) Of a pyramid of the same dimensions?

3. What is the volume of a cylinder the diameter of whose base is 12 in. and whose altitude is 27 in.? Of a cone having the same base and altitude?

4. How many cubic feet in a circular granite column whose circumference is 3 ft. and whose altitude is 16 ft.? In a cone having the same base and altitude?

5. Find the surface and volume of a sphere whose diameter is 18 in.

6. Find the surface and volume of a sphere whose circumference is 18 in.

287. Similar Solids.

1. Similar solids are such as have the same shape.

2. The volumes of similar solids are to each other as the cubes of their like dimensions.

EXERCISES.

288. *1.* The sides of two cubes are to each other as 3 to 5. If the smaller one contains 27 cu. in., what is the volume of the larger?

STATEMENT: cu. in. cu. in.

$$3^3 : 5^3 :: 27 : ?$$

2. A marble whose diameter is 1 in. is what part of a marble whose diameter is 2 in.? 3 in.? 4 in.?

3. A sphere whose radius is 2 in. is what part of a sphere whose radius is 4 in.? 6 in.? 9 in.?

4. How many baseballs 3 in. in diameter equal in volume a football 9 in. in diameter?

5. Which would you rather have, 4 oranges 3 in. in diameter, or 2 oranges 4 in. in diameter? Why?

PROGRESSIONS.

289. Write the number 3. Now write a number two larger than 3, then a number two larger than 5, and so on, till you have written six numbers. Write 50, then 47, then 44, and so on till you have written six numbers.

Write the number 2. Now write a number three times as great, then a number three times as great as the second, and so on till you have written six numbers. Now write 729; then write a number one third as great, and so on till you have written six numbers.

You have now written four *series* of numbers, for when several numbers increase or diminish by a constant difference, or by a constant ratio, they constitute a series; they are said to be in *Progression.* You will notice that there are two kinds of progression; the first is called *arithmetical* progression, and the second is called *geometrical* progression. The common difference may be a whole number or a fraction; and the same is true of the common ratio.

If the numbers making the series are constantly growing larger, the series is an *Ascending* series; if they are constantly growing smaller, they constitute a *Descending* series. Of course, by reversing the order of the terms, any ascending series will become a descending series, and *vice versa.*

The first term of a series and the last are the *extremes;* all the other terms are *means.*

ARITHMETICAL PROGRESSION.

290. When a series increases or diminishes by a *common difference*, we have an arithmetical progression.

It will be evident that, in an arithmetical progression, the *average term* will be found by taking half the sum of the extremes. Hence:

First. To find the sum of an arithmetical series, multiply half the sum of the extremes by the number of terms.

Second. To find any term of a series, multiply the common difference by the number of the term *less 1;* add the product to the first term in an ascending series, or subtract it from the first term in a descending series.

Third. To find the common difference, take the difference of any two consecutive terms.

EXERCISES.

291. *1.* Find the eighth term of the series 2, 5, 8, etc. Find the sum of the series.

2. Find the eighth term of the series 38, 35, 32, etc. Find the sum of the series.

3. The first term of a series is 7, the last term is 47, and the number of terms 11. What is the sum?

4. How many strokes will a clock that strikes the hours only, make in 12 hrs.?

5. A boy works 10 hrs. He receives 1 ct. for the first hour's work, 3 cts. for the second, 5 cts. for the third, and so on. How much will he receive for the last hour's work? How much for the whole time?

6. A body will fall $16\frac{1}{12}$ ft. in the first second of its fall, $48\frac{1}{4}$ ft. in the second, $80\frac{5}{12}$ in the third second, and so on. How far will it fall in a minute?

GEOMETRICAL PROGRESSION.

292. When the terms of a series increase or diminish by a *constant ratio,* we have a geometrical progression.

First. To find any term of a geometrical progression, raise the ratio to a power *one less* than the number of the term; multiply the first term by this power of the ratio.

Second. To find the sum of a geometrical series, multiply the last term by the ratio, subtract the first term from the product, and divide the remainder by the ratio less 1.

Third. To find the *general ratio,* find the ratio of any two consecutive terms.

EXERCISES.

293. *1.* Find the fifth term of a series whose first term is 2 and the ratio 3. What is the sum of the series?

2. What is the sum of a series whose first term is 5, ratio 6, and number of terms 7? Whose first term is 3, ratio 7, and number of terms 6?

3. If a man working 10 hrs. a day receives 1 ct. for the first hour, 2 cts. for the second, and so on, each hour receiving twice as much as for the preceding hour, how much does he receive for the last hour? How much for the day's labor?

METRIC SYSTEM OF MEASURES AND WEIGHTS.

294. About one hundred years ago the French introduced the metric system of measures and weights. This is a great improvement on all other systems, because it is purely decimal. All numbers in the metric system are simple numbers. This system is extensively used by scientific men and by government officials, but in this country it is rarely used in ordinary business transactions.

The standard of the metric system is the *Meter*, which was supposed to be one ten-millionth of the distance from the equator to the pole, measured on the earth's surface at the sea-level. The meter is 39.37+ inches in length, or it is a little longer than the pendulum rod, which is the standard of our common system.

The meter is the unit of *linear measure*, the square meter is the unit of *surface measure*, and the cubic meter is the unit of *solid measure*, or the *measure of volume*. The unit of measures of capacity, both for solids and for liquids, is the *Liter*. The liter is the cube of one tenth of a meter, and is about equal to nine tenths of a quart, dry measure. The unit of weight is the *Gram*. The gram is the weight of a cubic centimeter of distilled water at a temperature of about 39° Fahr. The gram is equal to about 15½ grains of Troy weight.

The names of the units in the metric system are constructed very ingeniously. For the names of the multiples of the principal unit in any "table," the following Greek prefixes are placed before the name of that unit: Deka=10, hekto=100, kilo=1,000, and myria=10,000. Thus, dekameter signifies ten meters, etc. For the divisions of the principal unit, the following Latin prefixes are put before its name: Deci=10, centi=100, and milli=1,000. Thus, decimeter means one tenth of a meter, etc. In the abbreviations for different denominations, a capital letter signifies a Greek prefix, and a small letter signifies a Latin prefix; thus, *Dm* means dekameter, and *dm* means decimeter. *Dg* means dekagram, and *dg* means decigram. There is good authority for pronouncing millim'eter or mil'lime'ter. The same is true as to the accent in several of the words of these tables.

295. Long or Linear Measure.

The *unit* of Linear Measure is the Meter=39.37 inches.

TABLE.

10 millimeters (mm) - - -	=1 centimeter (cm)
10 centimeters - - - - -	=1 decimeter (dm)
10 decimeters - - - - -	=1 meter (m)
10 meters - - - - - - -	=1 dekameter (Dm)
10 dekameters - - - - -	=1 hectometer (Hm)
10 hectometers - - - - -	=1 kilometer (Km)
10 kilometers - - - - -	=1 myriameter (Mm)

The kilometer is about .62 of a mile.

The square hektometer, or the hektare, is used in measuring land. It is equal to about 2½ acres.

296. Measures of Capacity.

The *unit* of capacity is the Liter=61.027 cubic inches=1.76 pint.

TABLE.

10 milliliters (^{ml}) - - - - -	=1 centiliter (^{cl})
10 centiliters - - - - - - -	=1 deciliter (^{dl})
10 deciliters - - - - - - -	=1 liter (^{l})
10 liters - - - - - - - -	=1 dekaliter (^{Dl})
10 dekaliters - - - - - - -	=1 hektoliter (^{Hl})
10 hektoliters - - - - - -	=1 kiloliter (^{Kl})

297. Measures of Weight.

The *unit* of weight is the Gram=15.43234 grains, or 15$\frac{1}{2}$ grains, nearly.

TABLE.

10 milligrams (^{mg}) - - - - -	=1 centigram (^{cg})
10 centigrams - - - - - -	=1 decigram (^{dg})
10 decigrams - - - - - - -	=1 gram (^{g})
10 grams - - - - - - - -	=1 dekagram (^{Dg})
10 dekagrams - - - - - -	=1 hektogram (^{Hg})
10 hektograms - - - - - -	=1 kilogram (^{Kg})

The kilogram is a little more than 2$\frac{1}{5}$ lbs. Avoirdupois weight. The unit for heavy masses is the *Tonneau*, or 1,000 kilograms, which is about one tenth greater than our ton.

EXERCISES.

298. *1.* 2^{Km}, 7^{Hm}, 8^{Dm}, 6^{m}, 3^{dm}, 4^{cm}, 1^{mm}, written as meters, is 2786.341^{m}; hence, it may be read in two ways.

2. 3^{Kg}, 8^{Hg}, 0^{Dg}, 1^{g}, 6^{dg}, 7^{cg}, 2^{mg}, written as grams, is 3801.672^{g}.

Read the following in two ways:

3.	437.59^{m}	*6.*	1576.932^{m}	*9.*	4810.007^{g}
4.	508.76^{l}	*7.*	3008.407^{g}	*10.*	7000.76^{l}
5.	941.185^{g}	*8.*	5700.054^{l}	*11.*	4703.408^{m}

Meters, liters, and grams are changed to higher denominations by moving the decimal point to the left, and to lower denominations by moving the point to the right. Thus: $3718.562^{m}=371.8562^{Dm}=3.718562^{Km}=37185.62^{dm}=3718562^{mm}$.

EXERCISES.

299. *1.* How many kilograms in 4178.37^{g}? How many centigrams?

2. How many hektoliters in 3947.792^{l}? How many centiliters?

3. How many kilometers in 9100.14^{m}? How many centimeters?

4. Express as meters, and find their sum: 786^{cm}, 9148^{dm}, and 4056^{Hm}.

5. Express in grams and add, 4375^{Dg}, 8156^{dg}, and 5000^{Kg}.

6. What is the cost of 12 meters of silk at $3.15 a yard? (Take the meter as 39.37 in.)

7. From Bloomington, Ill., to Chicago is 126 mi. What will a railroad ticket cost at 2 cts. a kilometer?

8. Find the cost of 6 lbs. of butter at $.55 a kilogram.

9. When coal is worth $7.70 a ton, what is the price per tonneau?

MISCELLANEOUS PROBLEMS.

300. *1.* The sum of two numbers is 4,607, and their difference is 819. What are the numbers?

2. How many times can 358 be subtracted from 3,222? What is the shortest method of solution? Why?

3. If a farmer has 186 sheep in one pasture and 248 in a second, how many must he change from the second to the first that there may be the same number in each?

4. A young man receives a salary of $18.50 a week. He pays $8.75 for his board and $4.85 for other expenses. In how many weeks can he pay a debt of $279.30?

5. If $9\frac{1}{2}$ T. of hay cost $140\frac{5}{8}$, how many tons can be bought for $74?

6. A person having $138\frac{3}{4}$ A. of land, sold $\frac{1}{5}$ of it to one man and $\frac{3}{7}$ of it to another. What was the value of the remainder at $68\frac{5}{8}$ an acre?

7. A boy multiplied by $\frac{4}{7}$ instead of $\frac{4}{5}$, and obtained $\frac{6}{11}$ for a result. What ought he to have obtained?

8. A man expended $148.50 in the purchase of rye at $.97 a bushel, oats at $.43 a bushel, and corn at $.58 a bushel, buying the same quantity of each. How many bushels in all did he purchase?

9. A drover bought 138 calves for $1,200.6 and sold them for $1,704.3. How much did he gain per head?

10. A dealer bought 1,729 bu. of wheat at $.67 a bushel. He sold part of it at the same price, receiving $585.58 for it. How many bushels had he left?

11. What is the value of a piece of land 146 rds. long and 95 rds. wide, at $78⅜ an acre?

12. A store building rents for $560 a year, which is 12½ % of its value. What is its value?

13. A dealer bought 23,864 T. of coal at $5.65 a ton. He sold 30 % of it at $6.75 a ton, 45 % of it at $7.10 a ton, and the remainder at $7.65 a ton. How much did he gain?

14. A gentleman gave to each of his three children ⅛, ⅓, and ¼ of his estate, respectively; and to his wife the remainder, which was $3,910. What was the share of each child?

15. A man bought a farm for $9,112.65, and sold it for $43.40 an acre, thereby gaining $1,173.15. What was the cost per acre?

16. A gentleman bought three pieces of property. For one he paid $3,297.8; for the second, 2.7 times as much as for the first; and for the third, 1.9 times as much as for the second. How much did he pay for all?

17. If a regiment of 960 men lose 320 men in battle and 80 men by desertion, what is the per cent of loss in each case? What per cent of the regiment is left for duty?

18. A man's family expenses are $946\frac{3}{4}$, which is $\frac{7}{8}$ of his income. How much does he save?

19. If 9 be added to both terms of the fraction $\frac{14}{15}$, will its value be increased or diminished? How much?

20. If 9 be subtracted from both terms of the fraction $\frac{14}{15}$, will its value be increased or diminished? How much?

21. How many square yards of plastering in the four walls and ceiling of a room 16 ft. long, 14 ft. wide, and 9 ft. high?

22. If 100 % is gained by selling an article for $1, what per cent would be gained by selling it for $2?

23. One half of the money received by a newsboy is profit. What per cent does he gain?

24. A field $68\frac{1}{2}$ rds. long contains $27\frac{3}{5}$ A. How wide is it?

25. What is the simple interest on $756, from June 18, 1895, to Oct. 27, 1896, at $5\frac{1}{2}$ %?

26. If $\frac{4}{6}$ of a farm is worth $1,400, what is $\frac{5}{12}$ of it worth? (One half of $1,400. Why?)

27. If 4 men eat 64 lbs. of bread in 2 wks., how many pounds will 16 men eat in 7 wks., at the same rate?

28. The product of three factors is 121,545; two of the factors are 37 and 45. What is the third factor?

29. Two men who are 1,704 mi. apart, travel toward each other, one 34 mi. a day and the other 37 mi. a day. In how many days will they meet?

30. A coal dealer sold $\frac{4}{5}$ of the coal he had on hand for \$2,576, at the rate of \$7 a ton. How many tons had he?

31. What is the value of my house, if $\frac{2}{3}$ of its value is \$340 more than $\frac{1}{4}$ of its value?

32. A house and lot were sold at a loss of \$162.50, thereby losing $2\frac{1}{2}\%$ of the cost. Find the cost.

33. A house valued at \$5,860 is insured for $\frac{3}{4}$ of its value, at $\frac{3}{4}\%$. Find the premium.

34. In the number 83.174, the value of the 8 is how many times the value of the 4?

35. If a teacher who receives a yearly salary of \$800, pays \$260 a year for board, and \$275 for other expenses, what per cent of his salary does he save?

36. What is the W. $\frac{1}{2}$ of the S. W. $\frac{1}{4}$ of the N. E. $\frac{1}{4}$ of a section of land worth at \$175 an acre?

37. A man having $\$19\frac{1}{2}$, gave $\frac{2}{3}$ of it for clover seed at $\$3\frac{1}{3}$ a bushel. How many bushels did he buy?

38. A lady paid $\frac{10}{13}$ of her money for a piano. She then bought a rocker for $\$14\frac{1}{2}$, which was $\frac{1}{3}$ of the money she had left. How much did she pay for the piano?

39. What is the bank discount on \$90 for 60 days, at $7\frac{1}{2}\%$?

40. What is the worth of a steer weighing $\frac{5}{8}$ of a ton, at $5\frac{1}{2}$ cts. a pound?

41. How many tiles 9 in. square will be required to cover a court 81 ft. long and 63 ft. wide?

42. A clerk receives a salary of $1,040 a year. He pays 18¾ % of it for board, 1½ % for washing, 20 % for clothing, 8½ % for other expenses, 5 % for books and other reading matter, and 25 % of the amount then left for entertainments and amusements. How much had he left?

43. A, B, and C formed a partnership. A furnished ⅜ of the capital, B ⅛ of it, and C the remainder. Their gains were $4,557. What was the share of each?

44. A note for $360, with interest at 6 %, is dated April 25, 1894. The following payments were made: $100, Aug. 15; $140, Dec. 1. How much was due Feb. 12, 1895? (Solve by Mercantile Rule.)

45. Three merchants shipped a cargo of iron by sea. A sent 185 T., B 210 T., and C 305 T. During a storm the sailors threw overboard 196 T. What per cent of the loss should each merchant sustain? How many tons should each merchant lose?

46. A drover sold cattle at $26.35 a head, receiving $1,554.65 for them. How much would he have received for them if he had sold them at $34.70 a head?

47. What is the number from which if 39⅝ be subtracted, ⅜ of the remainder is 97½?

48. A man owning 70 % of a ship, sells 30 % of his share. What per cent of the ship does he sell?

49. A merchant, after paying 62½ % of his debts, finds that $3,900 will pay 80 % of the remainder. What was the whole amount of his indebtedness?

50. A piece of land 150 ft. long and 120 ft. wide contains what per cent of an acre?

51. What is the true discount of $750 due in 1 yr. 8 mo. at 7%?

52. The tax to be raised in a certain town is $1,350. The taxable property is valued at $108,000. What is the rate of tax?

53. The surface of the six faces of a cube is 4,056 sq. in. What is the length of one edge?

54. Two men start from the same point. One travels 45 mi. east, and the other 35 mi. south. How far apart are they?

55. Give two ways for changing the form of a fraction without changing its value, and explain why the value is not changed.

56. A piece of work can be performed by A in 6 days, and by B in 8 days. In what time can both do it working together?

57. A and B working together can do a piece of work in 6 days. A alone can do it in 10 days. In what time can B do it?

58. The number of youth of school age in a certain city is 109,953, which is 38% of the number of inhabitants. What is the population of the city?

59. I bought $\frac{4}{5}$ of a barrel of apples, kept $\frac{5}{8}$ of them, and sold the remainder for $\$3\frac{1}{10}$. How much was a barrel worth at the same rate?

60. The pound sterling is worth $4.8665. How much U. S. coin will it require to pay a debt of £624 16s. 8d.?

61. The area of one face of a cube is 324 sq. in. Find its solid contents.

62. After $3\frac{1}{3}\%$ of a flock of sheep had been killed by dogs, and 91 had been sold to a butcher, $\frac{11}{15}$ of the original flock were left. What was the number in the flock at first?

63. How many feet of lumber in 10 boards 14 ft. long, 8 in. wide; 7 joists 6 in. by 2 in., 16 ft. long; 2 sills 8 in. by 8 in., 18 ft. long?

64. A lot was sold for $870, which was 20% more than it cost. Find the cost.

65. A merchant sold a lot of damaged sugar at a loss of $18\frac{3}{4}\%$, receiving $952.64 for it. Find the cost.

66. Divide $780 among A, B, and C so that B shall have twice as much as A, and C as much as A and B together.

67. Divide $571 among A, B, and C so that B shall have $54 more than A, and C $46 more than B.

68. A has $2,864. $87\frac{1}{2}\%$ of his money equals 70% of B's money. How much has B?

69. How many cubic feet in the hold of a vessel which contains 2,600 bu. of wheat?

70. A invested $18,000 in a mill. If $900 be added to $\frac{5}{6}$ of A's investment the sum will equal $\frac{3}{4}$ of B's investment. How much has B invested?

71. When a captain's observation of the sun shows noon, the ship's chronometer, keeping Greenwich time, reads 20 min. past 3.00 P. M. What is the longitude of the vessel?

72. What is the length of a street that is 66 ft. wide and contains an acre of land?

73. What is the area of a circle whose circumference is 136 ft.?

74. In what two ways may a fraction be multiplied by a whole number?

75. In a mixture of grain there are 75 bu. of corn, 45 bu. of oats, and 30 bu. of rye. What per cent of the mixture is each?

76. A dealer bought wheat for $1,707.16. He sold it for $1,911, gaining $.08 on each bushel. What was the cost per bushel?

77. What was the value of the cotton sold by a commission merchant whose commission at $2\frac{1}{2}\%$ amounted to $145?

78. How many bushels of oats at $\$\frac{3}{1\,6}$ a bushel will pay for $\frac{5}{8}$ of a ton of hay at $17\frac{1}{4}$ a ton?

79. A insured his house, worth $2,800, for $\frac{4}{5}$ of its value, at $\frac{3}{4}\%$. What would the insurer lose if the house should burn?

80. If a boy buys peaches at the rate of 5 for 2 cts. and sells them at the rate of 4 for 3 cts., how many must he buy and sell to gain $4.20?

81. A merchant sold broadcloth for $240, thereby losing 20% of the cost. For what should he have sold it to gain 15%?

82. Find the cube root of 48,228,544.

83. How many rods of fence will inclose 90 A. in the form of a square?

84. A capitalist sent his agent $3,600 to invest in wheat, after deducting his commission of 3½%. What was the agent's commission?

85. A invested $680 in railroad stock, which is $90 less than ⅞ of what he invested in land. How much did he invest in both?

86. If 28 men use 420 lbs. of beef in 3 wks., how many pounds will 18 men use in 7 wks.? (Solve by proportion and by analysis.)

87. Three men hired a coach for $15 to convey them to their respective homes, which were at distances from the place of starting as follows: A's, 18 mi.; B's, 24 mi.; and C's, 30 mi. What ought each to pay?

88. How many acres of land in the form of a square may be inclosed by 192 rds. of fence?

89. A note for $1,480, dated Aug. 6, 1893, was paid June 1, 1895, with interest at 7½%. What was the amount paid?

90. What is the present worth of a debt of $380, due in 14 mo. 12 days, discounted at 5%?

91. How many yards of carpet ¾ of a yard wide will be required to carpet a room 13½ ft. wide and 16 ft. long, the carpet to run lengthwise of the room?

92. An agent sold goods to the amount of $2,870, and received $57.40 commission. What was the rate of commission?

93. B sold a carriage for $234, which was $\frac{2}{3}$ of the cost. What would he have received for it if he had gained 25 %?

94. A merchant sold tea for 118 % of its cost and made a profit of 9 cts. a pound. Find the selling price.

95. An estate is valued at $15,876. The house is valued at $\frac{3}{11}$ of the value of the land. What is the value of each?

96. A owns $\frac{5}{12}$ of a store; B, $\frac{4}{15}$ of it, and C the remainder. The profits were $2,760. What was each man's share?

97. A insured his house for $3,460, his store for $5,820, and his stock of goods for $4,750. What was the premium at $\frac{1}{2}$ %?

98. $\frac{3}{4}$ % of a number is 168. What is the number?

99. A hotel is insured for $\frac{2}{3}$ of its value, at $1\frac{1}{2}$ % premium. The premium amounts to $150. Find the valuation of the hotel.

100. The parallel sides of a field in the shape of a trapezoid are 64 rds. and 70 rds. respectively. The perpendicular distance between them is 42 rds. How many acres in the field?

101. How many square feet in the surface of a cube of granite whose edges are $3\frac{1}{2}$ ft. in length?

102. A man sold a house for $4,900, which is $16\frac{2}{3}$ % more than it cost him. What per cent would he have gained if he had sold it for $5,100?

103. The signal service reports $3\frac{1}{2}$ in. of rain as falling in 24 hrs. How many cubic yards of water fell on an acre of ground?

104. If by selling land at \$68 an acre, 15 % is lost, at what price must it be sold to gain $18\frac{3}{4}$ %?

105. A's tax was \$41.07. What was the assessed valuation of his property if the rate of taxation was $1\frac{1}{2}$ %?

106. If $9\frac{5}{8}$ A. of meadow produce $20\frac{1}{8}$ T. of hay, how many tons will $5\frac{9}{10}$ A. produce at the same rate?

107. $37\frac{1}{2}$ % of $66\frac{2}{3}$ % of a number is 591. What is the number?

108. A grocer has a false balance by which he gives only 14 oz. to the pound. What is the real value of a tub of butter which he sells for \$20.48?

109. If the wheel of a bicycle is 2.9 ft. in diameter, how many times must it revolve in running a mile?

110. Property worth \$7,600 is insured for $\frac{3}{4}$ of its value at $\frac{3}{4}$ % premium. How much will the owner lose in case of total destruction by fire within a year after it is insured?

111. Five men bought a ship worth \$38,970. A paid 20 % of the whole cost, B $16\frac{2}{3}$ % of it, and the others paid the remainder equally. How much did each of the others pay?

112. In order to have an annual income of \$1,800, what sum of money must I invest at 6 %?

113. How many rods of fence will be required to inclose a garden in the form of a right-angled triangle whose area is $2\frac{2}{5}$ A., and whose base is 24 rds.?

114. Find the solid contents of a cylinder whose diameter is 10 in., and whose altitude is 18 in. What is the volume of a cone whose dimensions are the same?

115. A commission merchant deducts $2\frac{1}{4}\%$ commission and remits $4,715.46. What was the amount of the sales?

116. A man's fuel for a year cost him $156\frac{3}{4}$. $\frac{4}{11}$ of it he paid for wood at $4\frac{1}{2}$ a cord, and the remainder for coal at $7\frac{1}{8}$ a ton. How many cords of wood and how many tons of coal did he use?

117. The difference in time of two places is $31\frac{1}{2}\%$ of a day. What is their difference in longitude?

118. Find the area of an isosceles triangle whose base is 36 yds., and whose perpendicular distance to vertex is 30 yds.

119. What is the altitude of an isosceles triangle whose area is 2,752 sq. ft., and whose base is 64 ft.?

120. Find the length of the equal sides of an isosceles triangle whose base is 72 in., and whose altitude is 60 in. (The altitude of an isosceles triangle bisects the base.)

121. What is the altitude of an isosceles triangle whose base is 48 ft., and whose sides are 50 ft.? What is its area?

122. Find the area of an isosceles triangle whose altitude is 45 in., and whose sides are 80 in.

123. A grocer made a profit of $116.10 on 258 bbls. of flour, which he sold at $4.80 a barrel. What did it cost him per barrel?

124. A father gave $33\frac{1}{3}\%$, 25%, and $16\frac{2}{3}\%$ of his property, respectively, to each of his three children. He then had \$36,000 left. How much did each child receive?

125. In what two ways may a fraction be divided by a whole number?

126. A dealer sold 327 T. of coal at \$3.45 a ton, and a number of tons of another kind at \$6.85 a ton. He received \$3,018.75 for all. How many tons did he sell?

127. A merchant bought a cargo of flour for \$1,638.63, and sold it for $\frac{8}{7}$ of its cost, thereby losing \$.17 a barrel. How many barrels did he buy?

128. If 40% of the selling price of an article is profit, what is the per cent of gain on the cost?

129. A steam frigate sailing at the rate of $14\frac{5}{8}$ mi. an hour, gave chase to a pirate vessel $6\frac{1}{4}$ mi. ahead, sailing at the rate of $12\frac{3}{4}$ mi. an hour. How long before the frigate will overtake the pirate?

130. D bought $\frac{7}{16}$ of a manufacturing business for \$2,759.05, and E bought $\frac{1}{16}$ of the same business at the same rate. What did E's interest cost him?

131. If 48 horses eat 180 bu. of oats in 10 days, how many bushels will 38 horses eat in 18 days?

132. A piano which cost \$480 was sold at a gain of $16\frac{2}{3}\%$. The marked price was \$600. What per cent reduction was made on the marked price?

133. How shall a dealer mark reapers that cost \$112 so that he may fall $12\frac{1}{2}\%$ and still make a profit of $18\frac{3}{4}\%$?

134. A and B can mow a field in 7 days. A, B, and C can mow it in 5 days. If they receive $25, what ought C to receive?

135. The premium on a fire insurance policy at $\frac{7}{8}\%$ was $189. Find the face of the policy.

136. A schoolhouse is to be built at a cost of $9,600. The property of the town is assessed at $2,560,000. What will be the rate of tax, and what will be E's tax on property valued at $1,800?

137. Find the volume of a sphere whose radius is 3 in. Find the surface of the same sphere.

138. Make a drawing and locate the E. $\frac{1}{2}$ of the S. E. $\frac{1}{4}$ of the N. W. $\frac{1}{4}$ of Sec. 6. How many acres in it? How many rods of fence would be required to inclose it?

139. An importer paid duties amounting to $386.75. If the duty was 25% of the value of the goods, what was their value?

140. 18 shares of bank stock, par value $100 each, were sold at $3\frac{1}{8}\%$ premium. How much was received for them?

141. What is the difference between the true discount and the bank discount of $3,000 for 90 days at 6%?

142. If a barrel of flour will last one family $7\frac{1}{2}$ mo., a second family 9 mo., and a third $11\frac{1}{4}$ mo., how many months will it last all three families together?

143. The boundaries of a square and a circle are each 28 ft. Which has the greater area, and how much?

144. An agent charged a commission of $57.07 for selling goods, for which he received $1,756. What was his rate of commission?

145. B sold a horse for $342, which was 5 % less than he asked for it, and his asking price was 12½ % more than the horse cost. What per cent was gained?

146. The list price of a certain stove is $45. The retail dealer is allowed commercial discounts of 20 %, 8⅓ %, and 4 %. What does he pay apiece for the stoves?

147. Find the cost of a draft on New York for $2,800 at ⅝ % premium.

148. If a man works 8 hrs. a day he can complete a piece of work in 17½ days. How many hours a day must he work to complete it in 12½ days?

149. A sea captain paid $387, at 1½ %, for insuring ⅘ of the value of his ship. Find the value of the ship.

150. One city is in 95° E. longitude, another in 14° 20′ W. When it is noon at the latter place, what is the time at the former place? When it is noon at the former place what is the time at the latter?

151. A man's real estate is valued at $3,400, and his personal property at $1,560. If he pays a tax of 1⅜ %, what is his total tax?

152. If a ball weighing 32 lbs. is 6 in. in diameter, what is the weight of a similar ball 7 in. in diameter?

153. A farmer wishes to set out an orchard of 3,721 trees so that the number of rows shall be equal to the number of trees in a row. What will be the number of rows?

154. A man bequeathed $9,375 to his wife, which was 31¼ % of the sum he bequeathed to his children, and the sum bequeathed to his children was 62½ % of his estate. How much had he left?

155. A square court whose side is 48 yds., is paved with 36,864 square tiles. Find the dimensions of the tiles.

156. Gunpowder is composed of 15 parts of saltpeter, 2 of sulphur, and 3 of charcoal. How many pounds of each are there in 96 lbs. of powder?

157. A merchant reduced the price of a piece of cloth 10 cts. per yard, and thereby reduced his profit on the cloth from 20 % to 16⅔ %. What was the cost per yard?

158. A dealer bought a watch for $90, and set such a price on it that, after falling $12, he still made 20 % on the purchase. What per cent did he abate from the asking price?

159. What is the area of an equilateral triangle whose sides are 42 in.? What are the solid contents of a triangular prism whose altitude is 7 ft. and whose base is the same as the equilateral triangle?

160. What is the area of a circle whose diameter is 14 in.?

161. How many barrels of water (31½ gals.) will a cylindrical cistern hold that is 8 ft. in diameter and 14 ft. deep?

162. A merchant having 8 bu. 3 pks. of cranberries, lost by decay 2 pks. 5 qts. What per cent did he lose?

163. If a tax of $128 be paid on property valued at $16,000, what is the assessed valuation of a residence that is taxed $11.68, at the same rate?

164. Find the depth in feet of a cubical cistern that holds 600 bbls. ($31\frac{1}{2}$ gals.)

165. Change $\frac{3}{8}$, $\frac{7}{32}$, $\frac{11}{40}$, $\frac{3}{125}$, and $\frac{21}{1000}$ to decimals and add the results.

166. What common fractions can be changed to pure decimals?

167. At what price must hay which cost $14.50 a ton be sold to gain 20%?

168. 18 men and 11 boys together earn $169.20 a week. If a boy's pay is half a man's, what are the daily wages of a man, and also of a boy?

169. If $\frac{1}{6}$ of a quantity of goods valued at $1,800 is sold at a loss of $8\frac{1}{3}$%, for what must the remainder be sold to gain $12\frac{1}{2}$% on the whole?

170. If 24 men can dig a drain in 16 days, what per cent of the 24 men could dig it in 20 days?

171. A town must have $164,900 for expenses. If the cost of collecting is 3%, how much money must be raised?

172. When stock is 14% premium, how much stock can be bought for $3,990?

173. When stock is 11% discount, how much can be bought for $3,115?

174. A certain cube contains 421,875 cu. in. Find the diagonal of one of its faces. Find the entire surface of the cube.

175. A owns $\frac{5}{8}$ of a ship, and B owns the remainder. B sells $37\frac{1}{2}\%$ of his interest for \$18,000. What is A's interest worth at the same rate?

176. The altitude of a pyramid having a square base is 40 in.; the length of each side of the base is 60 in. Find the slant height of each face, and the entire surface and contents of the pyramid. (The slant height is a hypotenuse.)

177. Two men starting at different points travel toward each other till they meet at Washington, D. C., 77° W. longitude. One finds his watch 4 hrs. 20 min. slow, and the other finds his 6 hrs. 50 min. fast. What was the longitude of the starting point of each?

178. What is the surface and volume of a watermelon 14 in. in diameter, supposing it to be a perfect sphere?

179. Which contains the more land, 4 circular parks 40 rds. in diameter, or 2 others 60 rds. in diameter?

180. Bought 60 shares of telephone stock, par value \$100, at 40% below par; and sold them at once, at 15% below par. How much did I gain? What per cent did I gain?

181. A merchant, deducting 20% from his marked price, sold at cost. What per cent profit would he have made at the marked price?

182. A merchant buys cloth at 25% discount, and sells it at full price. What per cent profit does he make?

183. \$80 is $31\frac{1}{4}\%$ of what a man paid for a carriage. The carriage was sold for \$336. What per cent was gained?

184. An architect furnished plans and superintended the erection of a building which cost $38,600. He received a commission of $2\frac{3}{4}\%$ of the cost. How much was his commission?

185. Divide the product of 12, 60, 36, 35, and 38 by the product of 7, 30, 18, 24, and 19.

186. A purchased two farms, each containing an exact number of acres, for $21,021 and $4,165, respectively. He paid for each the largest number of dollars per acre that could be paid for both farms. How many dollars did he pay per acre? How many acres did he buy?

187. A man bought 1,275 bbls. of flour at $4\frac{3}{4}$ a barrel. He sold part of it at the same price, receiving $3,990 for it. How many barrels had he left? He sold the remainder at $5\frac{1}{4}$ a barrel. How much did he gain on the transaction?

188. If a cylindrical cistern 8 ft. in diameter holds 160 bbls. of water, how many barrels will a similar cistern 10 ft. in diameter hold?

189. A boy worked one year (52 wks.), receiving 25 cts. for the first week, 50 cts. for the second, 75 for the third, and so on through the year. How much did he receive for the last week's work? How much for the whole year?

190. A man worked a year, receiving 50 cts. for the first month, twice as much for the second as for the first, twice as much for the third as for the second, and so on through the year. How much did he receive for the last month? How much for the whole year?

191. A man lost 18¾% of his money. He then earned $66 and had 150% as much money as he had at first. How much did he have at first?

192. A man received from a bankrupt $1,441, which was 91⅔% of the sum due. How much was due?

193. A farmer raised 1,092 bu. of wheat, and the crop was 1,400% of the seed. How many bushels did he sow?

194. There are three wheels which revolve in 28, 36, and 44 sec., respectively. What is the fewest number of minutes and seconds in which all will make an exact number of revolutions?

195. Find the interest on $2,600 for 2 yrs. 9 mo. at 7%.

196. When the interest on $3,000 for 3 yrs. is $720, what is the rate per cent?

197. How long must $1,600 be loaned at 6% to gain $384?

198. What principal put at interest for 3½ yrs., at 5½%, will produce $404.25?

199. A speculator bought stock at 18% below par, and sold it at 15% above par. He gained $1,155. How much did he invest?

200. I invest and sell at a loss of 20%. I invest the proceeds again and sell at a gain of 20%. Do I gain or lose on the two speculations, and how many per cent?

201. If I buy stocks at 20% premium and sell them at 10% discount, what per cent do I lose?

ANSWERS.

Page 12.

XII; XV;
XXIV;
XXXVII;
XLV; LIX;
LXIII;
LXXVI;
LXXXVIII;
XCI; C;
CIX;
CXXV;
CXL;
CCLXXIV;
CCCCL;
DXCIX;
MCX

Page 18.

Sec. 30.

1. 59; 100;
200; 510;
801
2. 400; 218;
97; 520
3. 608; 246;
72; 350

Page 19.

4. 86; 411;
222; 560
5. 500; 707;
273; 80
6. 759; 904;
67; 517
7. 880; 606;
434; 99
8. 314; 612;
888; 76
9. 1000; 3001;
5070; 8065
10. 4400; 2010;
9016; 6606;
6258; 3512

Page 20.

1. 2025
41,300
18,014
2. 7474
36,050
53,508
3. 15,487
10,070
65,312
4. 48,800
71,701
12,056
5. 87,087
92,911
300,045
6. 99,426
513,000
600,400

Page 21.

7. 302,076
200,016
118,600
8. 444,444
600,006
900,000
9. 700,000
200,202
515,500
10. 670,070
330,700
7,000,200
11. 3,081,000
9,500,000
12,000,053
12. 10,010,010
27,400,011
89,806,047
13. 300,000,300
507,300,009
14. 997,003,014
1,300,000,000
5,000,000,154
13,000,406,000
15. 318,001,000,000
603,000,000,000
831,000,080,000
16. 4,000,000,000,000
37,000,000,064,000
405,000,002,000,000
9,000,009,000,000,009

Page 30.

1. 225
2. 215
3. 504
4. 310
5. 405
6. 392
7. 397
8. 484
9. 3617
10. 3701
11. 3151
12. 3309
13. 3224
14. 3778
15. 51,574
16. 37,257
17. 33,715
18. 35,923
19. 21,210
20. 3,336,011
21. 3,792,794
22. 3,637,761
23. 3,673,494

Page 34.

36. 14,942
37. 732,409

Page 35.

38. 22,853,264
39. 1,760,953,701
40. 91,077,222,829
41. 7240 lbs.
42. 1178 trees
43 $33,360

Page 36.

44. 47,624 men
45. 16,300 bricks
46. 3338 mi.
47. $26,352
48. 24,585 bu.
49. 29,062 ft.

Page 37.

50. 89,600 sq. mi.
51. 51,188 kegs
52. 12,690 mi.
53. 168,665 sq. mi.
54. 5,368,705 pop.

Page 38.

55. 300,701 lbs.
56. 4824 Music Hall
8000 Mor. Temple
58,000 St. Peter's
57 128,590,574 lbs.
105,984,384 lbs.
58. 7,438,396 bales

Page 39.

59. 13,965,373 T.
60. 3,384,671 carp

Page 39.

1. 2879
2. 3164
3. 3413
4. 2508
5. 2583
6. 2520

Page 40.

7. 6727
8. 14,399
9. 7149
10. 16,760
11. 6169
12. 14,006
13. 23,759
14. 24,193
15. 9171
16. 16,930
17. 12,378
18. 12,192
19. 9884
20. 15,089
21. 8,365,096
22. 8,990,484
23. 9,424,677

Page 47.

2. 158
3. 387
4. 128
5. 191
6. 78
7 305
8. 348
9. 309
10. 457
11. 76
12. 183
13. 410
14. 360
15. 149
16. 400
17. 89
18. 408
19. 156
20. 117
21. 309
22. 293
23. 295
24. 385
25. 94
26. 709
27. 370
28. 324
29. 135
30. 588
31. 406
32. 2867
33. 4589
34. 2849
35. 2792
36. 3976
37. 1904
38. 2180
39. 2707
40. 4107
41. 489
42. 509
43. 2080
44. 5008
45. 700
46. 48
47. 1003
48. 2149
49. 2816

Page 48.

2. 38,778
3. 38,889
4. 17,675
5. 14,524
6. 15,869
7. 44,689
8. 40,697
9. 87,476
10. 48,669
11. 10,899
12. 59,262
13. 14,998
14. 4051
15. 38,001
16. 384,438
17. 349,947
18. 638,609
19. 494,047
20. 266,667
21. 510,999
22. 730,616
23. 396,576
24. 299,728
25. 599,309
26. 192,203
27. 299,786
28. 95,347
29. 625,030

Page 52.

37. $1520
38. $3875
39. $3150
40. 18,699 men
41. 128 yrs.
42. 22,868
43. 1,004,479

Page 53.

45. 418,882
46. 6,777,840
47. 55,780,323
48. 3,881,103
49. 145 yrs.
50. 1265 mi.
51. 107,420 sq.mi.
52. $5,696,533
53. $2,474,808

Page 54.

54. 1,090,617 T.
55. 2,277,525
56. 739,339
57. 4,864,096 oz.
58. 6,548,425 bu.
59. $708,382,866
60. 1589 mi.
61. 8155 mi.
62. 17,773
63. 10,568

Page 64.

2. 992
3. 1416
4. 4081
5. 5072
6. 4368
7. 3411
8. 2439
9. 3556
10. 2874
11. 6570
12. 4375
13. 4768
14. 4962
15. 1236
16. 6524
17. 3320
18. 6489
19. 3976
20. 1704
21. 3516
22. 23,680
23. 45,395
24. 66,536
25. 34,032
26. 66,672
27. 42,273
28. 42,000
29. 32,072
30. 11,682
31. 188,544
32. 628,520
33. 403,781
34. 376,008
35. 177,069
36. 716,886
37. 544,248

Page 65.

2. 1360
2040
3400
4080
6120
6. 9400
14,100
23,500
28,200
37,600
9. 108,000
162,000
324,000
432,000
10. 1,512,000
11. 25,707,000
12. 21,450,000
13. 27,900,000

Page 66.

Sec. 63.

2. 1081
3 702
4. 504
5. 1961
6. 2128
7 2210
8. 3555
9. 2450
10. 4902
11. 7178
12. 7654
13. 9604
14. 19,749
15. 26,448
16. 56,024
17. 32,980
18. 49,980
19. 80,634
20. 579,150
21. 242,304
22. 699,654

Page 66.

Sec. 64.

1. 1666 pickets
2. 14,508 times
3. 84,000 ft.

Page 67.

4. 49,920 bu.
5. 45,990 mi.
6. 3591 gals.
7. 20,955 trees
8. 913,920 lbs.
9. 12,740 bu.
10. 1,791,800 cts.
11. 280,448 yds.
12. 114,560 rds.
13. $213,192.

Page 68.

14. 984,960 letters
15. 597,505 lbs.
16. 93,366 yds.
17. 1,312,000 A.
18. 4,514,400 ft.
19. 1,099,260 oz.
20. 63,180 T.
21. 17,360 mi.
22. 288,288 rev.

Page 69.

23. 92,628,000 mi.
24. 50,889,600 mi.
25. 292,284,246 lbs.
14,333,356 gals.
26. 327,471,196 gals.
75,509,135 lbs.
16,691,493 oz.

Page 69–*cont.*

27. $1,331,782,506
28. $343,514,472
29. 3,341,572,000 fishes
30. 18,489,025

Page 70.

1. 103,550
2. 273,681
3. 210,984
4. 413,184
5. 621,120
6. 416,816
7. 401,940
8. 163,878
9. 487,976
10. 215,303
11. 666,460
12. 580,138
13. 2,705,384
14. 1,522,620
15. 3,818,367
16. 2,978,019
17. 1,566,720
18. 10,047,608
19. 29,048,940
20. 22,489,248
21. 35,568,456
22. 23,221,058
23. 21,626,025
24. 57,317,200
25. 23,722,624
26. 56,960,268
27. 24,261,567
28. 42,086,040
29. 18,288,337
30. 278,527,200
31. 198,634,590
32. 294,361,021
33. 2,386,315,596
34. 2,767,142,380
35. 2,783,524,608
36. 3,765,586,199
37. 4,826,307,384
38. 5,191,006,023
39. 47,928,659,020
40. 40,880,789,600
41. 348,798,921,452
42. 266,141,837,376

Page 82.

30. 18,947 bbls.
31. 2640 rev.
32. $38,475
33. 23,040 A.
34. $6750
35. $19,439
36. 650 bu.

Page 83.

37. 365 A.
38. 483
39. 98
40. 468
41. 38 farms
42. 90 lbs.
43. 4835 gals.
44. $1736
45. $735
46. $76
47. 365 days
48. 5 mi. 2662 ft.

Page 84.

49. 269 persons and 2208 remainder
50. $98
51. 68 States
52. 85 A. and 13,682 A. remainder
53. 1,589,041 mi. and 35 mi. over
54. 86 schooners
55. 273 gals. per cow and 523 gals. over; 52 lbs. butter per cow and 7461 lbs. over
56. 896 lbs. per A. and 43,641 lbs. over
57. $271 and $10,206 over

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Sec. 74.

1. 245
2. 238
3. 128
4. 124
5. 673
6. 638
7. 834
8. 427
9. 574
10. 437
11. 247
12. 4138
13. 16,857
14. 2768
15. 6559
16. 4809

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17. 4812
18. 6238
19. 5263
20. 4169
21. 93,716
22. 209,758
23. 189,572
24. 26,485
25. 92,467
26. 74,809
27. 106,748
28. 9451
29. 9485
30. 79,649

Page 85.

Sec. 75.

1. 136
2. 141, 10 rem.
3. 143, 2 rem.
4. 167
5. 172, 4 rem.
6. 230, 7 rem.
7. 463
8. 286, 8 rem.
9. 315, 14 rem.
10. 312
11. 814
12. 439, 38 rem.
13. 1262
14. 725, 26 rem.
15. 1587, 25 rem.
16. 752
17. 582, 11 rem.
18. 827
19. 368
20. 836
21. 359
22. 508
23. 809
24. 716

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Sec. 75.

25. 486
26. 3752
27. 3792
28. 18,362
29. 5094
30. 6407
31. 7158
32. 2583
33. 736
34. 582, 25 rem.
35. 893
36. 761, 352 rem.
37. 969, 344 rem.
38. 734
39. 1108, 284 rem.
40. 1064
41. 956, 100 rem.
42. 985
43. 876, 110 rem.
44. 924
45. 896, 2547 rem.
46. 897
47. 683
48. 891, 3223 rem.
49. 1004, 276 rem.
50. 2007
51. 784

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Sec. 76.

1. 183
2. 139
3. 947
4. 173
5. 137
6. 337
7. 146
8. 476
9. 634
10. 431
11. 247
12. 437
13. 3164
14. 5709
15. 7189
16. 6378
17. 3576
18. 5498
19. 5772
20. 952, 4 rem.
21. 8375
22. 3498
23. 63,849
24. 58,297
25. 72,648
26. 42,869
27. 83,596
28. 37,849
29. 34,695
30. 67,538
31. 63,758
32. 78,334
33. 93,147

Page 87.

1. $3895
2. 4380 bu.
3. $1926
4. 1025 bu.
5. 132,699
6. $395

Page 88.

7. 337 mi.
8. $20,710
9. 58
10. 7,948,951
11. 6003
12. $45
13. 1568 bu.
14. 6228 votes
    6444 votes
15. 78 bbls.
16. $64

Page 89.

17. $5684
18. $21,890
19. 2520 poles
20. 90 times
    2 times
21. 8 wks.
22. 329
23. 356 cd.
    $4 a cord
24. 8360 more rev.
25. 8784 hrs.; 47 books and 56 pages

Page 96.

5. 2, 3, 17;
3, 5, 7;
2, 2, 3, 3, 3;
2, 2, 2, 3, 5;
2, 3, 3, 7;
2, 2, 2, 2, 2, 2, 2;
11, 13;
2, 3, 5, 5;
2, 3, 3, 3, 3;
1, 197;
2, 2, 2, 5, 5;
7, 31

6. 13, 17;
2, 11, 11;
1, 277;
2, 149;
1, 331;
7, 7, 7;
3, 11, 11;
3, 3, 7, 7;
1, 449;
2, 3, 5, 19;
5, 5, 5, 5;
2, 2, 5, 5, 7

7. 5, 11, 13;
5, 13, 13;
7, 7, 19;
2, 11, 47;
2, 3, 191;
3, 13, 31;
2, 3, 5, 43;
2, 3, 13, 17;
2, 3, 11, 73;
2, 3, 7, 11, 17;
13, 19, 37;
3, 11, 17, 17

8. 5, 7, 11, 29;
2, 2, 2, 2, 11, 71;
3, 3, 3, 7, 7, 13;
3, 3, 3, 3, 7, 61;
7, 11, 17, 31;
2, 2, 7, 11, 13, 17;
2, 2, 2, 7, 7, 7, 29;
2, 2, 11, 11, 13, 13;
2, 2, 2, 2, 2, 5, 5, 5, 5;
2, 2, 2, 2, 3, 3, 5, 5, 7, 7;
3, 3, 3, 3, 5, 5, 11, 11;
3, 7, 13, 23, 127

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1. 84
2. 360
3. 180
4. 54
5. 1008
6. 1848
7. 720
8. 18,018
9. 405
10. 1320
11. 588
12. 196

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13. 180
14. 390
15. 1080
16. 7560
17. 7200
18. 12,096
19. 5880
20. 660
21. 5280
22. 3780
23. 194,040
24. 40,320
25. 240 apples
26. $585
27. 720 ft.
28. 360 words

Page 101.

1. 4
2. 5
3. 9
4. 6
5. 8
6. 7
7. 11
8. 12
9. 14
10. 21
11. 5
12. 9
13. 18
14. 78
15. 45
16. 90
17. 24
18. 133
19. 154
20. 221
21. 15
22. 242
23. 38
24. 62
25. 3
26. 16
27. 42
28. 29

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29. 72
30. 25
31. 42
32. 23
33. 6
34. 49
35. 40
36. 75
37. 33
38. 26
39. 57
40. 138
41. 15 trees
42. 16 ft.
90 boards
130 boards
43. 21 bu.
39 bins
44. $31

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2. $\frac{2}{15}$; $\frac{11}{12}$; $\frac{12}{15}$; $\frac{1}{10}$
3. $\frac{10}{33}$; $\frac{14}{57}$; $\frac{6}{11}$; $\frac{21}{55}$
4. $\frac{17}{75}$; $\frac{8}{91}$; $\frac{33}{56}$; $\frac{50}{117}$

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5. $\frac{77}{145}$; $\frac{55}{183}$
6. $\frac{137}{165}$; $\frac{103}{113}$
7. $\frac{108}{116}$; $\frac{290}{174}$

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8. $\frac{287}{303}$; $\frac{200}{10}$
9. $\frac{818}{818}$; $\frac{821}{350}$
10. $\frac{272}{408}$; $\frac{46}{417}$
11. $\frac{1389}{461}$; $\frac{111}{450}$
12. $\frac{6001}{605}$; $\frac{557}{100}$

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Sec. 98.

2. $\frac{29}{47}$
3. $\frac{5}{6}$
4. $\frac{10}{11}$
5. $\frac{17}{10}$
6. $\frac{31}{51}$
7. $\frac{3}{4}$
8. $\frac{5}{7}$
9. $\frac{122}{135}$
10. $\frac{59}{75}$
11. $\frac{29}{61}$
12. $\frac{1}{3}$
13. $\frac{2}{3}$
14. $\frac{11}{17}$
15. $\frac{272}{450}$
16. $\frac{1}{3}$
17. $\frac{15}{4}$
18. $\frac{26}{51}$
19. $\frac{3}{4}$
20. $\frac{11}{15}$
21. $\frac{5}{7}$

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8. $24\frac{1}{3}$
9. $62\frac{1}{15}$
10. $31\frac{4}{15}$
11. $157\frac{13}{15}$
12. $18\frac{9}{13}$
13. $18\frac{3}{5}$
14. $77\frac{4}{5}$
15. $159\frac{5}{6}$
16. $7\frac{58}{85}$
17. $91\frac{1}{3}$
18. $295\frac{8}{15}$
19. $73\frac{5}{16}$
20. $751\frac{13}{51}$
21. 32
22. $172\frac{231}{275}$
23. 29

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Sec. 105.

5. $\frac{189}{13}$
6. $\frac{213}{13}$
7. $\frac{371}{15}$
8. $\frac{707}{19}$
9. $\frac{648}{35}$
10. $\frac{871}{37}$
11. $\frac{1375}{31}$
12. $\frac{518}{9}$
13. $\frac{2487}{38}$
14. $\frac{2053}{41}$
15. $\frac{8615}{113}$
16. $\frac{12809}{75}$
17. $\frac{9814}{57}$
18. $\frac{4841}{60}$
19. $\frac{23005}{146}$
20. $\frac{24025}{317}$

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Sec. 108.

11. $\frac{72}{180}$; $\frac{105}{180}$; $\frac{132}{180}$; $\frac{100}{180}$
12. $\frac{80}{144}$; $\frac{128}{144}$; $\frac{108}{144}$; $\frac{96}{144}$
13. $\frac{60}{70}$; $\frac{45}{70}$; $\frac{56}{70}$; $\frac{56}{70}$
14. $\frac{33}{48}$; $\frac{12}{48}$; $\frac{32}{48}$; $\frac{40}{48}$
15. $\frac{50}{60}$; $\frac{52}{60}$; $\frac{42}{60}$; $\frac{55}{60}$
16. $\frac{200}{440}$; $\frac{209}{440}$; $\frac{209}{440}$; $\frac{264}{440}$
17. $\frac{48}{78}$; $\frac{52}{78}$; $\frac{24}{78}$; $\frac{54}{78}$
18. $\frac{84}{100}$; $\frac{75}{100}$; $\frac{72}{100}$; $\frac{72}{100}$
19. $\frac{112}{168}$; $\frac{145}{168}$; $\frac{105}{168}$; $\frac{96}{168}$
20. $\frac{615}{615}$; $\frac{246}{615}$; $\frac{738}{615}$
21. $\frac{330}{360}$; $\frac{348}{360}$; $\frac{328}{360}$; $\frac{250}{360}$
22. $\frac{550}{880}$; $\frac{470}{880}$; $\frac{255}{880}$; $\frac{478}{880}$
23. $\frac{880}{880}$; $\frac{825}{880}$; $\frac{488}{880}$; $\frac{528}{880}$
24. $\frac{880}{888}$; $\frac{808}{888}$; $\frac{324}{888}$; $\frac{800}{888}$; $\frac{840}{888}$
25. $\frac{2730}{4095}$; $\frac{2275}{4095}$; $\frac{2510}{4095}$; $\frac{2340}{4095}$; $\frac{2485}{4095}$; $\frac{587}{4095}$
26. $\frac{102377}{115555}$; $\frac{117326}{115555}$; $\frac{123333}{115555}$
27. $\frac{12}{24}$; $\frac{18}{24}$; $\frac{18}{24}$; $\frac{20}{24}$; $\frac{21}{24}$; $\frac{20}{24}$

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Sec. 113.

1. 8; .06; .63; 6.3
2. .009; .039; .403

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3. .0003; .0015; .0507
4. .0241
5. .4011; 1.800; .00018
6. .00704; .01027
7. .60005; .000005; .000409
8. .001016; .0000072; .0004012
9. .00000019; .00000604
10. 4.05
11. 78.3; 83.56
12. 302.092; 45.908
13. 814.0057; 6000.00098
14. .000003039; 86.0111

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Sec. 116.

1. .25; .4; .375; .05
2. .1875; .08; .175; .12
3. .75; .45; .48; .325
4. .8; .625; .46; .3125
5. .525; .046875; .3375; .78
6. .6875; .03125; .192; .495

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Sec. 116.

7. .25; .234375; .775; .6
8. .5625; .808; .6796875; 37.5
9. .9; $.833\frac{1}{3}$; .308; 8.6

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10. .9375; .6336; .63625; 25.05

11. 18.75; .75; .12125; .404

12. .444$\frac{4}{9}$; .81818$\frac{2}{11}$; .5833$\frac{1}{3}$; .851851$\frac{13}{27}$; .2153846$\frac{2}{13}$

13. .66$\frac{2}{3}$; .28; .06; 14.4

14. .083$\frac{1}{3}$; .2; 17.4; 37.125

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Sec. 118.

1. $\frac{3}{5}$; $\frac{9}{10}$; $\frac{7}{40}$; $\frac{43}{1000}$

2. $\frac{2}{15}$; $\frac{1}{40}$; $\frac{3}{4}$; $\frac{27}{1000}$

3. $\frac{38}{10000}$; $\frac{18}{25}$; $\frac{3}{10000}$

4. $\frac{5}{8}$; $\frac{8}{10000}$; $\frac{201}{800}$

5. $\frac{14}{25}$; $\frac{96}{125}$; $\frac{561}{1000}$

6. $\frac{51}{800}$; $\frac{1}{100}$; $\frac{7}{625}$

7. $4\frac{3}{10}$; $9\frac{7}{100}$; $5\frac{3}{250}$

8. $\frac{1}{8}$; $\frac{11}{1000}$; $14\frac{1}{10}$

9. $\frac{3}{1000}$; $\frac{5}{8}$; $\frac{41}{500}$

10. $39\frac{12}{50}$; $\frac{301}{100000}$

11. $1\frac{1}{10000}$; $100\frac{47}{100}$

12. $\frac{4}{5}$; $\frac{21}{25}$; $\frac{1001}{10000}$

13. $\frac{1}{100}$; $\frac{201}{625}$

14. $\frac{1}{5000}$; $\frac{11}{50}$

15. $5\frac{27}{125}$; $3\frac{7}{16}$; $\frac{27}{50}$

16. $\frac{3}{40}$; $\frac{231}{5000}$; $2\frac{27}{125}$

Page 125.

12. 2189 far.

13. 5571 far.

14. 3886 far.

15. 10,120 far.

16. 7712 far.

17. 39 far.

18. 13,464 far.

19. 17,183 far.

Page 126.

2. £3 14s. 6d. 3 far.

3. £18 8d.

4. £25 14s. 1d.

5. £27 6s. 8d. 2 far.

6. £1990 18s.

7. £55

Page 128.

11. 657 in.

12. 936 in.

13. 77 ft.

14. 1219 in.

15. 2289 rds.

16. 3489 rds.

17. 273 ft.

18. 4536 in.

Page 129.

20. 6 yds. 1 in.

21. 2 rds. 2 yds. 2 ft. 2 in.

22. 6 rds. 4 yds. 1 ft. 10 in.

23. 8 rds. 3 yds. 2 ft. 9 in.

24. 13 mi. 1 fur. 18 rds.

25. 27 mi. 3 fur. 7 rds.

26. 33 mi. 4 fur. 30 rds.

27. 89 mi. 3 fur. 37 rds.

28. 125 mi.

Page 129.

1. 277.2 in.

2. 966.24 in.

3. 4904 l.

4. 63,900 l.

Page 130.

5. 96,099 l.

6. 38 rds. 18 l.

7. 63 ch. 47 l.

8. 6 mi. 86 l.

9. 14 mi. 3 ch.

10. 35 mi.

Page 132.

2. 1015 sq. in.

3. 23,393 sq.in.

4. 184,464 sq. in.

5. 313,767 sq. in.

6. 37,713 sq.in.

7. 100,040 sq. rds.

8. 435,200 sq. rds.

9. 716,920 sq. rds.

10. 1,075,215 sq. rds.

11. 1,331,340 sq. rds.

Page 133.

13. 5 sq. yds. 8 sq. ft.

14. 7 sq. yds. 1 sq. ft.

15. 33 A. 40 sq. rds.

16. 160 A.

17. 13 sq. yds.

18. 29 sq. yds.

19. 480 A. 140 sq. rds.

20. 3 Sec. 160 A.

21. 8 sq. mi. 25 A. 79 sq. rds.

22. 8 Sec. 616 A. 74 sq. rds.

Page 135.

15. 9 A..

16. 952 sq. yds.

17. $33

18. $4680

19. $16.2

Page 136.

20. $129.6

21. 100 rds.

Page 140.

3. 41,369 cu. in.

4. 395,712 cu. in.

5. 653,184 cu. in.

6. 840,442 cu. in.

Page 141.

7. 793,020 cu. in.
8. 33,621 cu. in.
9. 3,525,120 cu. in.
10. 18 cu. ft.
11. 70 cu. yds. 22 cu. ft.
12. 16 cu. ft. 95 cu. in.
13. 6 cu. yds. 21 cu. ft. 786 cu. in.
14. 775 cu. yds. 20 cu. ft.
15. 786 cu. yds. 15 cu. ft.
16. 13 cu. yds. 13 cu. ft. 13 cu. in.
17. 22 cu. yds. 9 cu. ft.
18. 30 cu. yds. 9 cu. ft. 625 cu. in.
19. 40 cu. yds. 12 cu. ft.

Page 141.

2. 180 cu. ft.
3. 219 cu. yds. 15 cu. ft.
4. 1152 cu. ft.
5. 8617 gals. and 129 cu. in.

Page 142.

6. 2592 cu. ft.
7. 131,784 cu. in.; 76 cu. ft. 456 cu. in.
8. 46,656 cubes
9. 12,960 bricks
10. 3 ft.
11. 16 in.
12. 19 ft. 6 in.
13. 5 ft.

Page 143.

1. 9 cd.
2. 168 cd. ft.
3. 6 ft.
4. 8 ft.
5. 1,050,624 cu. in.
6. 518,400 cu. in.; 2 cd. 2 cd. ft. 12 cu. ft.
7. $540

Page 144.

1. 336 ft.
2. 128 ft.
3. 576 ft.
4. 400 ft.
5. 600 ft.
6. 2400 ft.
7. $90

Page 146.

14. 148 gi.; 230 gi.
15. 10 gals.; 171 gals. 1 qt. 1 pt.

Page 147.

16. 360 days
17. $25.2
18. 27 bbls.
19. $18

Page 148.

12. 519 pts.
13. 47 bu. 2 pks. 1 qt.
14. $288
15. $2538
16. 54 bbls.
17. 78 bu.

Page 150.

13. 4680 gr.
14. 1600 pwt.
15. 69,133 gr.
16. 106,488 gr.

Page 151.

17. 19 pwt.
18. 1 lb. 7 oz. 19 pwt.
19. 10 oz. 20 gr.
20. 10 lbs. 18 pwt.
21. 23 lbs.
22. 30 lbs. 9 oz. 13 pwt.
23. $1125.6
24. 36 spoons

Page 153.

12. 429 oz.
13. 547 lbs.
14. $320
15. $69.2
16. $176
17. $10
18. $60.81
19. 10 T. 103 lbs. 15 oz.
20. 254,334 oz.

Page 155.

2. 324,000″
3. 432,000″
4. 648,000″
5. 1,080,000″
6. 96,050″
7. 273,300″
8. 1440°; 180°
9. 2520°; 90°

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11. 13°; 24″
12. 1 cir. 178° 15′
13. 14° 54′ 44″
14. 148° 54′ 18″
15. 9 signs
16. 1½ quadrants
17. 8 times
18. 11 ft.

Page 158.

16. 31,556,930 sec.
17. 10 yrs. 297 days 1 hr. 22 min.
18. 129,600 min.
19. 2 yrs. 10 wks. 3 days 6 hrs.
20. 3,155,673,600 sec.
21. 7 yrs.

Page 160.

10. $5
11. 18,164 sheets
12. 7 bales
13. 2450 wks. 5 days
14. 144 rows.

Page 163.

1. 4352 cu. ft.
2. 581 bu.
3. 9 oz. 12 pwt.
4. 4 hrs. 30 min.
5. $\frac{11}{16}$ wk.
6. 2 rds. 8 l. 2.64 in.
7. 2 qts. 1 pt.
8. £.4416$\frac{2}{3}$
9. 37′ 30″
10. $\frac{131}{1888}$ cu. yd.
11. .421875 of a bu.
12. 312 lbs. 8 oz.
13. $\frac{13}{33}$ gal.
14. 213 rds. 5 ft. 6 in.
15. .006944″
16. 37 A.

Page 165.

3. $4.4
4. $49.1
5. 4.17 lbs.
6. $1072.01
7. 1382.315 mi.
8. 20.082 cd.
9. 153.14 rds.
10. $139.75

Page 166.

11. 230.33 bu.
12. 235 A.
13. $28 65
14. 66.8997 gals.
15. 30.47063°
16. 17.74514 T.
17. 11.460098
18. 17.7768808
19. 797.7777698

Page 170.

32. 40.
33. $19\frac{1}{16}$
34. $34\frac{1}{3}$
35. $54\frac{11}{12}$
36. 31
37. $51\frac{7}{16}$
38. $68\frac{9}{7}$
39. $75\frac{3}{10}$
40. $119\frac{1}{3}$
41. $98\frac{11}{15}$
42. $77\frac{39}{41}$
43. $33\frac{3}{4}$
44. $89\frac{25}{36}$
45. $57\frac{1}{11}$

Page 171.

4. 161 gals. 3 qts.
5. 4 lbs. 7 oz. 15 pwt. 8 gr.

Page 172.

6. 870 A.; 1 Sec. 230 A.
7. £60 10s. 8d.
8. 12° 24′
9. 83 sq. yds. 8 sq. ft. 140 sq. in.

Page 173.

11. 307 bu. 3 pks. 5 qts.
12. 2 yrs. 77 days 7 hrs. 30 min. 20 sec.
13. 18 mi. 20 l.
14. 12 T. 1385 lbs. 6 oz.
15. 89 sq. rds. 23 sq. yds. 2 sq. ft. 64 sq. in.
16. 60 cu. yds. 1 cu. ft. 208 cu. in.
17. 66 cd. 1 cd. ft. 4 cu. ft.
18. 51 gals. 3 qts. 1 gi.
19. 17 mi. 151 rds. 3 yds. 4 in.

Page 174.

1. $7583.15
2. $121\frac{1}{3}$
3. 437 A. 75 sq. rds. $9\frac{1}{3}$ sq. yds.
4. $55\frac{5}{8}$ yrs.
5. 29 wks. 6 days 23 hrs. $35\frac{9}{10}$ min.
6. 45.7192 T.
7. 16 T. 1156 lbs. $11\frac{11}{16}$ oz.
8. $164\frac{7}{8}$ yds.

Page 175.

9. 25.217582
10. 968 bu. 1 qt.
11. $68\frac{3}{4}$ cd.
12. 62 cd. 2 cd. ft. 12 cu. ft.
13. $187\frac{1}{8}$ mi.
14. 30 signs 9° 32′ 49″
15. 281 cu. yds. 10 cu. ft. 1444 cu. in.
16. 160 A.

Page 176.

1. $229
2. $2.29
3. $1322.73
4. 44.807 lbs.
5. 36.625 bbls.
6. 18.125 bu.
7. 2389.91 mi.
8. 463.125 A.

Page 177.

9. $334.13
10. 86977.694 oz.
11. $3.687
12. $12.135
13. $37.285
14. $15.108
15. $4
16. $14.135
17. $47.55

Page 177–*cont.*

18. $88.06
19. $32.99
20. $87.625
21. 4.96 gals.
22. 2.535 ft.
23. 9.76
24. .4563
25. .00421
26. 9.5683
27. 10.1112
28. .2223
29. 99.9999
30. 3.9039
31. 6.3
32. 89.72501
33. .00027
34. 22.9977

Page 180.

30. $30\frac{15}{16}$
31. $40\frac{7}{8}$
32. $44\frac{29}{32}$
33. $45\frac{4}{15}$
34. $40\frac{13}{72}$
35. $12\frac{81}{100}$
36. $27\frac{23}{25}$
37. $72\frac{9}{17}$
38. $34\frac{11}{21}$
39. $217\frac{1}{72}$
40. $199\frac{59}{72}$
41. $332\frac{13}{15}$
42. $188\frac{17}{18}$
43. $\frac{21}{100}$

Page 181.

2. 25 gals. 2 qts. 1 pt. 2 gi.
3. 17 cd. 5 cd. ft. 11 cu. ft.
4. £15 13s 4d. 1 far.
5. 1 yd. 3 in.
6. 1 lb. 7 oz. 13 pwt. 3 gr.
7. 3 T. 1534 lbs. 9 oz.
8. 571 cu. yds. 1 cu. ft. 1313 cu. in.
9. 16° 52′ 43″

Page 182.

10. 158 days 17 hrs. 52 min. 30 sec.
11. 28 A. 70 sq. rds. 15 sq. yds.
12. 4 mi. 49 ch. 2 rds. 18 l.
14. 49 yrs. 10 mo.

Page 183.

15. 7 yrs. 9 mo. 1 day
16. 155 yrs. 6 mo. 12 days
17. 90 yrs. 3 mo. 17 days
18. 63 yrs. 2 mo. 28 days
19. 3 yrs. 7 mo. 4 days
20. 5 yrs. 9 mo. 12 days 15 hrs. 35 min.

Page 183.

1. $155\frac{13}{40}$ A.
2. .022535
3. 83 A. 39 sq. rds. 18 sq. yds.
4. $\$37\frac{13}{14}$
5. $3396.9375

Page 184.

6. 3 T. 1062 lbs. 11 oz.
7. $57\frac{8}{21}$
8. 103.125 lbs.
9. $\frac{17}{40}$
10. 275 cu. yds. 16 cu. ft. 192 cu. in.
11. 8.0226
12. $83\frac{47}{50}$
13. 4 rds. 2 ft. 8 in.
14. 18 cd. 5 cd. ft. 10 cu. ft. 1712 cu. in.
15. $24\frac{11}{18}$
16. 399.99996
17. $57\frac{18}{185}$
18. 23 sq. rds. 8 sq. yds. 5 sq. ft. 116 sq. in.

Page 185.

19. 16° 43′ 52″
20. $38\frac{91}{108}$
21. 7 lbs. 5 oz. 14 pwt. 23 gr.
22. $\frac{1}{104}$
23. Increase; $\frac{18}{185}$

Page 187.

5. 564 cts.
6. $173.25
7. $2231.7
8. $20.71
9. $139.5
10. $120.9285
11. $28.2375
12. $61.525
13. $37,040
14. $741
15. $14.04
16. $.0625
17. $.08
18. $.81
19. $.7
20. $20
21. $62.6
22. $111.65

Page 188.

23. 690 mi.
24. 880.11 cu. ft.
25. 5194.125 bu.
26. 2274.48 lbs.
27. 100.1 bu.
28. $140.1552
29. 390.885 lbs.

Page 188 – *cont.*

30. .0204204 of a rd.
31. 4891.231+ft.
32. $.04293
33. .20075 of a lb.

Page 189.

34. 188,430.55 cu. in.
35. 2143.2
36. .2853
37. 84.611
38. 9.893
39. 43.072
40. .0000505
41. 16.7098
42. .00021
43. 44,278.6013
44. 170.7552+
45. 1259.3339+
46. 12,427.3664+
47. 36,802.76
48. 1988.1998+
49. .1092
50. 134,778.165+
51. .0000134732
52. 20.664492

Page 192.

28. $7\frac{5}{11}$
29. 16
30. $9\frac{13}{35}$
31. $20\frac{7}{10}$
32. $35\frac{1}{4}$
33. $40\frac{6}{8}$
34. $231\frac{1}{4}$
35. 69
36. $50\frac{4}{15}$
37. $21\frac{1}{3}$
38. $90\frac{18}{15}$
39. $26\frac{7}{16}$
40. $337\frac{1}{2}$
41. $376\frac{1}{4}$
42. $97\frac{1}{4}$
43. $107\frac{8}{11}$

Page 196.

34. $61\frac{1}{4}$
35. $128\frac{4}{7}$
36. $5\frac{17}{21}$
37. 152
38. $165\frac{8}{15}$
39. 325
40. $25\frac{1}{8}$
41. $447\frac{8}{7}$
42. $548\frac{1}{6}$
43. $17\frac{5}{71}$
44. 253
45. $54\frac{19}{35}$
46. 248
47. 513

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26. $\frac{3}{5}$
27. $\frac{18}{35}$
28. $\frac{8}{11}$
29. $\frac{129}{147}$
30. $\frac{16}{115}$
31. $\frac{1}{3474}$

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32. $\frac{89}{140}$
33. $\frac{5}{17}$
34. $\frac{13}{30}$
35. $\frac{1}{115}$
36. $\frac{19}{47}$
37. $\frac{44}{355}$
38. $\frac{117}{135}$
39. $\frac{27}{76}$
40. $\frac{20}{1885}$
41. $\frac{1}{4}$
42. $\frac{5}{77}$
43. $\frac{1}{13}$

Page 202.

24. $6
25. $157\frac{1}{2}$
26. $497\frac{1}{2}$ gals.
27. $10,350\frac{19}{20}$
28. $587\frac{13}{16}$ mi.
29. $414\frac{24}{35}$ rds.

Page 203.

30. 2548
31. $2197\frac{1}{2}$
32. $2927\frac{1}{7}$
33. 10,025
34. $11191\frac{1}{6}$
35. 1864
36. $1719\frac{3}{7}$
37. $10,200\frac{3}{8}$
38. 8601
39. 909
40. $3420\frac{3}{44}$
41. $46,309\frac{9}{16}$
42. $73,812\frac{1}{36}$
43. $18,954\frac{11}{45}$
44. $6012\frac{9}{10}$
45. $8752\frac{23}{35}$

Page 204.

3. 9 lbs. 9 oz. 4 pwt.
4. 279 bu. 2 pks. 2 qts.
5. £234 7s. 2d. 1 far.
6. 24 hhds. 36 gals. 3 qts.
7. 66 T. 1000 lbs.
8. 23 cd. 1 cd. ft. 4 cu. ft.
9. 294 A. 7 sq. rds. $12\frac{1}{2}$ sq. yds.
10. 183° 28′ 30″
11. 52 rds. 5 yds. 1 ft.
12. 59 sq. rds. 8 sq. yds. 7 sq. ft. 108 sq. in.
13. 178 wks. 4 days 3 hrs. 45 min.
14. 307 T. 422 lbs. 8 oz.
15. 1923 mi. 18 ch. 1 rd.

Page 205.

16. 2223 bu. 1 pk. 6 qts. 1 pt.
17. 5421 cd. 6 cd. ft.
18. 283 lbs. 2 oz. 7 pwt. 15 gr.
19. £178 7s. 4d. 2 far.
20. 3418° 33′ 36″
21. 2893 gals. 3 qts.

Page 205 – *cont.*

22. 2725 cu. yds. 25 cu. ft. 1080 cu. in.

23. 875 gt. gross 2 gross 5 doz. 3 ones

24. 63 bales 3 bun. 1 rm. 18 qu. 16 sheets

25. 136 mi. 11 rds. 2 yds. 6 in.

26. 138 cen. 18 yrs. 49 days 23 hrs. 29 min. 36 sec.

Page 205.

1. $2.346

2. 917\frac{8}{10}$

3. 300 mi.

4. 224$\frac{3}{5}$ cu yds.

Page 206.

5. 2602$\frac{3}{5}$ lbs.

6. $732.09636

7. 156 bu. 1 pk. 5 qts.

8. 34 Sec. 27 A. 54 sq. rds. 20 sq. yds. 7 sq. ft. 18 sq. in.

9. $200

10. $1.5631

11. 2323$\frac{14}{31}$ bu.

12. .65625 of a bale.

13. 250$\frac{10}{13}$

14. 135 T. 1420 lbs. 2 oz.

15. .001032

16. 7796\frac{88}{100}$

17. .63612

18. 3503.85035

Page 207.

19. 1052 cu. yds. 25 cu. ft. 756 cu. in.

20. 146 wks. 21 hrs. 53 min. 45 sec.

21. 17,578$\frac{3}{5}$

22. 2786$\frac{18}{25}$

23. 3579$\frac{17}{20}$

24. .0000008742

Page 209.

10. 67 yds.

11. $1.85

12. 135 bu.

13. 98 doz.

14. $1.35

15. 18 wks.

16. 24 boxes

17. 168 hrs.

18. $4.75

19. 176.75 bbls.

Page 210.

20. 25 dollars

21. 20.09 perches

22. $16.875

23. $.045

24. £70.8

25. $.045

26. .075 of a yd.

27. 385 bu.

28. 10.89 gals.

29. 4.875 marks

30. .908 of a gal.

31. $.0275

Page 211.

1. 128,000

2. 23.07

3. 73.088

4. .056

5. 63.445

6. .01

7. .003

8. .001

9. 48

10. .03

11. .0049

12. 481.6

13. 15.375

14. 1,000,000

15. .000001

16. 112

17. .0575

18. 700

19. .0000001

20. 39.0625

Page 215.

42. $\frac{11}{316}$

43. $\frac{4}{57}$ of a gal.

44. $\frac{6}{35}$

45. $\frac{2}{375}$ of a bu.

46. $\frac{8}{105}$

47. $\frac{7}{50}$ of a yd.

48. $\frac{13}{1054}$

49. $\frac{1}{655}$

50. $\frac{409}{3684}$

51. $\frac{8772}{58851117}$

52. $\frac{67}{5460}$

53. $\frac{3}{177}$

54. $7\frac{10}{39}$

55. $3\frac{43}{71}$

56. $\frac{5}{5383}$

57. $5\frac{58}{60}$

58. $\frac{15}{196}$

59. $\frac{7}{541}$

60. $\frac{5}{1611}$

61. $17\frac{58}{315}$

Page 219.

32. $27\frac{3}{41}$ mi.

33. $10\frac{188}{355}$ wks.

34. $5\frac{10}{13}$ T.

35. $34\frac{7}{8}$ bu.

36. $88\frac{13}{16}$

37. $89\frac{10}{19}$

38. $134\frac{6}{7}$

39. $6\frac{23}{25}$

40. $7\frac{328}{365}$

41. 110

42. $14\frac{46}{167}$

43. $87\frac{1}{3}$

44. $35\frac{87}{95}$

45. 111

46. $81\frac{1}{4}$

47. $4\frac{139}{167}$

48. $2\frac{58}{301}$

49. 1090

50. 580

51. $28\frac{6}{13}$

Page 222.

22. $1\frac{7}{16}$

23. $\frac{30}{31}$

24. $\frac{8}{9}$

25. $\frac{20}{23}$

Page 222 – *cont.*

26. $1\frac{7}{15}$
27. $5\frac{43}{81}$
28. $1\frac{13}{15}$
29. $\frac{6}{77}$
30. $1\frac{5}{9}$
31. $3\frac{4}{9}$
32. $26\frac{3}{8}$
33. $\frac{27}{55}$
34. $\frac{7227}{1458}$
35. $1\frac{13}{37}$
36. $\frac{37}{55}$
37. $\frac{888}{1051}$
38. $\frac{6}{65}$
39. $\frac{2}{5}$
40. $2\frac{9}{13}$
41. $\frac{88}{191}$
42. $6\frac{88}{231}$
43. $17\frac{12}{17}$

Page 223.

3. 45 cans
4. 61 gals. 1 qt. 1 pt.
5. 2 cu. yds. 25 cu. ft. 845 cu in.
6. 100 thimbles
7. 2° 16′ 8″
8. £2 5s. 9d. 2 far.
9. 25 loads
10. 2 bu. 3 pks. 7 qts.
11. 928 lbs. 14 oz.

Page 224.

12. 946 revolutions
13. 112 bbls.
14. 96 lots
15. 65 mi. 49 ch. 93 l.
16. 2 yrs. 228 days 23 hrs. 8 min.
17. 3 lbs. 2 oz. 5 pwt. 4 gr.
18. 2 bales 1 bun. 19 qu. 23 sheets
19. 11 A. 109 sq. rds. 21 sq. yds. 3 sq. ft.
20. 2 gt. gross 4 gross 10 doz. 10 ones

Page 224.

1. 38 ch.
2. 88.75 rds.
3. 82 vests

Page 225.

4. $\$7\frac{5}{8}$
5. 476.34 revolutions
6. 5 cd. 5 cd. ft. 5 cu. ft.
7. $27\frac{1}{2}$ bu.
8. 76 dollars
9. 4 days
10. 16 rds. $2\frac{1}{4}$ yds.
11. 398.56+bu.
12. 78 A. 116 sq. rds.
13. $\$85\frac{3}{5}$
14. $8\frac{23}{45}$
15. 3 wks. 6 days 21 hrs. 58 min. 38 sec.

Page 226.

16. .0112
17. 7 children
18. $1.45764
19. $8\frac{88}{311}$
20. .0066
21. 9 cu. yds. 19 cu. ft. 1700 cu.in.
22. 34 mi. 75 ch. 3 rds. 23 l.
23. .00095
24. $3\frac{118}{135}$
25. $\frac{2103}{1733}$

Page 227.

2. $\frac{5}{8}$
3. $1\frac{13}{16}$
4. 6
5. $2\frac{1}{3}$
6. $11\frac{1}{5}$
7. $\frac{5}{14}$
8. $\frac{5}{7}$
9. $8\frac{1}{16}$
10. $1\frac{32}{115}$
11. $\frac{8}{168}$
12. $\frac{72}{115}$
13. $\frac{25}{37}$

Page 231.

19. $7488
20. $517.3
21. $15,000
22. $10,946
23. Frank, $236.5
Ida, $150.5

Page 232.

24. $289\frac{1}{8}$ ft.
25. $96,000
26. $1583.05

Page 232.

1. $\$43\frac{3}{5}$
2. 38.739 mi.
47.655 mi.
3. $19,755
4. 224 A.
5. 142 sheep
6. $\$112\frac{13}{15}$

Page 233.

7. 8.31+bbls.
8. 418 boys
9. 600 pupils
10. $41.44
11. $8308
12. $\$\frac{5}{15}$
13. 30 yds. $22.2
14. 7
15. Daughters, 59 A. 120 rds.
Sons, 81 A. 100 rds.

Page 234.

16. $\$44\frac{1}{2}$
17. 330 T.
18. $4.9

Page 234 – *cont.*

19. $10,839\frac{23}{63}$
20. $.00302
21. $316\frac{4}{9}$ bu.
$395\frac{5}{9}$ bu.
22. 18 mi. 211 rds. 4 yds.
23. 624 A.
24. $\$16\frac{1}{8}$

Page 235.

25. $28,296
26. 20 wks.
27. $1375.05
28. $11\frac{197}{175}$ days
29. $.57\frac{1}{4}$
30. A, 14,966
B, 10,690
C, 2138
31. $\$7\frac{22}{165}$
32. $4292.25
33. 102.85+bu.
30.39+bbls.
34. $\$137\frac{17}{36}$

Page 243.

40. 3414.6 lbs.
41. $621
42. $18\frac{3}{4}\%$

Page 244.

43. $82\frac{3}{7}\%$
$121\frac{19}{36}\%$
44. 2700 bu.
45. $2600
46. $2528.89
47. $39\frac{1}{3}\%$
48. 2100 mi.
49. Daughter, $2000
Son, $2500
50. $83\frac{1}{3}\%$
51. $48,000
52. $\$1346\frac{2}{3}$

Page 245.

53. A, $66\frac{2}{3}\%$
B, $83\frac{1}{3}\%$
C, $88\frac{8}{9}\%$
D, 75%
E, $97\frac{2}{3}\%$
F, $69\frac{4}{9}\%$
54. $4700
55. $12,000
56. $54,280
57. $11,820
58. 518 apple
364 plum
217 pear
301 cherry
59. 328
60. Library, $7500
College, $11,250
Church, $2340
Hospital, $8850
Wife, $27,000
Each daughter, $16,530

Page 246.

61. Nitre, $75\frac{175}{1755}\%$
Charcoal, $15\frac{455}{1755}\%$
Sulphur, $9\frac{1183}{1755}\%$
62. $50,000
63. $1440
64. $4100
65. 1700 rds.
66. $6700

Page 254.

61. $438.242
62. $600

Page 255.

63. $72
64. $93.879
65. $253.75
66. $12,500
67. $1\frac{3}{8}\%$
$176.844
A, $77.6
B, $92.24
68. 14%
69. $13,600
$542.64
$14,961.36
70. $1243.2
71. $524.58

Page 256.

73. $456
74. $2896
75. $1624.32
76. $5.625
77. $42,180
78. $2847
$2\frac{1}{2}\%$
$87
79. $1783

Page 257.

80. $232
2%
81. $778.855
82. $.24
83. 13%
$86.445
84. $1618.75
85. Insurer, $39,280
Owner, $24,720
86. $2.6
87. $3125
$125

Page 258.

88. $1237.9296
89. 24%
90. $9690
91. A's, B's, and C's same
92. $7000
93. $280
$616
94. $14,463.75
95. A, $11,680
B, $5840

Page 259.

96. $1225
97. $6400
98. $4800
99. A, $4104
B, $3456

Page 259–*cont.*

100. $10,608
101. 33⅓ %
102. 25,000 bu.
103. $4000

Page 262.

1. $85.12
2. $171.9
3. $308.81+
4. $179.092+
5. $147.513+
6. $297.585+
7. $190.8
8. $90.228+
9. $39.225+

Page 263.

10. $790.4
11. $28.261+
12. $74.9
13. $253.203+
14. $58.65
15. $16.791+
16. $15.055+
17. $403.093+
18. $1445.783+
19. $253.416+
20. $701.739
21. $398.229+
22. $1474.333+
23. $3403.456+
24. $127.692+
25. $78.413+

Page 264.

2. $122.2
3. $94.92
4. $86
5. $100.815
6. $69.2
7. $42.2625
8. $72.387
9. $36.4203
10. $155
11. $86.224
12. $9.6264
13. $90.9544

Page 265.

14. $2745
15. $94.025
16. $1175.55
17. $964.105
18. $76.11955
19. $4275.528
20. $370.3268
21. $133.6625
22. $30.62309+

Page 266.

2. $21.481+
3. $40.782+
4. $66.333+
5. $246.458+
6. $77.246+
7. $239.82+
8. $6.558+
9. $292.166+
10. $199.833+

Page 267.

11. $110.717+
12. $293.245+
13. $1.216+
14. $166.075+
15. $233.496+
16. $145.872+
17. $7.324+
18. $183.263+
19. $1.782+
20. $50.649+
21. $1835.75+
22. $236.48+
23. $731.835+

Page 268.

24. $3582.541+
25. $1820.261+
26. $1015.81+
27. $751.931+
28. $481.667+
29. $71.324+
30. $68.592
31. $5300
32. $370.16
33. $844.563+
34. $253.312+
35. $1672.488+

Page 271.

2. $339.447+
3. $73.043+
4. $473.859+

Page 272.

5. $234.825+
6. $390.667

Page 272.

1. $80.30+

Page 273.

2. $5.888

Page 274.

2. $1878.4
3. $1274.672
4. $1648.046+
5. $1271.7
6. $2011.566+
7. $4036.32
8. $1310.024+
9. $2005.527+
10. $3168.66+

Page 275.

2. $150.339+
3. $199.736+
4. $76.289+
5. $138.603
6. $50.966+
7. $104.861+

Page 277.

2. $433.96+
3. $616.51+
$55.48+
4. $34.13+
5. $675.263+
$44.73+
$271.028+
$18.97+
$1333.005+
$23.495+

Page 278.

6. $267.86+
$287.29+
7. $573.97+
$588.25+
8. Principal
9. $915.14+

Page 279.

2. $12.121
$769.879
3. $11.025
$888.975
4. $4.851+
$370.748+

Page 279 – *cont.*

5. $10.4
$1189.6
6. $3.906
$492.094
7. $2.591+
$92.208+
8. $12.145+
$2637.854+
9. $5.754+
$870.245+

Page 280.

2. $464.88+
3. $905.81+
4. $96.64+
5. $556.78+
6. $1488.91+
7. $856.30+
8. $767.29+
9. $2861.80+
10. $653.18+

Page 281.

2. $1204.5
3. $690.69
4. $2404.8
5. $997.5
6. $1602
7. $949.05
8. $1891.84+
9. $12,736.094

Page 282.

11. $3000
12. $6824.40+
$6875.78+
$6822.70+
$6877.51+
14. $1431.15
15. $2565.55
16. $1741.367
17. $764.736

Page 288.

3. 77° 52′ 4″ W.
4. 1 hr. 5 min. 27½ sec. A. M. the following day
5. 70° 15′ 15″ W.
6. 33° 52′ 30″ W.
7. 90° 12′ 15″ W.
8. 5 hrs. 28 min. 29 sec. A. M.
9. 85° 26′ E.
10. 41 min. 1⅜ sec. P. M.
11. 88° 31′ 44″ W.
12. 92° 12′ W.

Page 291.

1. 45 men

Page 292.

2. 3920 bu.
3. 90 ft.
4. $68
5. $8320

Page 298.

1. 64 ft.
2. 1188 bricks
3. 252 men
4. $131.2
5. 3 men

Page 294.

6. 38⅜ rms.
7. 13⅓ days
8. 6 days
9. $360
10. $1600

Page 301.

1. 64
2. 145
3. 416
4. 310
5. .838
6. .035
7. 9.08
8. 8.6812+
9. 261.58+
10. $\frac{17}{18}$
11. $\frac{44}{115}$
12. 32¼
13. 165 rds.
14. 997 men

Page 302.

15. 312 rds.

Page 303.

1. 38
2. 97
3. 234
4. 20.3
5. 2.89
6. .45
7. .0809
8. 2.924+
9. $\frac{4}{15}$
10. $\frac{11}{35}$
11. 3⅜
12. 3⅐
13. 3 ft.
14. 7.49+ft.

Page 306.

1. 4⅝ sq. ft.
2. 7⅐ sq. ft. The square is $1\frac{21}{55}$ times as large
3. $7\frac{7}{15}$ sq. ft.

Page 309.

1. 20 in.
2. 39 ft.
3. 28 in.
4. 84 yds.
5. 18.6+in.
6. 9⅓ sq. ft.
7. 41⅜ A.
8. 172⅛ A.
9. 20 ft.
192 sq. ft.
10. 30.85+ft.

Page 310.

11. 110.8+sq. in.
37.7+sq. yds.
18.54+ sq. rds.
6.76+A.

Page 310.

1. 10.472 ft.
2. 15.91+ft.
3. 5 in.
4. 384.846 A.
5. 203.716 A.
6. 7 rds.

Page 311.

1. $33\frac{3}{4}$ sq. rds.
2. 45 sq. in.
3. $\frac{8}{16}$, $\frac{1}{4}$; $\frac{8}{16}$, $\frac{1}{5}$
4. 4 circles
5. 144; 36; 16
6. 10 in.
7. 72 sq. ft.
162 sq. ft.
882 sq. ft.

Page 313.

1. 260 cu. ft.
$86\frac{2}{3}$ cu. ft.
2. 103.8 cu. ft.
34.6 cu. ft.
3. 1.76+cu. ft.
.58+cu. ft.

Page 314.

Sec. 286.

4. 11.436+cu. ft.
3.812 cu. ft.
5. 7.068+sq. ft.
1.76+cu. ft.
6. 102.78 sq. in.
97.98 cu. in.

Page 314.

Sec. 288.

1. 125 cu. in.
2. $\frac{1}{8}$; $\frac{1}{27}$; $\frac{1}{64}$
3. $\frac{1}{8}$; $\frac{1}{27}$; $\frac{8}{125}$
4. 27
5. 2 oranges, 4 in.

Page 316.

1. 23; 100
2. 17; 220
3. 297
4. 78
5. 19 cts.; $1

Page 317.

6. 57,900 ft.

Page 317.

1. 162; 242
2. 279,935
3. $5.12
$10.23

Page 321.

1. 4.17837 Kg
417,837 cg
2. 39.47792 Hl
394,779.2 cl
3. 9.10014 Km
910,014 cm
4. 406,522.66 m
5. 5,044,565.6 g
6. $41.338
7. $4.06
8. $1.5
9. $8.47

Page 322.

1. 2713
1894
2. 9 times
3. 31 sheep
4. 57 wks.
5. 5 T.
6. $3538\frac{11}{14}$
7. $\frac{14}{33}$
8. 225 bu.

Page 323.

9. $3.65
10. 855 bu.
11. $6796\frac{3}{16}$
12. $4480
13. $35,378.38
14. $1700
$2040
$2550
15. $38.45
16. $29,119.574
17. $33\frac{1}{3}$ %
$8\frac{1}{3}$ %
$58\frac{1}{3}$ %

Page 324.

18. $270\frac{1}{3}$
19. Increased $\frac{1}{10}$
20. Diminished $\frac{1}{10}$
21. $84\frac{3}{5}$ sq. yds.
22. 300 %
23. 100 %
24. 64 rds.
25. $56.479+
26. $700
27. 896 lbs.
28. 73
29. 24 days

Page 325.

30. 828 T.
31. $3570
32. $6500
33. $26.37
34. 20,000
35. $33\frac{1}{3}$ %
36. $3500
37. $4\frac{1}{8}$ bu.
38. $394\frac{3}{8}$
39. $1.18\frac{1}{8}$
40. $68.75
41. 9072 tiles

Page 326.

42. $358.41
43. A, $1953
B, $1519
C, $1085
44. $132.657+
45. A, $26\frac{3}{7}$ %; 51.8 T.
B, 30 %; 58.8 T.
C, $43\frac{4}{7}$ %; 85.4 T.
46. $2047.3
47. $267\frac{1}{8}$
48. 21 %
49. $13,000
50. $41\frac{29}{111}$ %

Page 327.

51. $78.35+
52. $1\frac{1}{4}$ %
53. 26 in.
54. 57.008+mi.
56. $3\frac{3}{7}$ days

Page 327 – *cont.*

57. 15 days
58. 289,350
59. $\$3\frac{3}{10}$
60. $3040.751+

Page 328.

61. 5832 cu. in.
62. 390
63. $397\frac{1}{3}$ ft.
64. $725
65. $1172.48
66. A, $130
B, $260
C, $390
67. A, $139
B, $193
C, $239
68. $3580
69. 3235.58+cu. ft.
70. $21,200
71. 50° W.

Page 329.

72. 40 rds.
73. 5.40+sq. rds.
75. Corn, 50 %
Oats, 30 %
Rye, 20 %
76. $.67
77. $5800
78. $57\frac{1}{2}$ bu.
79. $1985
80. 1200
81. $345
82. 364

Page 330.

83. 480 rds.
84. $121.73+
85. $1670
86. 630 lbs.
87. A, $\$3\frac{3}{4}$
B, $5
C, $\$6\frac{1}{4}$
88. $14\frac{2}{3}$ A.
89. $1682.266+
90. $358.49
91. 32 yds.
92. 2 %

Page 331.

93. $260
94. 59 cts.
95. $3402 house
$12,474 land
96. A, $1150
B, $736
C, $874
97. $70.15
98. 22,400
99. $15,000
100. $17\frac{47}{80}$ A.
101. $73\frac{1}{8}$ sq. ft.
102. $21\frac{3}{7}$ %

Page 332.

103. $470\frac{5}{9}$ cu. yds.
104. $95
105. $2738
106. $12\frac{1414}{1515}$ T.
107. 2364
108. $17.92
109. 579.54+ times
110. $1942.75
111. $8227
112. $30,000
113. 96 rds.

Page 333.

114. Cylinder, 1413.72 cu. in.
Cone, 471.24 cu. in.
115. $4824
116. $11\frac{7}{8}$ cd.
14 T.
117. 112° 30′
118. 540 sq. yds.
119. 86 ft.
120. 69.97+in.
121. 43.86+ft.
116.96 sq. yds.
122. 20.669 sq. ft.
123. $4.35

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124. $48,000
$36,000
$24,000
126. 603 T.
127. 357 bbls.
128. $66\frac{2}{3}$ %
129. $3\frac{3}{4}$ hrs.
130. $2522.56
131. $256\frac{1}{4}$ bu.
132. $6\frac{2}{3}$ %
133. $152

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134. $\$7\frac{1}{7}$
135. $21,600
136. $\frac{3}{8}$ %
$6.75
137. 113.0976 cu. in.
113.0976 sq. in.
138. 20 A.; 240 rds.
139. $1547
140. $1869.75
141. $2.165+
142. 3 mo.
143. Circle 13.384 sq. ft. more

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144. $3\frac{1}{4}$ %
145. $6\frac{7}{8}$ %
146. $31.68
147. $2817.5
148. $11\frac{1}{8}$ hrs.
149. $38,700
150. 7 hrs. 17 min. 20 sec. P. M.
4 hrs. 42 min. 40 sec. A. M.
151. $68.2
152. $50\frac{23}{27}$ lbs.
153. 61

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154. $8625
155. 9 in.

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156. Saltpeter, 72 lbs.
Sulphur, 9.6 lbs.
Charcoal, 14.4 lbs.
157. $3
158. 10 %
159. 5.304 sq. ft.
87.128+cu ft.
160. 153.9384 sq. in.
161. 167.116+ bbls.
162. 7½ %

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163. $1460
164. 13.6+ft.
165. .91975
167. $17.4
168. Man, $1.2
Boy, $.6
169. $1750
170. 80 %
171. $170,000
172. $3500
173. $3500
174. 106.066 in.
26.041 sq. yds.

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175. $60,000
176. 50 in.
66⅔ sq. ft.
27⅗ cu. ft.
177. 142° W.
25° 30′ E.
178. 4.276 sq. ft.
1436.7584 cu. in.
179. 2 parks
180. $1500
41⅔ %
181. 25 %
182. 33⅓ %
183. 31¼ %

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184. $1061.5
185. 20
186. $49
514 A.
187. 435 bbls.
$217½
188. 312½ bbls.
189. $13
$344.5
190. $1024
$2047.5

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191. $96
192. $1572
193. 78 bu.
194. 46 min. 12 sec.
195. $500.5
196. 8 %
197. 4 yrs.
198. $2100
199. $3500
200. Lose 4 %
201. 25 %

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